Quartic fermions in IIA superspace

Ομιλία αφιερωμένη στη μνήμη του Γιάννη Μπάκα

Dimitrios Tsimpis

Recent developments in high energy theory and cosmology
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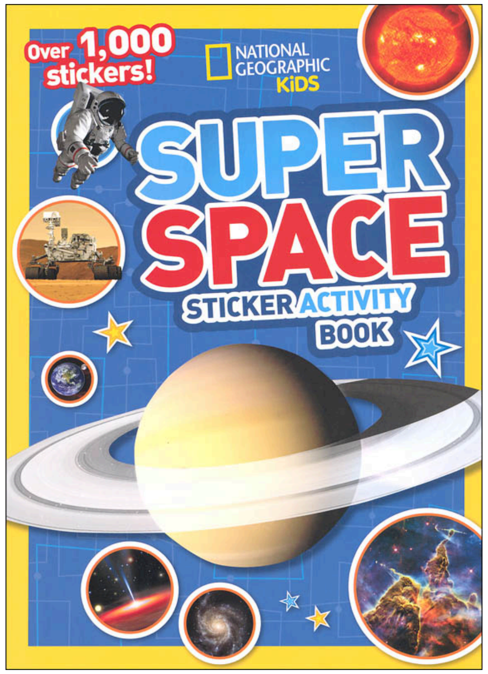
Based on:

- Bertrand Souères & DT, Phys.Rev.D (2018)
- Stefan Theisen & DT, (2013)
- DT, JHEP (2005)



IIA supereravities

- Dimensional reduction of CJS supergravity

  Giani & Pernici, Phys.Rev.D (1984)
  Campbell & West, Nucl.Phys.B (1984)
  Huq & Namazie, Class.Quant.Grav. (1985)

- Romans Supergravity

  Romans, Phys.Lett.B (1986)

- HLW Supergravity

  How, Lambert & West, Phys.Lett.B (1998)

- 10d (massive) superspace

  DT, JHEP (2005)

Effective action

- The vacuum is given by:

$$\left.\frac{\delta S_{\text{eff}}}{\delta \phi}\right|_{\langle\phi\rangle} = 0$$

- The type IIA 10d string effective action:

$$\mathcal{L}_{\text{eff}} = \frac{1}{g_s^2}\left(\mathcal{L}_{\text{IIA}} + \alpha'^3\left(I_0(R) - \frac{1}{8}I_1(R) + \cdots\right) + \mathcal{O}(\alpha'^4)\right)$$

$$+\left(\alpha'^3\left(I_0(R) + \frac{1}{8}I_1(R) + \cdots\right) + \mathcal{O}(\alpha'^4)\right)$$

$$+\mathcal{O}\left(g_s^2\right)$$

- At low energies ($\alpha' \to 0$), $\mathcal{L}_{\text{IIA}}$ is perturbatively exact in g_s

Motivation: fermionic terms

- Fermion condensation may form in the bosonic vacuum

$$\langle\lambda\rangle = 0 \; ; \quad \langle\bar{\lambda}\lambda\rangle := \int [\mathcal{D}\phi](\bar{\lambda}\lambda)e^{-S[\phi]} \neq 0$$

- Potential applications, e.g. to cosmology
- Single quartic dilatino term in $\mathcal{L}_{\rm IIA}$ (c.c. ?)
- Quartic terms identical in massive and massless IIA (conjectured by Romans; easily shown in superspace)
- Could be « simply » read off from the original literature

