The quantization of the hadronic string

with Dorin Weissman

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In memory of Ioannis Bakas

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On integrable models from pp-wave string backgrounds

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Abstract

We construct solutions of type IIB supergravity with non-trivial Ramond-Ramond 5-form in ten dimensions by replacing the transverse flat space of pp-wave backgrounds with exact $N = (4,4)$ $c = 4$ superconformal field theory blocks. These solutions, which also include a dilaton and (in some cases) an anti-symmetric tensor field, lead to integrable models on the world-sheet in the light-cone gauge of string theory. In one instance we demonstrate explicitly the emergence of the complex sine-Gordon model, which coincides with integrable perturbations of the corresponding superconformal building blocks in the transverse space. In other cases we arrive at the supersymmetric Liouville theory or at the complex sine-Liouville model. For axionic instantons in the transverse space, as for the (semi)-wormhole geometry, we obtain an entire class of supersymmetric pp-wave backgrounds by solving the Killing spinor equations as in flat space, supplemented by the appropriate chiral projections; as such, they generalize the usual Neveu-Schwarz five-

Introduction

- Four decades ago Chodos and Thorn suggested a model of a bosonic string with two massive particles on its ends.
- Their motivation probably was the physics of a flux tube and quark and an anti-quark on its ends.
- I encountered this model as an approximation of the stringy hadron in holographic model (HISH)
- Through the years this model has been addressed but never fully quantized and clearly not in four dimension. This is what we have recently acheieved
- On root to the quantization of the model we have further developed an alternative method of renormalization
- The hadronic sting is better behaved that the ordinary bosoinc string

Outline

- Introduction
- HISH Holography Inspired Stringy Hadron
- The classical string with massive endpoints
- Fluctuations and gauge choice
- Transverse fluctuations
- Planar modes
- Quantizing the non-critical string
- The quantum Regge trajectory
- Generalization to asymmetric case
- Higher order in the perturbation expansion
- Summary and open question

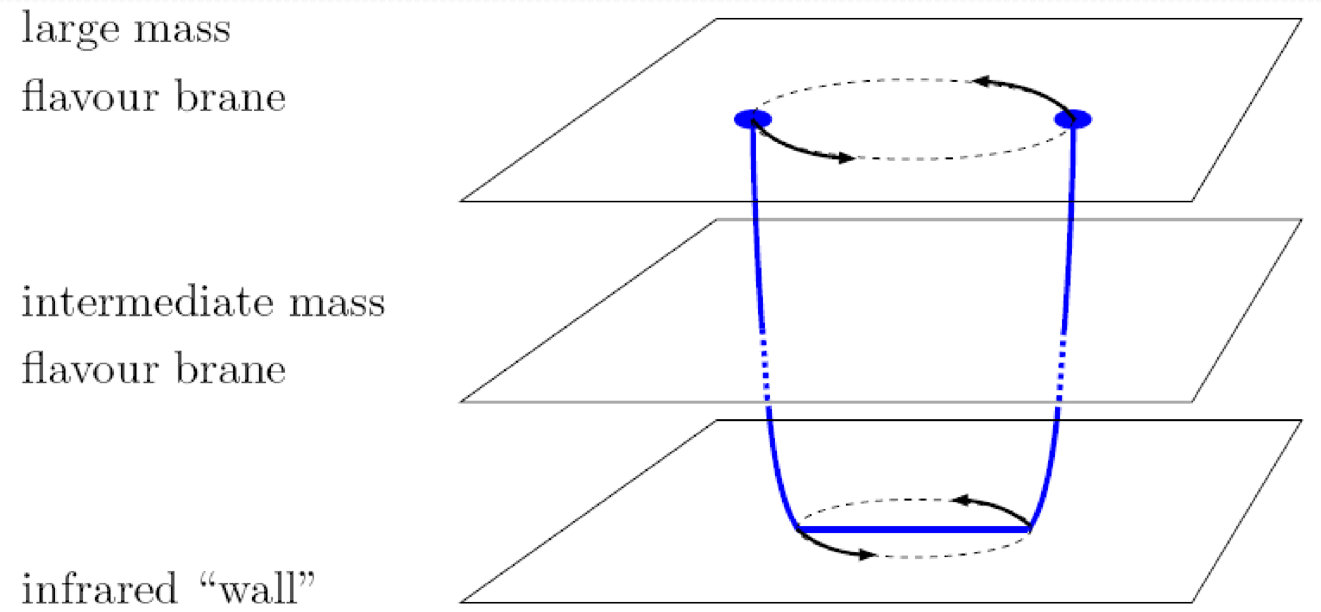
A brief review of Holography Inspired stringy hadron model

HISH- Holography Inspired Stringy Hadron

- The construction of the HISH model is based on the following steps.
- (i) Analyzing string configurations in confining holographic string models that correspond to hadrons.
- (ii) Devising a transition from the holographic regime of large Nc and large λ to the real world that bypasses expansions in $\frac{1}{N_c}$ and $\frac{1}{\lambda}$
- (iii) Proposing a model of stringy hadrons in **flat four dimensions with massive endpoint particles** that is **inspired** by the corresponding holographic model
- (iv)Dressing the endpoint particles with **baryonic vertex**, **charge, spin etc**
- (v) Confronting the outcome of the models with **experimental data** .

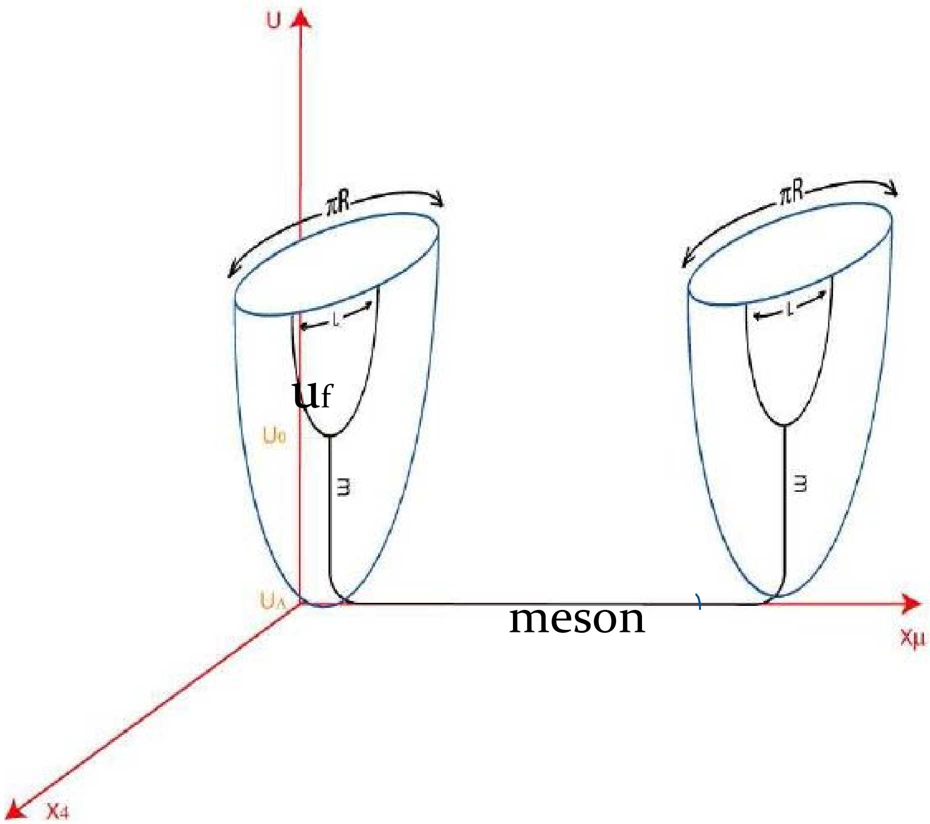
(i) The structure of a holographic meson

- The structure of a **holographic meson** is a **rotating string** that starts and ends on flavor branes (the same or different). For instance a heavy quarqonium is



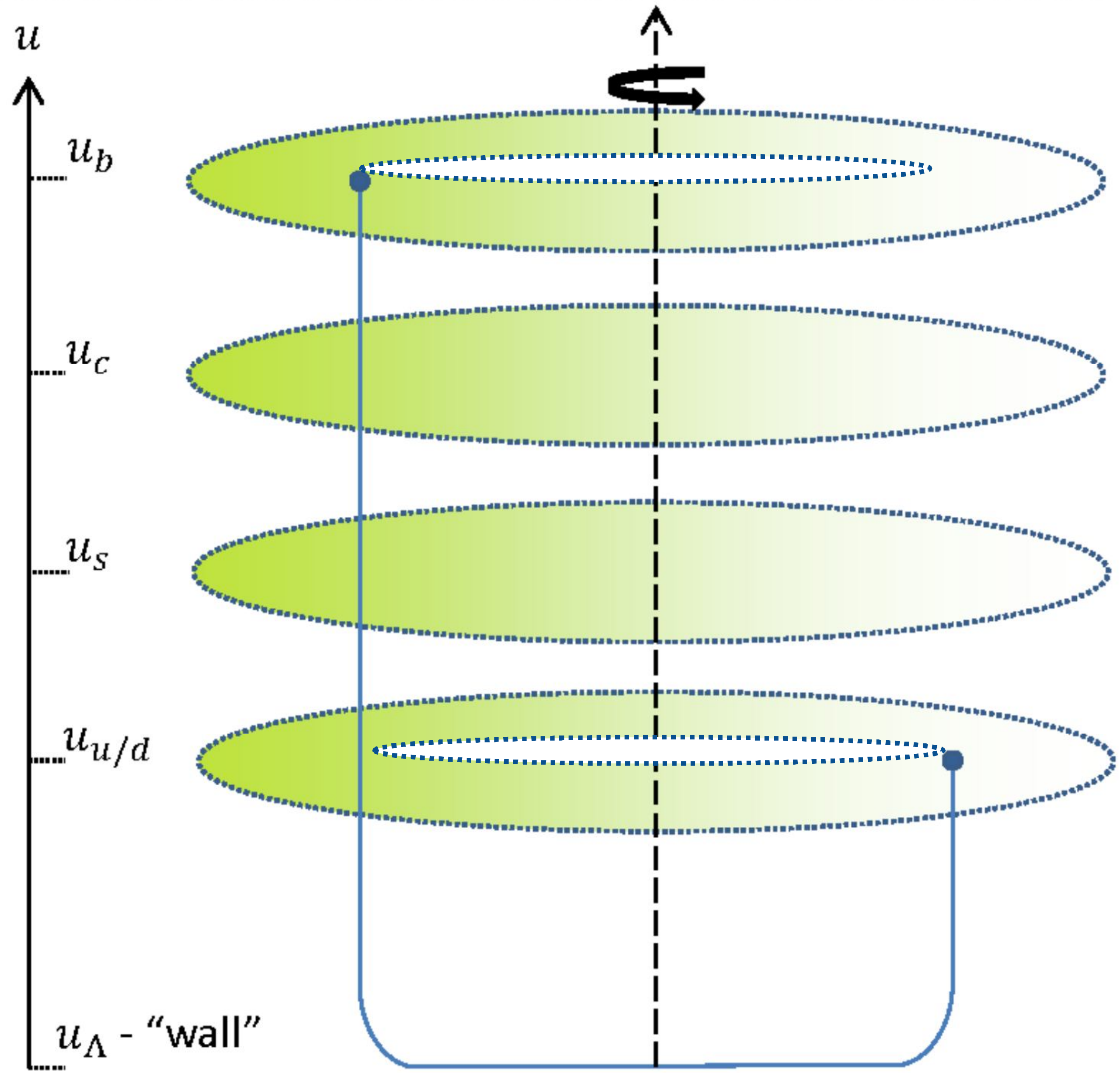
Stringy meson in U shape flavor brane setup

- In the generalized Sakai Sugimoto model the meson looks like



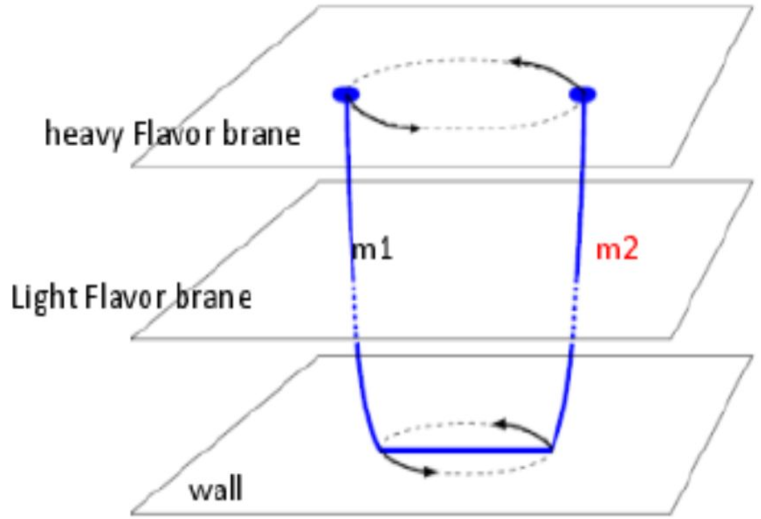
- We now rotate this string configuration

Example: The B meson

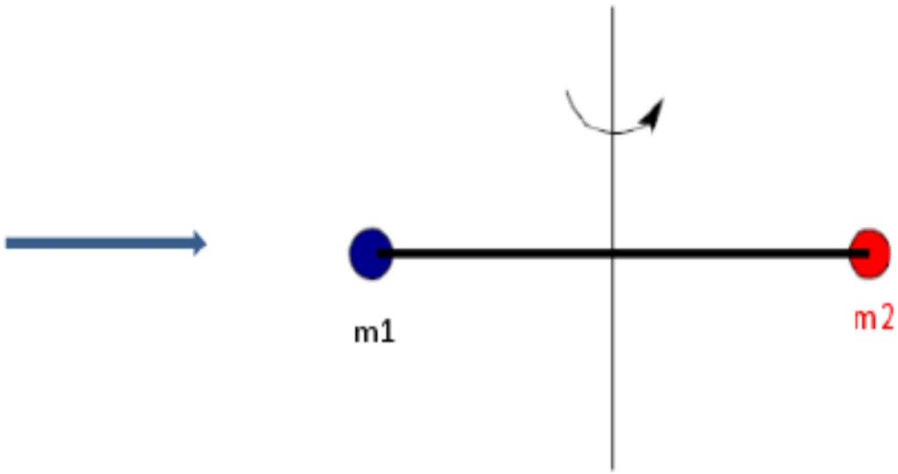


HISH

- The vertical segments of the holographic hadronic string can me mapped to massive particles at the



Rotating holographic string



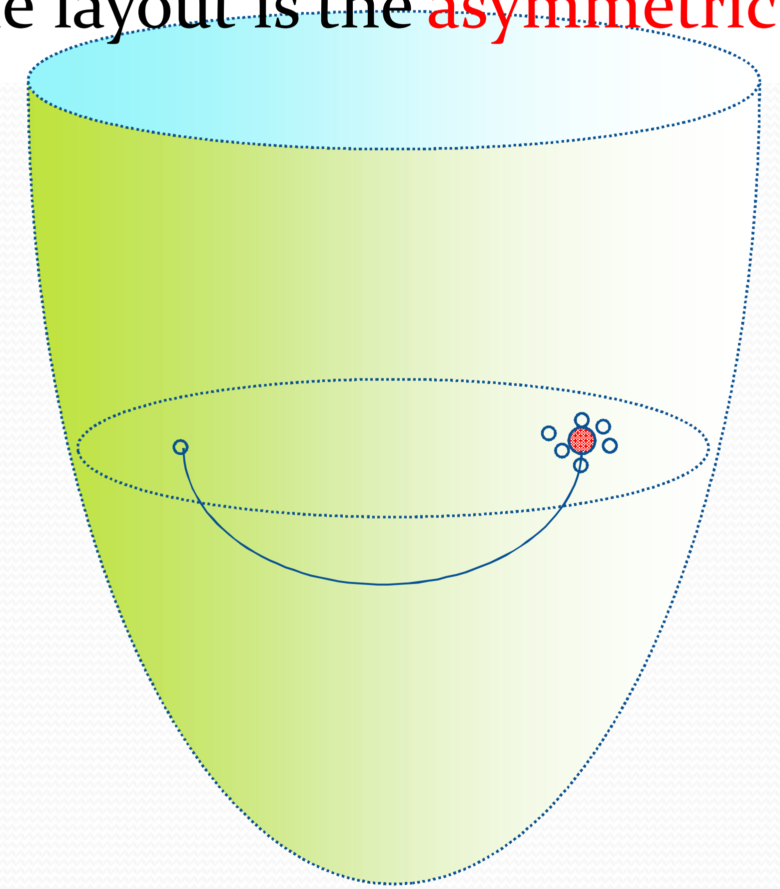
Rotating sting in flat space-time with massive endpoints

end of the string with mass

$$m_{sep} = T \int_{u_0}^{u_f} g(u)$$

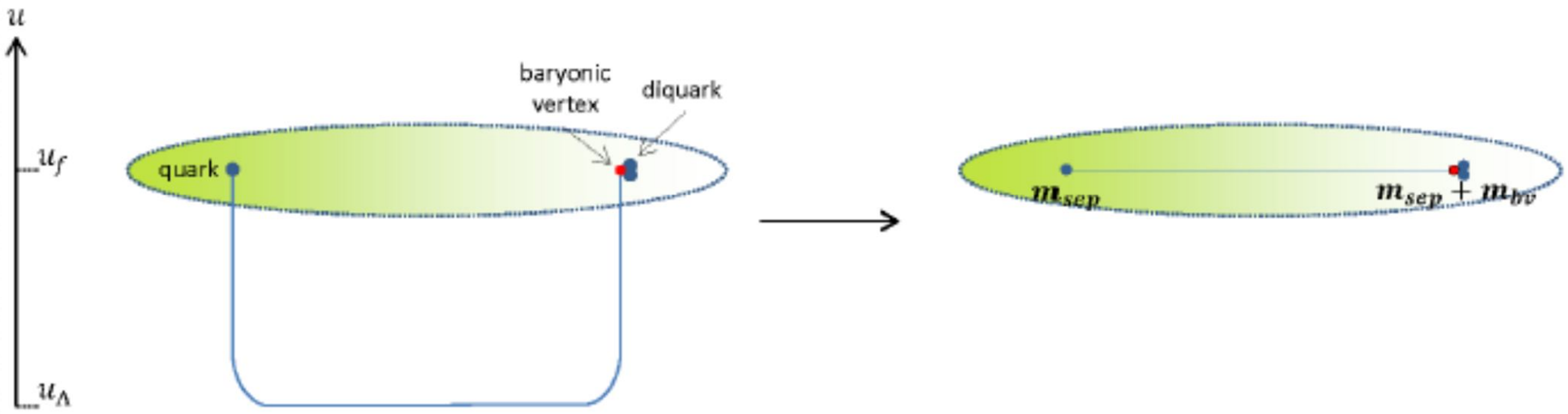
HISH Baryon

- In holography a baryon is a baryonic vertex which is a wrapped Dp brane on a p cycle and is connected with Nc strings to a flavor brane.
- The preferable layout is the asymmetric one.