The quartic fermion terms are

$$
\begin{aligned}
e^{-1}L_{(4)} = {} & \tfrac{1}{4}(\bar{\psi}_{\mu_1}\Gamma^{\mu_1}\psi^{\mu_3})(\bar{\psi}_{\mu_2}\Gamma^{\mu_2}\psi_{\mu_3}) \\
& -\tfrac{1}{32}(\bar{\psi}_{\mu_1}\Gamma_{\mu_2}\psi_{\mu_3})(2\bar{\psi}^{\mu_1}\Gamma^{\mu_2}\psi^{\mu_3}+4\bar{\psi}^{\mu_1}\Gamma^{\mu_3}\psi^{\mu_2}-\bar{\psi}_{\nu_1}\Gamma^{\mu_1\mu_2\mu_3\nu_1\nu_2}\psi\nu_2) \\
& +\tfrac{1}{32}(\bar{\psi}_{\mu_1}\Gamma^{11}\psi_{\mu_2})(2\bar{\psi}^{\mu_1}\Gamma^{11}\psi^{\mu_2}+\bar{\psi}_{\nu_1}\Gamma^{11}\Gamma^{\mu_1\mu_2\nu_1\nu_2}\psi_{\nu_2}) \\
& -\tfrac{1}{64}(\bar{\psi}_{\mu_1}\Gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6}\psi_{\mu_2})(\bar{\psi}_{\mu_3}\Gamma_{\mu_4\mu_5}\psi_{\mu_6}) \\
& -\tfrac{3}{16}(\bar{\psi}^{\mu_1}\Gamma^{\mu_2\mu_3}\psi^{\mu_4})(\bar{\psi}_{[\mu_1}\Gamma_{\mu_2\mu_3}\psi_{\mu_4]}) \\
& -\tfrac{1}{16}(\bar{\psi}_{\mu_1}\Gamma^{11}\Gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}\psi_{\mu_2})(\bar{\psi}_{\mu_3}\Gamma^{11}\Gamma_{\mu_4}\psi_{\mu_5}) \\
& +\tfrac{1}{16}(\bar{\psi}^{\mu_1}\Gamma^{11}\Gamma^{\mu_2}\psi^{\mu_3})(\bar{\psi}_{\mu_1}\Gamma^{11}\Gamma_{\mu_2}\psi_{\mu_3}+2\bar{\psi}_{\mu_2}\Gamma^{11}\Gamma_{\mu_3}\psi_{\mu_1}) \\
& +\tfrac{1}{16}(\bar{\lambda}\Gamma^{\mu_1\mu_2\mu_3}\lambda)[\bar{\psi}_{\mu_1}\Gamma_{\mu_2}\psi_{\mu_3}-\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,\bar{\lambda}\Gamma^{11}\Gamma_{\mu_1\mu_2}\psi_{\mu_3}] \\
& +\tfrac{1}{2}(\bar{\lambda}\Gamma^{11}\Gamma^{\mu_1}\Gamma^{\mu_2}\psi_{\mu_1})(\bar{\lambda}\Gamma^{11}\psi_{\mu_2}) \\
& -\tfrac{1}{32}\sqrt{\tfrac{1}{2}}(\bar{\lambda}\Gamma^{11}\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3\mu_4\mu_5}\psi_{\mu_1})(\bar{\psi}_{\mu_2}\Gamma_{\mu_3\mu_4}\psi_{\mu_5} \\
& -\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,\bar{\lambda}\Gamma^{11}\Gamma_{\mu_2\mu_3\mu_4}\psi_{\mu_5}-\tfrac{1}{72}\bar{\lambda}\Gamma_{\mu_2\mu_3\mu_4\mu_5}\lambda) \\
& -\tfrac{3}{128}(\bar{\lambda}\Gamma^{\mu_1\mu_2\mu_3\mu_4}\lambda)(\bar{\psi}_{\mu_1}\Gamma_{\mu_2\mu_3}\psi_{\mu_4}-\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\bar{\lambda}\Gamma_{\mu_1\mu_2\mu_3}\psi_{\mu_4}) \\
& +\tfrac{1}{8}\sqrt{\tfrac{1}{2}}(\bar{\lambda}\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3\mu_4}\psi_{\mu_1})(\bar{\psi}_{\mu_2}\Gamma_{\mu_3}\Gamma^{11}\psi_{\mu_4} \\
& -\sqrt{\tfrac{1}{2}}\bar{\lambda}\Gamma_{\mu_2\mu_3}\psi_{\mu_4}+\tfrac{7}{72}\bar{\lambda}\Gamma^{11}\Gamma_{\mu_2\mu_3\mu_4}\lambda) \\
& +\tfrac{3}{16}\sqrt{\tfrac{1}{2}}(\bar{\lambda}\Gamma^{\mu_1}\Gamma^{\mu_2\mu_3}\psi_{\mu_1})(\bar{\psi}_{\mu_2}\Gamma^{11}\psi_{\mu_3}-3\sqrt{\tfrac{1}{2}}\bar{\lambda}\Gamma_{\mu_2}\psi_{\mu_3}+\tfrac{17}{24}\bar{\lambda}\Gamma^{11}\Gamma_{\mu_2\mu_3}\lambda) \\
& +\tfrac{1}{16}\sqrt{\tfrac{1}{2}}(\bar{\psi}_{\mu_1}\Gamma^{11}\psi_{\mu_2})(3\bar{\lambda}\Gamma^{\mu_3}\Gamma^{\mu_1\mu_2}\psi_{\mu_3}+5\sqrt{\tfrac{1}{2}}\bar{\lambda}\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda) \\
& -\tfrac{3}{32}(\bar{\lambda}\Gamma_{\mu_1}\psi_{\mu_2})(3\bar{\lambda}\Gamma^{\mu_3}\Gamma^{\mu_1\mu_2}\psi_{\mu_3}+\tfrac{13}{9}\sqrt{\tfrac{1}{2}}\bar{\lambda}\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda) \\
& +\tfrac{1}{1536}[\tfrac{1}{4}(\bar{\lambda}\Gamma^{\mu_1\mu_2\mu_3\mu_4}\lambda)(\bar{\lambda}\Gamma_{\mu_1\mu_2\mu_3\mu_4}\lambda)-\tfrac{821}{54}(\bar{\lambda}\Gamma^{\mu_1\mu_2\mu_3}\lambda)(\bar{\lambda}\Gamma_{\mu_1\mu_2\mu_3}\lambda) \\
& -\tfrac{82}{3}(\bar{\lambda}\Gamma^{11}\Gamma^{\mu_1\mu_2}\lambda)(\bar{\lambda}\Gamma^{11}\Gamma_{\mu_1\mu_2}\lambda)]\,. \qquad (24)
\end{aligned}
$$