HISH

- In HISH the holographic baryon is mapped into a single string that connects a quark on one side and a diquark on the other side



The classical string with massive endpoints

The classical string with massive endpoints

- We start with the action and equations of motion
- The action is the NG action plus two point-particle action terms

$$S = S_{st} + S_{pp}|_{\sigma=-\ell} + S_{pp}|_{\sigma=\ell}$$

$$\boxed{S_{st} = -T\int d\tau d\sigma\sqrt{-h} = -T\int d\tau d\sigma\sqrt{\dot{X}^2X'^2 - (\dot{X}\cdot X')^2}}$$

$$-\infty < \tau < \infty \quad -\ell \leq \sigma \leq \ell.$$

- The world sheet coordinates

and the induced metric is $h_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$

The relativistic particle action

$$\boxed{S_{pp} = -m\int d\tau\sqrt{-\dot{X}^2}}$$

The equations of motion

- The bulk equation of motion

$$\partial_\alpha(\sqrt{-h}h^{\alpha\beta}\partial_\beta X^\mu) = 0$$

- The boundary equation of motion

$$\boxed{T\sqrt{-h}\partial^\sigma X^\mu \pm m\partial_\tau\left(\frac{\dot{X}^\mu}{\sqrt{-\dot{X}^2}}\right) = 0}$$

- The + for $\sigma = \ell$ and – for $\sigma = -\ell$.

Rotation solution of the equations of motion

- A rotating solution in the (1,2) plane

$$X^0 = \tau, \qquad X^1 = R(\sigma)\cos(k\tau), \qquad X^2 = R(\sigma)\sin(k\tau)$$

For any choice of $R(\sigma)$

Substituting it to the boundary equation of motion

$$T\frac{\sqrt{(1-k^2R^2)R'^2}}{R'} \mp m\frac{k^2R}{\sqrt{1-k^2R^2}} = 0$$

Which translates into the tension being balanced by the

centrifugal force $$\boxed{\frac{T}{\gamma} = \frac{2\gamma m\beta^2}{L}}$$

The energy and angular momentum

- There are two natural choices of

$$R(\sigma)=\sigma: \qquad \frac{T}{mk}=\frac{k\ell}{1-k^2\ell^2} \qquad L=2\ell \qquad \beta=k\ell$$

$$R(\sigma)=\frac{1}{k}\sin(k\sigma): \qquad \frac{T}{mk}=\frac{\sin(k\ell)}{\cos^2(k\ell)} \qquad L=\frac{2}{k}\sin(k\ell) \qquad \beta=\sin(k\ell)$$

- The energy and angular momentum which are the Neother charges associated with shifts of and x^0 and θ.
- The contribution of the string

$$\boxed{E_{st}=-T\int_{-\ell}^{\ell}d\sigma\sqrt{-h}h^{\tau\alpha}\partial_\alpha t \qquad J_{st}=-T\int_{-\ell}^{\ell}d\sigma\sqrt{-h}\rho^2h^{\tau\alpha}\partial_\alpha\theta}$$

- The contribution of the particles

$$\boxed{E_{pp}=m\frac{\dot{t}}{\sqrt{-\dot{X}^2}} \qquad J_{pp}=m\frac{\rho^2\dot{\theta}}{\sqrt{-\dot{X}^2}}}$$

The energy and angular momentum

- Altogether for the rotating string E and J are

$$\boxed{\begin{aligned} E &= \frac{2m}{\sqrt{1-\beta^2}} + TL\frac{\arcsin\beta}{\beta} \\ J &= \frac{mL\beta}{\sqrt{1-\beta^2}} + \frac{1}{4}TL^2\frac{\arcsin\beta - \beta\sqrt{1-\beta^2}}{\beta^2} \end{aligned}}$$

- In the limit of small masses

$$\boxed{J = \frac{1}{2\pi T}E^2\left(1 - \frac{8\sqrt{\pi}}{3}\left(\frac{m}{E}\right)^{3/2} + \frac{2\pi^{3/2}}{5}\left(\frac{m}{E}\right)^{5/2} + \ldots\right)}$$

- In the limit of large masses

$$J_4 = \frac{2m^{1/2}}{T3\sqrt{3}}(E-2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T}(E-2m)^{5/2}$$

$$- \frac{1003}{\quad}(E-2m)^{7/2}$$

The quantum string with massive endpoints

Fluctuations and gauge choice

- Consider the quantum fluctuations around the rotating solution

$$X^\mu = X^\mu_{cl} + \delta X^\mu = (t\,,\rho\,,\theta\,,z^i) = (\tau + \lambda\delta t, R(\sigma) + \lambda\delta\rho, k\tau + \lambda\delta\theta, \lambda\delta z^i)$$

- Where $\lambda.$ is a formal expansion parameter later will be expressed in terms of $\frac{1}{mL}$
- We fix the reparameterization invariance using the orthogonal gauge

$$\boxed{\begin{aligned} &\frac{1}{2}(h_{\tau\tau} + h_{\sigma\sigma}) = \frac{1}{2}(\dot{X}^2 + X'^2) = 0 \\ &h_{\tau\sigma} = h_{\sigma\tau} = \dot{X}\cdot X' = 0 \end{aligned}}$$

- We further use the static gauge $\tau = X^0 \quad \Rightarrow \quad \delta t = 0$

$R(\sigma) = \sigma$ or $R(\sigma) = \frac{1}{k}\sin(k\sigma$

Transverse fluctuations

- Consider the fluctuations transverse to the plane of rotation. We truncate the action to second order

$$S_{st,\delta z} = -T\lambda^2 \int d\tau d\sigma \left[\frac{1}{2}\left(\sqrt{R'^2} g\right)^{-1} \delta z'^2 - \frac{1}{2}\left(\sqrt{R'^2} g\right) \delta \dot{z}^2\right]$$

$$S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2} \gamma \delta \dot{z}^2$$

- To properly normalize the kinetic term define

$$f_t \equiv (\sqrt{R'^2} g)^{1/2} \delta z$$

- For the case

$$R(\sigma) = \tfrac{1}{k}\sin(k\sigma)$$

$$S_{st,\delta z} = -T\lambda^2 \int d\tau d\sigma \left(\frac{1}{2} f_t'^2 - \frac{1}{2} \dot{f}_t^2\right)$$

$$S_{pp,\delta z} = m\lambda^2 \int d\tau \frac{1}{2} \gamma \dot{f}_t^2$$

The transverse fluctuations

- The world sheet Hamiltonian

$$H = \frac{1}{2} T \lambda^2 \left(\int_{-\ell}^{\ell} d\sigma (\dot{f}_t^2 + f_t'^2) + \frac{\gamma m}{T} \dot{f}_t^2 |_{\pm \ell} \right)$$

- The Mode expansion and canonical quantization

$$f_t = f_0 + i\sqrt{\mathcal{N}} \sum_{n \neq 0} \frac{\alpha_n}{\omega_n} e^{-i\omega_n \tau} f_n(\sigma)$$

- The canonical quantization condition

$$[f_t(\sigma), \pi_t(\sigma')] = i\delta(\sigma - \sigma')$$

- Can be achieved upon choosing $\mathcal{N} = \frac{1}{2T\ell^2\lambda^2}$.

The transverse fluctuations

- The fluctuations have to obey the bulk EOM

$$f_n'' + \omega_n^2 f_n = 0$$

- The boundary equations are $$T f_n' \mp \gamma m \omega_n^2 f_n = 0$$

- The eigenfquencies are subject to

$$\boxed{2\delta \cot(\delta) x \cos(2x) - (\delta^2 - \cot^2(\delta) x^2) \sin(2x) = 0}$$

- Where $x \equiv \omega_n \ell \qquad \delta \equiv k\ell = \arccos(\gamma^{-1})$

Energy and angular momentum

- E and J associated with the transverse modes are

$$E_{st,f_t} = T\lambda^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \frac{1}{\cos^2(k\sigma)} (f_t'^2 + \dot{f}_t^2)$$

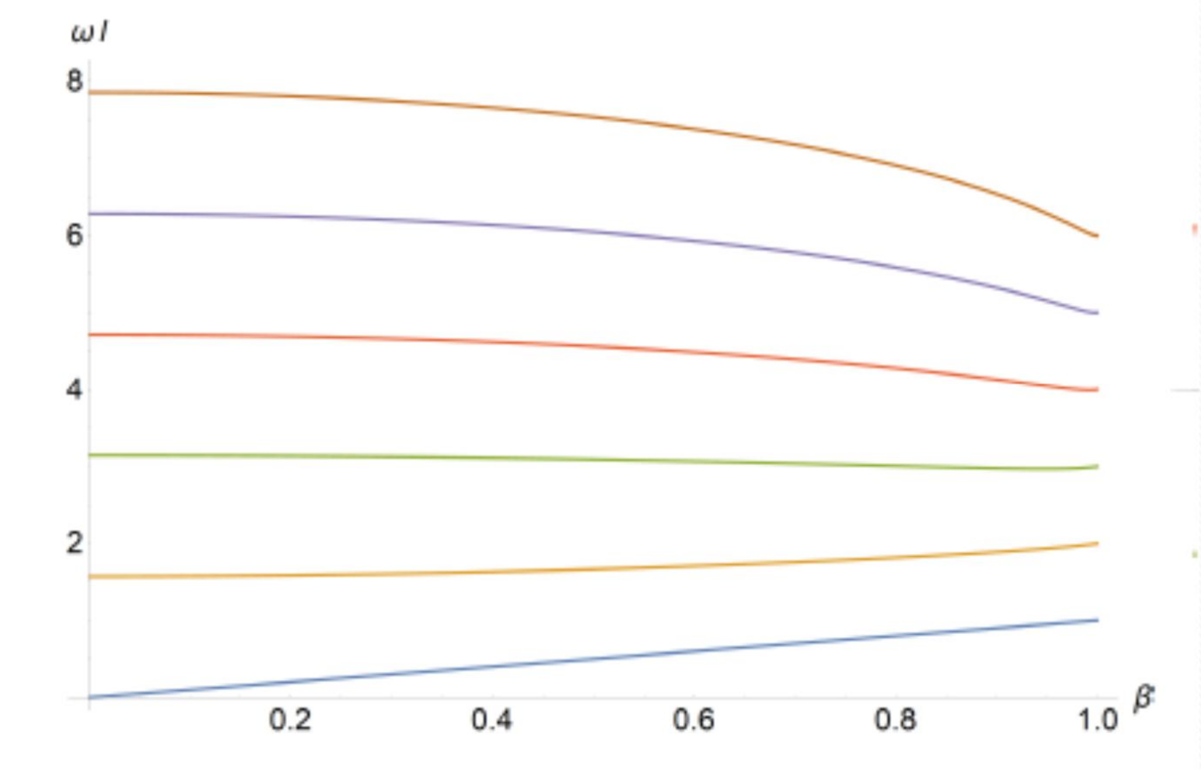
$$E_{pp,f_t} = \frac{1}{2} m\lambda^2 \gamma^3 \dot{f}_t^2$$