Quartic fermion terms

S.Theisen & DT, (2013)

- Unfortunately the literature:

  Giani & Pernici, Phys.Rev.D (1984)
  Campbell & West, Nucl.Phys.B (1984)
  Huq & Namazie, Class.Quant.Grav. (1985)
  Nicoletti & Orazi, Int.J.Mod.Phys.A (2011)

  does not seem to be in agreement!

- Calculate from scratch in superspace
  (quartic terms not given in explicit form)

Massive IIA (on-shell) superspace

DT, JHEP (2005)

- Superspace Bianchi identities (BI)

$$DT^A = E^B R_B{}^A \; ; \quad DR_A{}^B = 0$$

- Field redefinitions can be used to constrain various components of the supertorsion $T_{AB}{}^C$

$$T_{\alpha\beta}{}^c = -i(\gamma^c)_{\alpha\beta}$$

$$T^{\alpha\beta c} = -i(\gamma^c)^{\alpha\beta}$$

$$T_{\underline{\alpha}b}{}^c, \; T_{ab}{}^c = 0$$

Massive IIA (on-shell) superspace

- Solution to the BI (complete list of massive IIA's)

$$\text{Massless IIA:} \begin{cases} L = \frac{3}{4}(\mu\lambda) \\ L' = -\frac{3}{4}(\mu\lambda) \end{cases}$$