$$J_{st,f_t} = T\lambda^2 \int_{-\ell}^{\ell} d\sigma \frac{1}{2} \frac{\tan^2(k\sigma)}{k} (f_t'^2 + \dot{f}_t^2)$$

$$J_{pp,f_t} = \frac{1}{2} m\lambda^2 \gamma^3 \frac{\sin^2(k\ell)}{k} \dot{f}_t^2$$

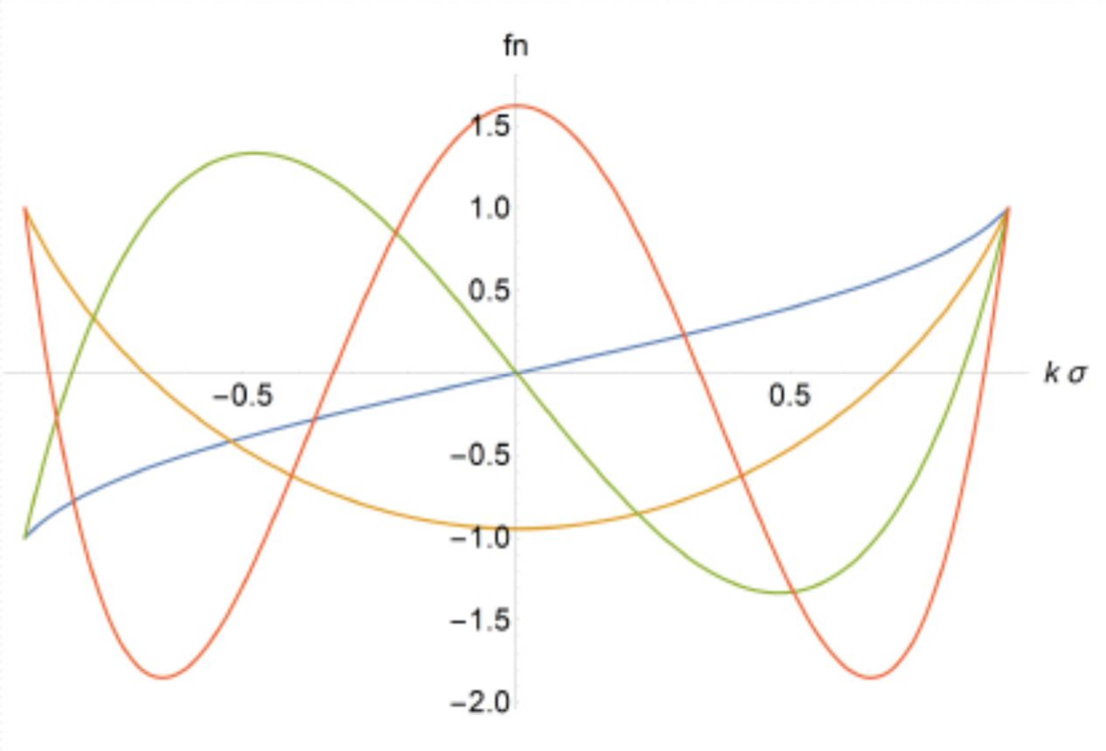
The eigenfrequencies

- The first few eigenfrequencies



The transverse modes

- The first transverse modes



The contribution of the transverse modes to the intercept

The intercept

- We would like to determine the impact of the fluctuations on E and J , namely the quantum correction of the trajectory
- We found before the classical E and J

$$E = E(m, T, \gamma) \qquad J = J(m, T, \gamma)$$

- The classical trajectory is defines by $J = J_{cl}(E)$
- We define the intercept as

$$\boxed{a = \langle \delta J - \frac{1}{k}\delta E \rangle \qquad \frac{1}{k} = \frac{1}{2}\frac{L}{\beta}}$$

- By substituting the expressions for Es,Epp,Js, Jpp we proved that

$$\boxed{a = -\frac{1}{k}\langle H \rangle}$$

The intercept

- Using the mode expansion and the orthogonality relations we get

$$H = -\frac{1}{2}\frac{T\lambda^2}{2}\frac{1}{T\ell\lambda^2}\sum_{n\neq 0}\alpha_{-n}\alpha_n = \frac{1}{2\ell}\sum_{n\neq 0}\alpha_{-n}\alpha_n$$

- Thus the contribution of the transverse modes is

$$\boxed{a_t = -\frac{1}{k}\langle H\rangle = -\frac{1}{2}\sum_{n>0}\frac{\omega_n\ell}{\delta}}$$

- For the massless case since $\delta = \frac{\pi}{2}$ and $\omega_n\ell = \frac{\pi}{2}n$

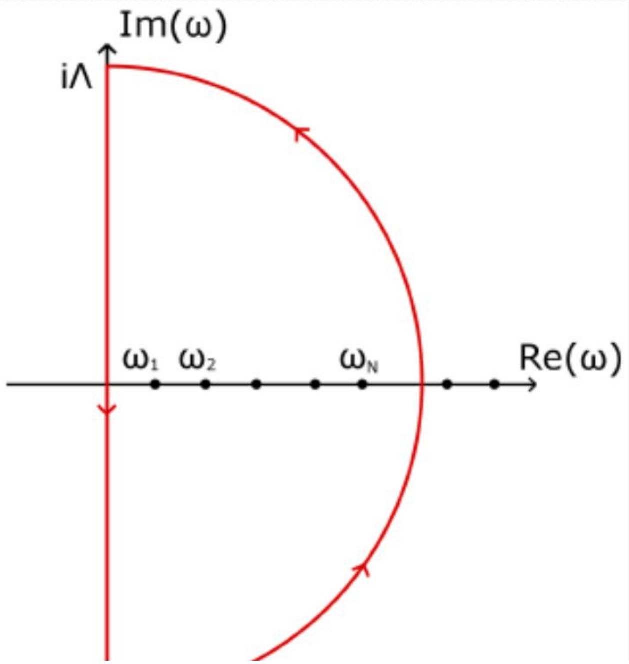
$$\boxed{a_t(m=0) = -\frac{1}{2}\sum_{n>0} n = \frac{1}{24}}$$

The renormalization of the sum of the eigenfrequencies

- We convert the infinite sum into a contour integral using Cauchy integral formula

$$\boxed{\frac{1}{2\pi i}\oint dz z\frac{d}{dz}\log f(z) = \frac{1}{2\pi i}\oint dz z\frac{f'(z)}{f(z)} = \sum_j n_j z_j - \sum_k \tilde{n}_k \tilde{z}_k}$$

- We will use a function $f(\omega)$ with only simple zeros at $\omega = \omega_n$ which are on the positive real axis



The renormalization of the sum of the eigenfrequencies

- The sum of the eigen-frequencies is the Casimir energy

$$E_C = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n \quad = -\frac{2\beta a}{L}$$

- The semi-circle regularizes the Casimir energy

$$E_C^{(reg)} = \frac{1}{2}\sum_{n=1}^{N(\Lambda)}\omega_n$$

- We renormalize the result in the same way that we do for the Casimir effect

$$\boxed{E_C^{(ren)} = \lim_{\Lambda\to\infty}\left(E_C^{(reg)}(m,T,L) - E_C^{(reg)}(m,T,L\to\infty)\right)}$$

The renormalization of the sum for the massless case

- For the ordinary string with no endpoint particles

$$f(\omega) = \sin(\pi\omega\ell) = 0$$

- Usually we use the zeta function renormlization

$$\sum_{n=1}^{\infty} n = \zeta(-1) = -\frac{1}{12}$$

- Using our method

$$\boxed{E_C(m=0) = \frac{1}{2}\sum_{n=1}^{\infty}\omega_n = \frac{1}{4\pi i}\oint \omega \frac{f'(\omega)}{f(\omega)} d\omega = \frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega}$$

$$\frac{1}{4i}\oint \omega\ell\cot(\pi\omega\ell)d\omega = -\frac{1}{4}\int_{-\Lambda}^{\Lambda} y\ell\coth(\pi y\ell)dy + \frac{1}{4}\Lambda^2\ell\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{2i\theta}\cot(\pi\Lambda\ell e^{i\theta})d\theta$$

The intercept for the massless case

- The regularized energy reads

$$E_C^{(reg)} = \left(-\frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell}\right) + \frac{1}{2}\Lambda^2 \ell = \frac{\Lambda^2 \ell}{4} - \frac{1}{24\ell} = \frac{\Lambda^2 L}{8} - \frac{1}{12L}$$