$$\text{Romans:} \begin{cases} L = \frac{1}{2}me^{2\phi} + \frac{3}{4}(\mu\lambda) \\ L' = -\frac{1}{2}me^{2\phi} - \frac{3}{4}(\mu\lambda) \end{cases}$$

$$\text{HLW:} \begin{cases} L = \frac{3}{2}m + \frac{3}{4}(\mu\lambda) \\ L' = \frac{3}{2}m - \frac{3}{4}(\mu\lambda) \end{cases}$$

given in terms of two scalar superfields at mass dimension one

Massive IIA (on-shell) superspace

- Fermionic superfield equations of motion

$$\gamma^b T_{ab} = -4\widetilde{T}_a - 9\gamma_a\widetilde{T}\,, \qquad e_m{}^a e_n{}^b\, T_{ab}^\alpha|_{\theta=0} = \nabla_{[m}\psi_{n]}^\alpha + \mathcal{O}(\psi)$$

and

$$i\gamma^a D_a\lambda = -\frac{24}{5}(L-L')\mu - \frac{16}{3}L_{(2)}(\gamma^{(2)}\mu)$$
$$-12K_{(1)}(\gamma^{(1)}\lambda) + 3K_{(3)}(\gamma^{(3)}\lambda) + \frac{3}{40}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)$$
$$i\gamma^a D_a\mu = \frac{24}{5}(L-L')\lambda - \frac{16}{3}L_{(2)}(\gamma^{(2)}\lambda)$$
$$-12K_{(1)}(\gamma^{(1)}\mu) - 3K_{(3)}(\gamma^{(3)}\mu) + \frac{3}{40}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)$$

Massive IIA (on-shell) superspace

where

$$\widetilde{T} = \frac{272}{225} L\mu - \frac{8}{25} L'\mu + \frac{8}{9} L_{(2)}(\gamma^{(2)}\mu) + \frac{8}{45} L_{(4)}(\gamma^{(4)}\mu)$$

$$+ \frac{8}{9} K_{(1)}(\gamma^{(1)}\lambda) - \frac{16}{45} K_{(3)}(\gamma^{(3)}\lambda) - \frac{11}{450}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda)$$

and

$$\widetilde{T}_a = -\frac{3i}{20}(\gamma_a^{(1)} D_{(1)}\mu) - \frac{1}{5} L_{(2)}(\gamma_a^{(2)}\lambda) + \frac{2}{5} L_{(4)}(\gamma_a^{(4)}\lambda)$$

$$+ \frac{1}{5} K_{(1)}(\gamma_a^{(1)}\mu) - \frac{3}{20} K_{(3)}(\gamma_a^{(3)}\mu) + \frac{3}{160}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu)$$

Integrating the eom

B. Souères & DT, Phys.Rev.D (2018)

- Identify the bosonic forms

$$F := -\frac{16}{3}\hat{L}_{(2)}|_{\theta=0} \; ; \quad H := 24ie^{2\phi}\hat{K}_{(3)}|_{\theta=0} \; ; \quad G := 192e^{2\phi}\hat{L}_{(4)}|_{\theta=0}$$

from the superform BI

$$
\begin{aligned}
0 &= \mathrm{d}\hat{K}_1 \\
0 &= \mathrm{d}\hat{L}_2 + \frac{18}{5}\, me^{2\phi}\, \hat{K}_3 \\
0 &= \mathrm{d}\hat{K}_3 + 4i\, \hat{K}_1 \wedge \hat{K}_3 \\
0 &= \mathrm{d}\hat{L}_4 + \frac{2i}{3}\, \hat{L}_2 \wedge \hat{K}_3 - 4i\, \hat{K}_1 \wedge \hat{L}_4
\end{aligned}
$$

Integrating the eom

- Solution

$$\hat{K}_a = K_a$$

$$\hat{L}_{ab} = L_{ab} + \frac{3}{8}\,\mu\gamma_{ab}\lambda$$

$$\hat{K}_{abc} = K_{abc} - \frac{1}{8}\,\mu\gamma_{abc}\mu + \frac{1}{8}\,\lambda\gamma_{abc}\lambda$$

$$\hat{L}_{abcd} = L_{abcd} + \frac{1}{32}\,\mu\gamma_{abcd}\lambda$$

and

$$\hat{K}_\alpha = \frac{i}{2}\,\lambda_\alpha \quad \hat{L}_\alpha{}^\beta = -\frac{3}{16}\,\delta_\alpha^\beta \quad \hat{K}_{ab\alpha} = \frac{i}{12}\,(\gamma_{ab}\lambda)_\alpha \quad \hat{L}_{abc\alpha} = \frac{i}{96}\,(\gamma_{abc}\mu)_\alpha$$

$$\hat{K}^\alpha = \frac{i}{2}\,\mu^\alpha \quad \hat{L}^\alpha{}_\beta = -\frac{3}{16}\,\delta^\alpha_\beta \quad \hat{K}_{ab}{}^\alpha = -\frac{i}{12}\,(\gamma_{ab}\mu)^\alpha \quad \hat{L}_{abc}{}^\alpha = -\frac{i}{96}\,(\gamma_{abc}\lambda)^\alpha$$

$$\hat{K}_{a\alpha\beta} = -\frac{1}{24}\,(\gamma_a)_{\alpha\beta} \quad \hat{L}_{ab\alpha}{}^\beta = -\frac{1}{192}\,(\gamma_{ab})_\alpha{}^\beta$$

$$\hat{K}_a{}^{\alpha\beta} = \frac{1}{24}\,(\gamma_a)^{\alpha\beta} \quad \hat{L}_{ab}{}^\alpha{}_\beta = \frac{1}{192}\,(\gamma_{ab})^\alpha{}_\beta$$

Integrating the eom

- To extract the fermonic terms, we may add $c\,\Delta T^{\alpha}$ to the fermionic eom, before integrating,

  where:

$$\Delta T^{\alpha} := -\tilde{T}^{\alpha} + \frac{344}{225} L\mu + \frac{8}{9} L_{(2)}(\gamma^{(2)}\mu) + \frac{8}{45} L_{(4)}(\gamma^{(4)}\mu) + \frac{8}{9} K_{(1)}(\gamma^{(1)}\lambda) - \frac{16}{45} K_{(3)}(\gamma^{(3)}\lambda) - \frac{11}{450} (\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)$$

  vanishes on shell.

- Similarly we may add $c'\,\Delta T^{\alpha}\lambda_{\alpha}$ to the dilaton eom, before integrating, etc.
- The constants c, c' etc, are determined by consistency

Integrating the eom

- The diatonic action

$$S = S_b - 80 \int \mathrm{d}^{10}x\sqrt{\hat{g}}\Big\{(\bar{\Lambda}\Gamma^m\nabla_m\Lambda) + \frac{9}{25}e^{5\phi/4}m(\bar{\Lambda}\Lambda)$$
$$+ \frac{1}{8}e^{3\phi/4}F_{mn}(\bar{\Lambda}\Gamma^{mn}\Gamma_{11}\Lambda) + \frac{1}{40}e^{-\phi/2}H_{mnp}(\bar{\Lambda}\Gamma^{mnp}\Gamma_{11}\Lambda)$$
$$+ \frac{1}{160}e^{\phi/4}G_{mnpq}(\bar{\Lambda}\Gamma^{mnpq}\Lambda) + \frac{33}{10}(\bar{\Lambda}\Lambda)^2\Big\}$$

where:

$$S_b = \int \mathrm{d}^{10}x\sqrt{\hat{g}}\Big(\hat{R} + \frac{1}{2}(\partial\phi)^2 + \frac{8}{25}m^2e^{5\phi/2} + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2$$
$$+ \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2\Big) + \mathrm{CS}$$