- The corresponding force

$$F_C^{(reg)} = -\frac{d}{dL} E_C^{(reg)} = -\frac{\Lambda^2}{8} + \frac{1}{12L^2}$$

- The renormalized force

$$\boxed{F_C^{(ren)} = \lim_{\Lambda \to \infty} \left(F_C^{(reg)}(L) - F_C^{(reg)}(L \to \infty)\right) = \frac{1}{12L^2}}$$

- The renormalized energy

$$E_C^{(ren)} = -\frac{1}{12L} \qquad \Rightarrow \qquad a = \frac{1}{24}$$

Non rotating string with massive endpoints

- The eigenfrequencies for the static string are determined from

$$f(\omega) = 2\frac{m\omega}{T}\cos(\omega L) + (1 - \frac{m^2\omega^2}{T^2})\sin(\omega L) = 0$$

- Following the same procedure as for the massless case we get

$$E_C^{(ren)} = \frac{1}{2\pi}\int_0^\infty \log\left(1 - e^{-2Ly}\frac{(T-my)^2}{(T+my)^2}\right) dy = \frac{1}{2\pi L}\int_0^\infty \log\left(1 - e^{-2x}\frac{(q-x)^2}{(q+x)^2}\right) dx$$

- Where $q = \frac{TL}{m}$.
- In the limits of q=0 or q goes to infinity this reduces

$$E_C^{(ren)}(q=0) = E_C^{(ren)}(q \to \infty) = \frac{1}{2\pi L}\int_0^\infty dx \log(1 - e^{-2x}) = -\frac{\pi}{24L}$$

The renormatization for the rotating massive string

- For the case of our interest a rotating string with massive endpoints the eigenfrequencies

$$f(z) = 2z\beta^2\sqrt{1-\beta^2}\cos\left(\frac{2z\arcsin\beta}{\beta}\right) + (\beta^4 - (1-\beta^2)z^2)\sin\left(\frac{2z\arcsin\beta}{\beta}\right)$$

- The divergent of the integral for the regularized Ec

$$\left[-\frac{\Lambda^2 L\arcsin\beta}{\pi\beta} - \frac{2\Lambda}{\pi} + \frac{T}{2\gamma m}\log\frac{2\gamma m\Lambda}{T}\right] + \left[\frac{2\Lambda^2 L\arcsin\beta}{\pi\beta} + \frac{2\Lambda}{\pi}\right] =$$
$$= \frac{\Lambda^2 L\arcsin\beta}{\pi\beta} + \frac{T}{2\gamma m}\log\frac{2\gamma m\Lambda}{T}$$

- The key ingredient now is to rewrite it as

$$\boxed{\frac{\Lambda^2\tilde{L}}{\pi} + \frac{T}{2\tilde{m}}\log\frac{2\tilde{m}\Lambda}{T}}$$ where $\tilde{L} = L\frac{\arcsin\beta}{\beta}$

The renormatization for the rotating massive string

- The integral of the renormalized Casimir energy

$$E_C^{ren} = \frac{1}{\pi}\int_0^{\infty} \log\left(\frac{2y\beta^2\sqrt{1-\beta^2}\cosh\left(\frac{2y\arcsin\beta}{\beta}\right) + (\beta^4 + (1-\beta^2)y^2)\sinh\left(\frac{2y\arcsin\beta}{\beta}\right)}{\frac{1}{2}\left((1-\beta^2)y^2 + 2\beta^2\sqrt{1-\beta^2} + \beta^4\right)\exp\left(\frac{2y\arcsin\beta}{\beta}\right)}\right.$$

- The corresponding result for the intercept reads

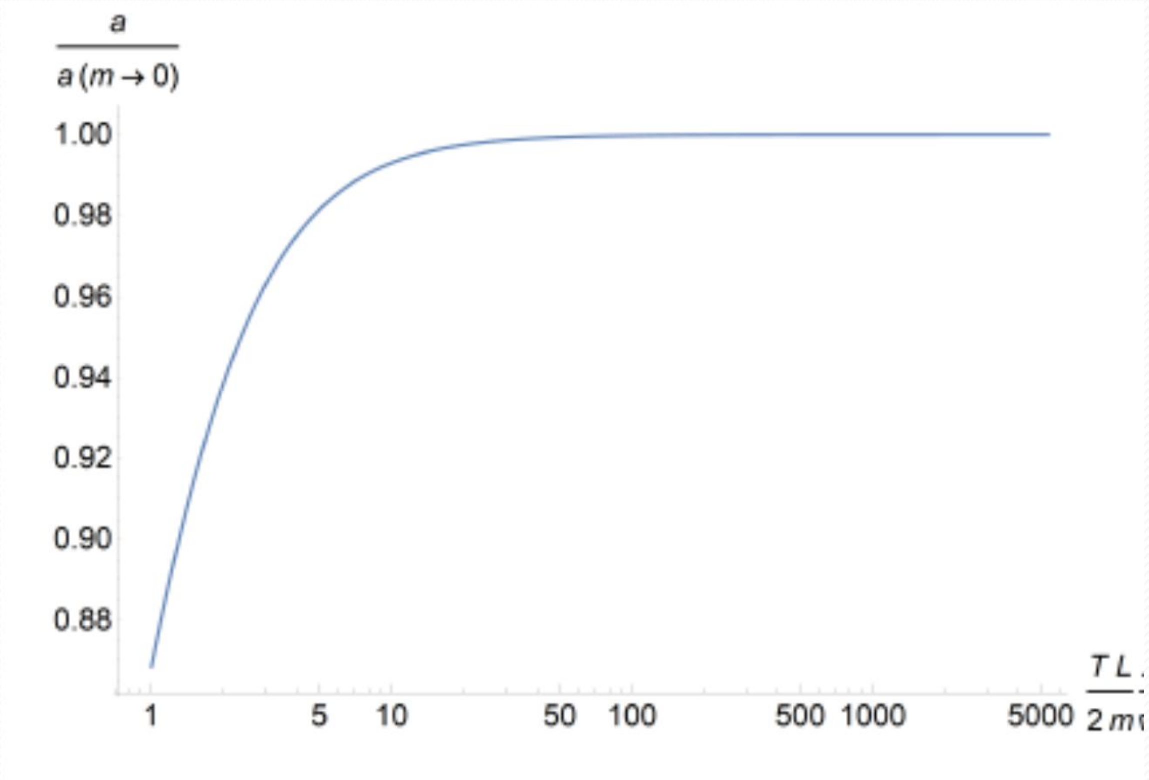
$$\boxed{a_t = -\frac{1}{2\pi\beta}\int_0^{\infty} \log\left[1 - \exp\left(-\frac{4\arcsin\beta}{\beta}y\right)\left(\frac{y-\gamma\beta^2}{y+\gamma\beta^2}\right)^2\right]}$$

- This goes back to the standard massless result

$$a_t(\beta\to 1) = -\frac{1}{2\pi}\int_0^{\infty} \log\left(1 - e^{-2\pi y}\right) = \frac{1}{24}$$

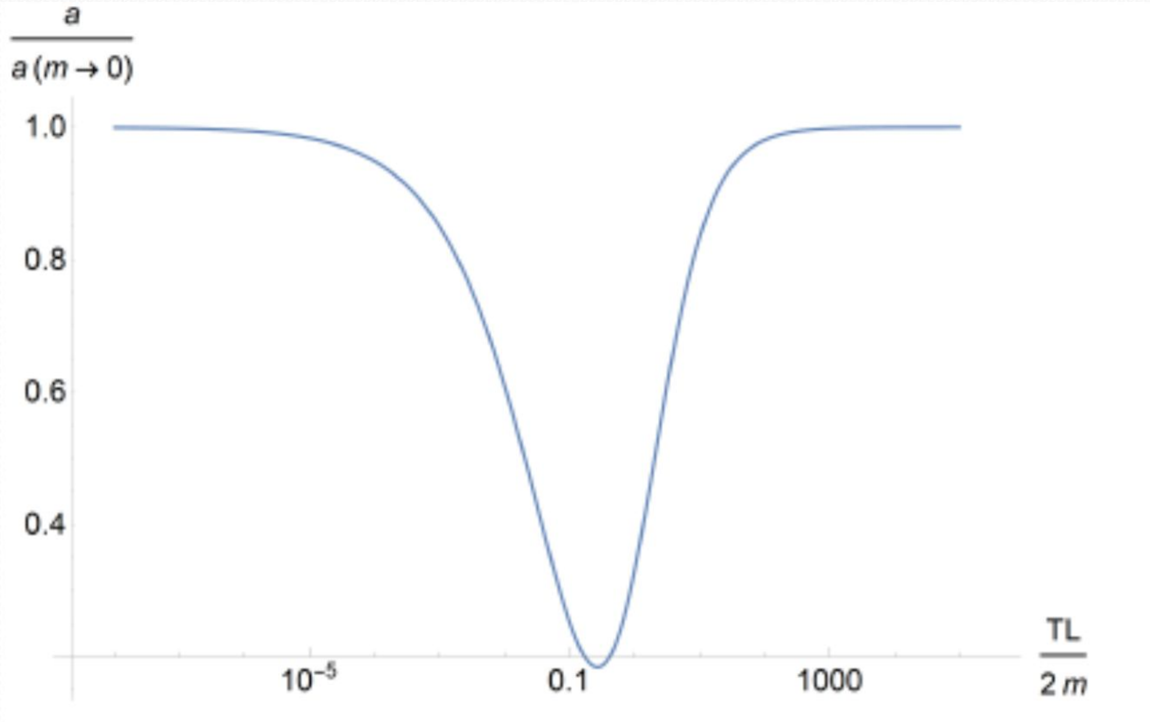
- For small masses $\boxed{a_t = \frac{1}{24} - \frac{11}{360\pi}\frac{1}{\gamma^3} + \ldots = \frac{1}{24} - \frac{11}{360\pi}(\frac{2m}{TL})^{3/2}}$

The contribution of the a trasverse mode to the intercep



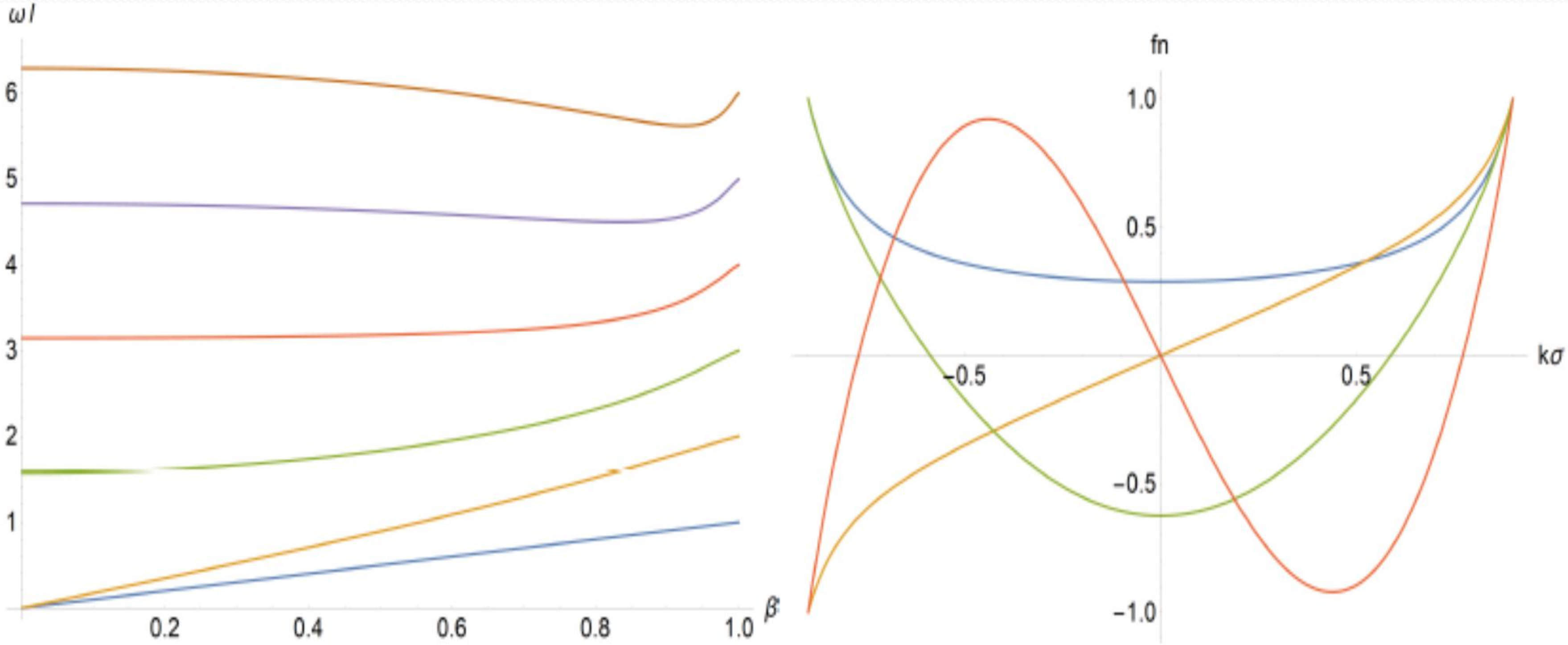
The contribution of the a trasverse mode to the intercep

- Compare with the static massive case



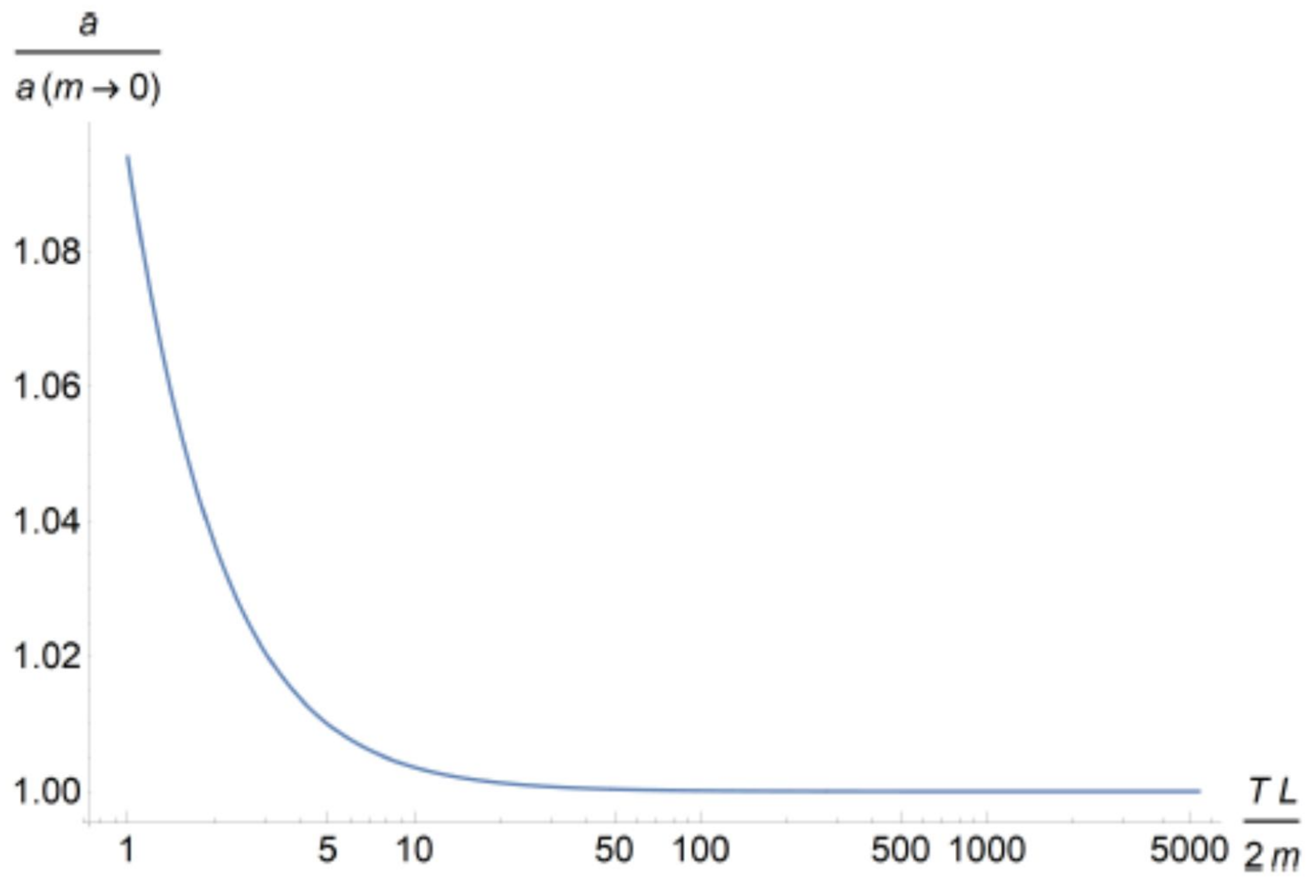
The contribution to the intercept of the planar mode

- We have done the analysis also for the planar mode
- The eigenmodes and eigenfrequencies



The planar mode

- The contribution of the planar mode to the intercept



The Quantization of the non-critical string

The Polchiski Strominger term

- It is well known that the quantization of the Polyakov string in non-critical dimensions is done by adding to the action the Liouville term
- Polchinsky and Strominger mapped this in the formulation of the NG string by taking a "composite Liouville " mode

$$\phi = -\frac{1}{2}\log\left(\partial_+ X \cdot \partial_- X\right)$$

- In the orthogonal gauge the PS term reads

$$\boxed{S_{PS} = \frac{B}{2\pi}\int d^2\sigma\partial_+\phi\partial_-\phi = \frac{B}{2\pi}\int d^2\sigma\frac{(\partial_+^2 X \cdot \partial_- X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial_- X)^2}}$$

The renormalization of the PS term

- The contribution of the PS to the intercept is achieved by substituting the classical solution into the PS action

$$\boxed{E_{PS} = \langle H_{PS} \rangle = -\int d\sigma \mathcal{L}_{PS} = \frac{B}{2\pi} \int_{-\ell}^{\ell} d\sigma k^2 \tan^2(k\sigma) = \frac{B}{\pi} k(\tan\delta - \delta)}$$

where . $\delta = k\ell$ The term $k \tan\delta$ diverges in the massless case since $\delta = \pi/2$

- For small masses it is finite but un-physically large
- Hellerman et all renormalized the PS term for the massless case by

$$S_{PS} \to S_{PS}^{(reg)} = \frac{B}{2\pi} \int d^2\sigma \frac{(\partial_+^2 X \cdot \partial\_X)(\partial_-^2 X \cdot \partial_+ X)}{(\partial_+ X \cdot \partial\_X)^2 + \alpha' \epsilon^4 (\partial_+^2 X \cdot \partial_-^2 X)}$$

- And adding a counterterm

$$S_{ct} \propto \frac{1}{\epsilon} \int d\tau (\ddot{X} \cdot \ddot{X})^{1/4}$$

The renormalization of the PS term

- The result of the renormalization of the PS term in the massless case is that

$$a_{PS}(m=0) = \frac{26-D}{24}$$

- As a result the total intercept is in any D dimensions

$$\boxed{a(m=0) = \frac{D-2}{24} + \frac{26-D}{24} = 1}$$

- The massive endpoints serve as a regulator. Never the less we have to perform a subtraction since we subtract anyhow for the other divergenseis and also since we want to connect smoothly to the subtraction at m=o