Gravitino frame

- Supersymmetry canonically associates the metric

$$g_{mn}^{(\beta)} := e^{2\beta\phi}\,\hat{g}_{mn}$$

with the gravitino

$$\psi_m^{(\beta)} := \Psi_m - \beta\,\Gamma_m\Lambda$$

where

$$\beta = \begin{cases} -\frac{3}{4}, & \text{superspace-frame gravitino} \\ 0, & \text{Einstein-frame gravitino} \\ \frac{1}{4}, & \text{string-frame gravitino}\ . \end{cases}$$

Comparing to the literature

- To compare with the 24 quartic-fermion terms in Giani & Pernici, Phys.Rev.D (1984) we must set

$$\psi_m^{GP} := \beta\sqrt{2}\,\Gamma_{11}\Gamma_m\lambda^{GP} \;, \quad \widehat{\Psi}_m^{GP} := c\,\Gamma_{11}\Gamma_m\lambda^{GP} \;, \quad c := \sqrt{2}(\beta + 1/12)$$

therein, resulting in

$$(\bar\lambda\Gamma_{mn}\Gamma_{11}\lambda)^2(\frac{26\sqrt{2}}{3}c^3 - \frac{29}{4}c^4) + (\bar\lambda\Gamma_{mnpq}\lambda)^2(\frac{1}{\sqrt{2}}c^3 - \frac{21}{8}c^4)$$
$$+ (\bar\lambda\Gamma_{mnp}\lambda)^2(\frac{7}{3\sqrt{2}}c^3 - 5c^4)$$
$$+ (\bar\lambda\Gamma_{mnp}\Gamma_{11}\lambda)^2(-\frac{2}{3}c^2 + \frac{7}{\sqrt{2}}c^3 + \sqrt{2}c^3 - 6c^4)$$
$$= (32c^2 - 276\sqrt{2}c^3 + \frac{1773}{2}c^4)(\bar\lambda\lambda)^2$$

Setting $\beta = -3/4$ we recover the dilatonic (superspace) action

Applications

- Turning on diatonic condensates in the Einstein-frame action

$$S^E = S_b + \int \mathrm{d}^{10}x\sqrt{\hat{g}}\Big\{(\bar{\Lambda}\Gamma^m\nabla_m\Lambda) - \frac{21}{20}e^{5\phi/4}m(\bar{\Lambda}\Lambda) + \frac{3}{512}(\bar{\Lambda}\Lambda)^2 - \frac{5}{32}e^{3\phi/4}F_{mn}(\bar{\Lambda}\Gamma^{mn}\Gamma_{11}\Lambda) + \frac{1}{128}e^{\phi/4}G_{mnpq}(\bar{\Lambda}\Gamma^{mnpq}\Lambda)\Big\}$$

allows for (formal) dS solutions

- Ten-dimensional dS solutions on Kähler-Einstein

$$-\hat{R}_{mn} = \frac{3}{2^{12}}(\bar{\Lambda}\Lambda)^2\hat{g}_{mn}$$

- Four-dimensional dS solutions on Kähler-Einstein

$$(\bar{\Lambda}\Lambda) = \Re(A)\ ; \quad (\bar{\Lambda}\Gamma_{(2)}\Lambda) = \Re(A)\,J\ ; \quad (\bar{\Lambda}\Gamma_{(4)}\Lambda) = \Im(A)\,\mathrm{vol}_4 + \Re(A)\,\frac{1}{2}J^2$$

parameterized by one complex constant A

Conclusions

- The ambiguity of the quartic fermion terms has been resolved
- Superform formulation of (massive) IIA
- Dilatino condensates have not been considered in (massive) IIA
- Positive quartic dilatino term
  (promising application to cosmological settings)
- Systematic study of IIA condensates
- What is the origin of the condensate ?
  (calculate in a controlled setting)