The renormalization of the PS term

- We can re-write the PS term as

$$\boxed{E_{PS} = \frac{B}{\pi}\left(\frac{T}{\tilde{m}} - \frac{2\delta^2}{\tilde{L}}\right)}$$

- Using the length and mass measured in the Lab frame

$$\tilde{m} = \gamma m \qquad \tilde{L} = L\frac{\arcsin\beta}{\beta} = \frac{2}{k}\sin\delta \times \frac{\delta}{\sin\delta} = \frac{2\delta}{k}$$

- Based on the boundary equation of motion

$$k\tan\delta = \frac{T\cos\delta}{m} = \frac{T}{\gamma m}$$

- WE renormalize by subtracting from the force for the string of length L the force of the string with $L \to \infty$

The contribution of the PS term to the intercept

- Thus the renormalized contribution of the

$$\boxed{a_{PS} = -\frac{1}{k}E_{PS}^{(ren)} = \frac{26-D}{12\pi}\delta = \frac{26-D}{12\pi}\arcsin\beta}$$

- In the limit of $\beta \to 1$ $\quad a_{PS} = \frac{26-D}{24}$

- As a function of $2m/TL$ it reads

$$a_{PS} = \frac{26-D}{12\pi}\arccos\left(\sqrt{\frac{2m}{2m+TL}}\right) = \frac{26-D}{24}\left[1 - \frac{2}{\pi}\left(\frac{2m}{TL}\right)^{1/2} + \frac{2}{3\pi}\left(\frac{2m}{TL}\right)^{3/2} + \dots\right]$$

- Thus the total intercept is

$$\boxed{a = (D-3)a_t + a_p + a_{PS} \approx 1 - \frac{26-D}{12\pi}(\frac{2m}{TL})^{1/2} + \frac{199-14D}{240\pi}(\frac{2m}{TL})^{3/2}}$$

The Quantum Regge Trajectories

The quantum spectrum

- The vacuum is taken to be the classical rotating string
- We can build excited states by applying on the vacuum.
- The worldsheet Hamiltonian is related to E and J via

$$H = \delta E - k\delta J .$$

- In the massless case the spectrum constitute the
- Regge trajectories

$$\boxed{J + N = \alpha' E^2 + a}$$

The spectrum of the massive string

- In the massive case the transverse and planar numbers

$$N_t = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n \qquad N_p = \sum_{n=1}^{\infty} \alpha^p_{-n}\alpha^p_n$$

- The massive modified Regge trajectory can be written as

$$\boxed{J + \frac{1}{\beta}(N_t + N_p) = J_{cl}(E) + (D-3)a_t + a_p + a_{PS}}$$

- The eigenvalues of Nt and Np on a generic state

$$N_t = \sum^{\infty} \omega_n^{(t)} \ell N_n^{(t)} \qquad N_p = \sum^{\infty} \omega_n^{(p)} \ell N_n^{(p)}$$

The radial excited states

- The first two radially excited states are

N	State	$N_t + N_p$	No. of states
1	$\alpha^i_{-1}\|0\rangle$	$\omega_1^{(t)}\ell$	$D-3$
	$\alpha^p_{-1}\|0\rangle$	$\omega_1^{(p)}\ell$	1
2	$\alpha^i_{-2}\|0\rangle$	$\omega_2^{(t)}\ell$	$D-3$
	$\alpha^p_{-2}\|0\rangle$	$\omega_2^{(p)}\ell$	1
	$\alpha^i_{-1}\alpha^j_{-1}\|0\rangle$	$2\omega_1^{(t)}\ell$	$(D-3)^2$
	$\alpha^i_{-1}\alpha^p_{-1}\|0\rangle$	$\omega_1^{(t)}\ell + \omega_1^{(p)}\ell$	$D-3$
	$\alpha^p_{-1}\alpha^p_{-1}\|0\rangle$	$2\omega_1^{(p)}\ell$	1

The Generalization to the asymmetric case

Asymmetric string

- So far we have assumed that the two endpoint particles casrry the same mass. We generalize it now to the case of two different masses
- We are interested in asymmetric scenarios since they can, as we will see negative intercept.
- Negative intercept is what one finds for all the hadronic trajectories if it is defined in the relation of the orbital and not the total angular momentum.
- For instance has a 0.5 and S=1 s so for L=J-S we get a= -0.5
- The extreme case of one massless end and one infinitely heavy end is a Dirichlet-Neumann case which can analyzed as the usual case wih $\omega_n = n - \frac{1}{2}$

Asymmetric string

- The intercept for such a case is

$$a_{ND} = -\frac{1}{2}\sum_{n=1}^{\infty}(n - \frac{1}{2}) = -\frac{1}{48}$$

- Instead of 1/24 for the DD or NN cases.
- In the asymmetric case there two arms with different length $\frac{T\ell_i}{m_i} = \beta_i^2\gamma_i^2$

The intercept for the asymmetric string

- The contribution to the intercept from transverse modes

$$a_t = -\frac{1}{2\pi}\int_0^\infty dy \log\left(1 - e^{-2(\arcsin\beta_1 + \arcsin\beta_2)y}\frac{(y-\beta_1^2\gamma_1)(y-\beta_2^2\gamma_2)}{(y+\beta_1^2\gamma_1)(y+\beta_2^2\gamma_2)}\right)$$

- The contribution from the plannar modes

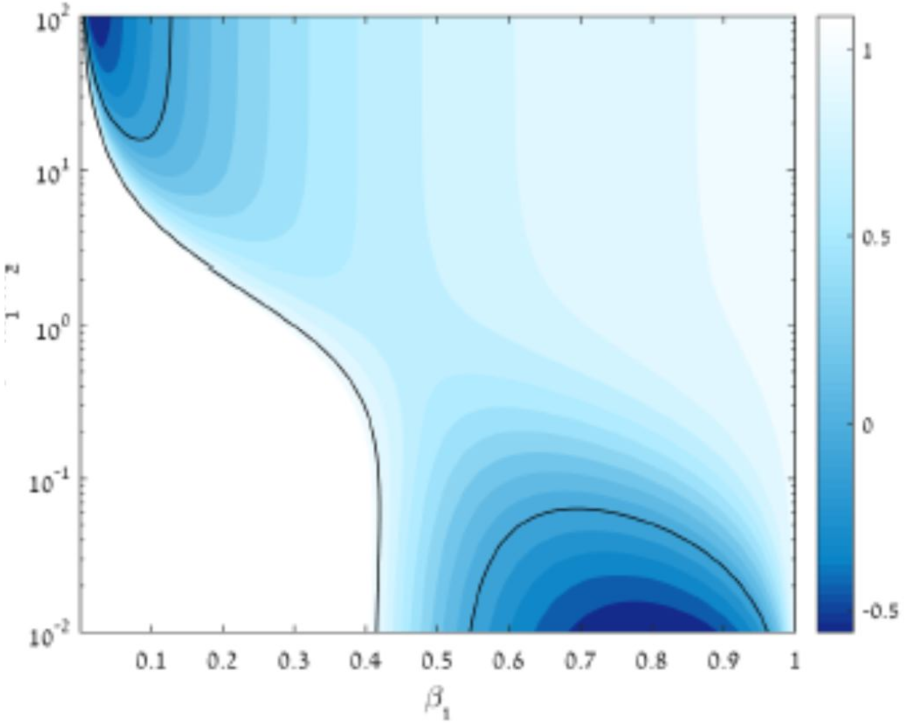
$$\boxed{\begin{aligned} a_p = &-\frac{1}{2\pi}\int_0^\infty dy \\ &\log\left[1 - e^{-2(\arcsin\beta_1 + \arcsin\beta_2)y}\left(\frac{y^2 - 2y\gamma_1\beta_1 + \gamma_1^2(1+\beta_1^2)}{y^2 + 2y\gamma_1\beta_1 + \gamma_1^2(1+\beta_1^2)}\right)\left(\frac{y^2 - 2y\gamma_2\beta_2 + \gamma_2^2(1+\beta_2^2)}{y^2 + 2y\gamma_2\beta_2 + \gamma_2^2(1+\beta_2^2)}\right)\right]\end{aligned}}$$

- The contribution of the PS mode

$$\boxed{a_{PS} = \frac{26-D}{24\pi}(\delta_1 + \delta_2) = \frac{26-D}{24\pi}(\arcsin\beta_1 + \arcsin\beta_2)}$$

The asymmetric intercept

- The intercept as a function of $r = m_1/m_2$. and β_{11}



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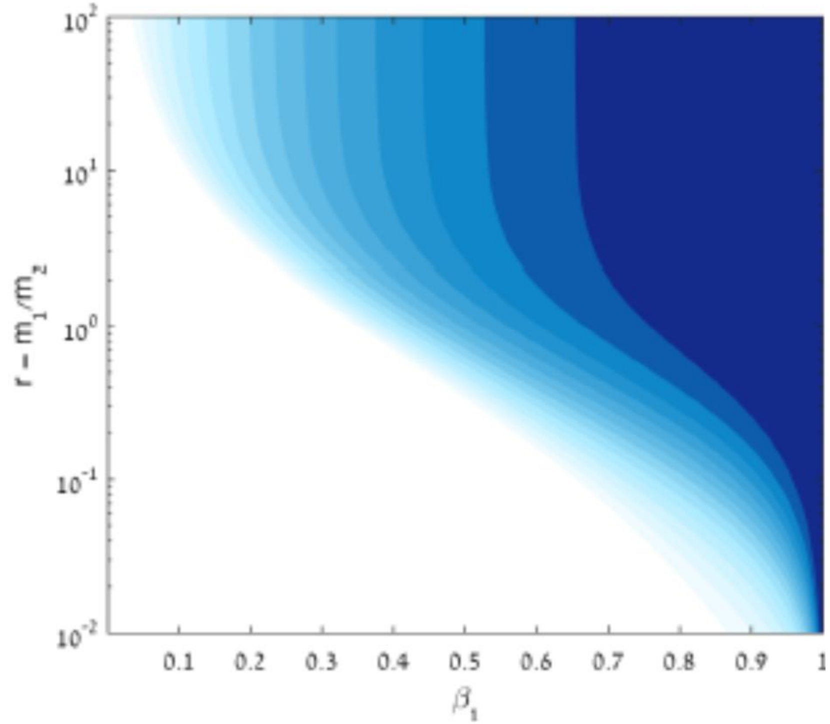
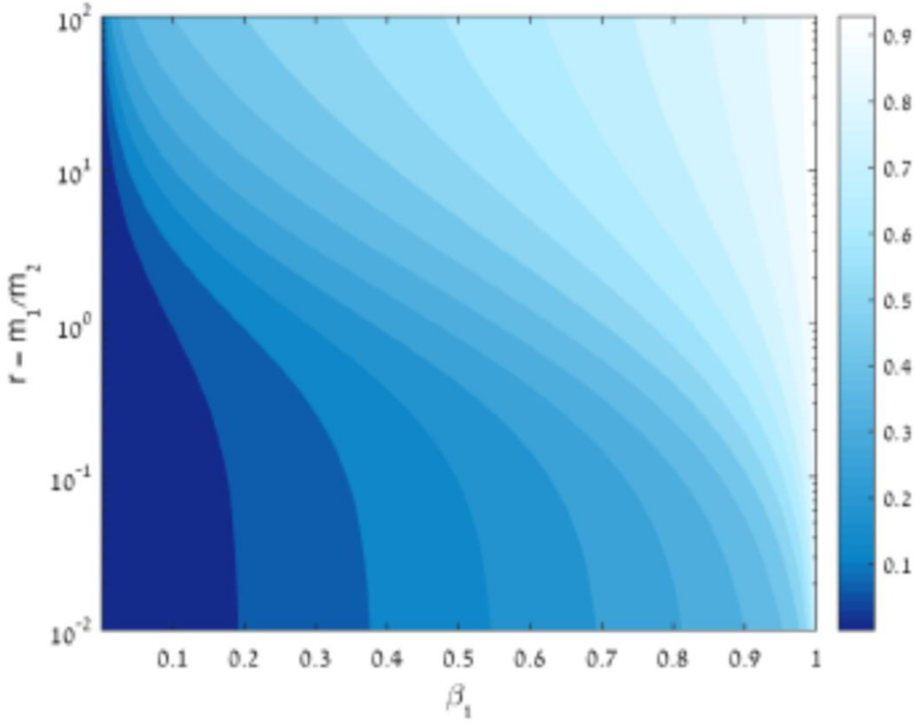


Transverse

planar

The asymmetric intercept

- The intercept



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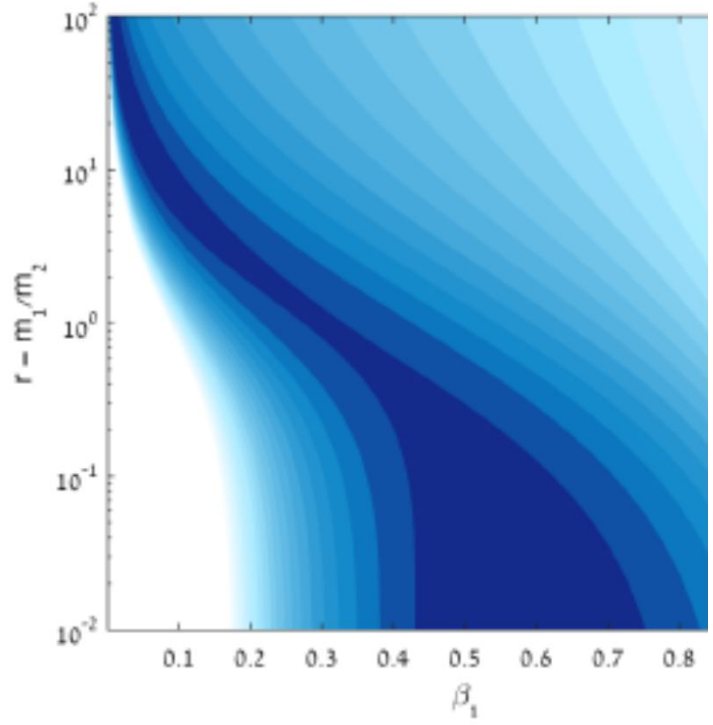


(4)

contribution of PS

full intercept

Summary and open questions

- We quantized the bosonic string with massive endpoints.
- This is in fact a quantization of the hadronic string
- The trajectories that follows from the model fit very nicely the hadronic spectra
- The decay width of hadron were also computed and their fit to the data is also good
- We are currently computing the analog of the Veneziano amplitude for scattering of massive strings.
- Adding charges and spins are next to do facing the challenge of getting negative intercept
- Many more hadronic data to explain

HISH

- For strings with massive endpoints one determine the solution of the classical EOM that corresponds to a rotating string
- The classical energy and angular momentum

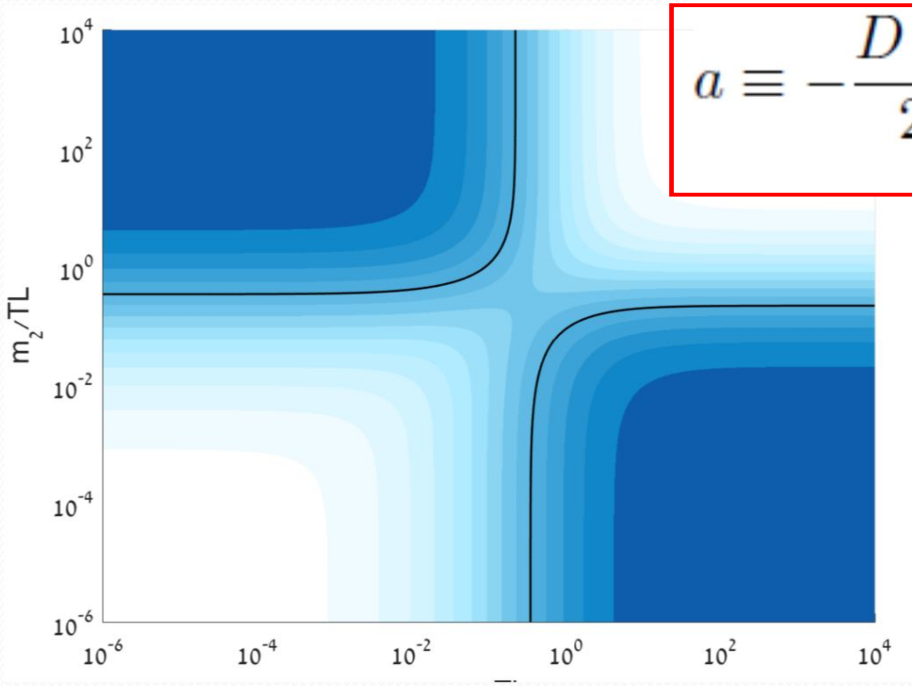
$$E = \sum_{i=1,2} \left(\gamma_i m_i + T\ell_i \frac{\arcsin \beta_i}{\beta_i} \right)$$

$$J = \sum_{i=1,2} \left[\gamma_i m_i \beta_i \ell_i + \frac{1}{2} T \ell_i^2 \left(\arcsin \beta_i - \beta_i \sqrt{1-\beta_i^2} \right) \right]$$

- The quantum intercept for a static string $J \to J - a.$

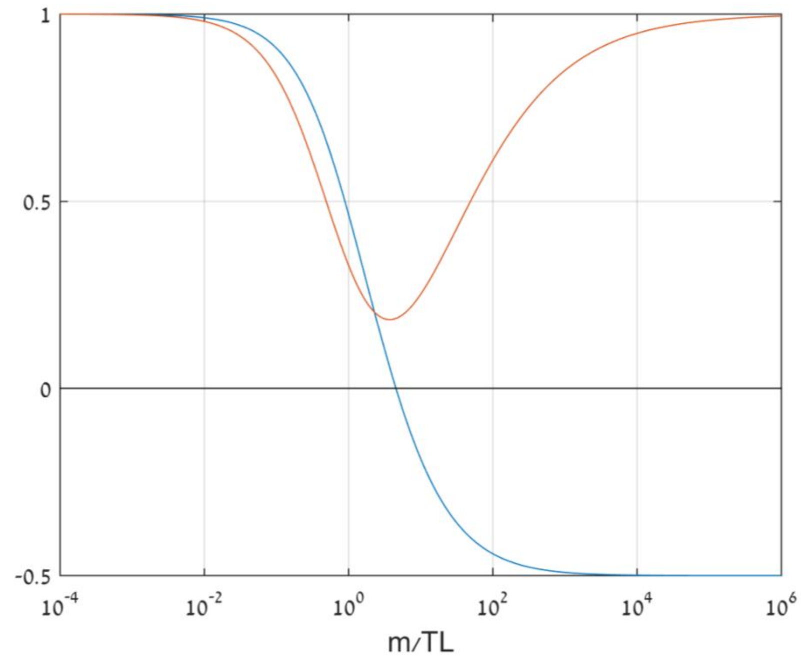
$$\hat{a}(q_1, q_2) = \frac{1}{2\pi^2} \int_0^\infty dz \log \left[1 - e^{-2z} \left(\frac{q_1 - z}{q_1 + z} \right) \left(\frac{q_2 - z}{q_2 + z} \right) \right]$$

The intercept for a string with massive endpoints



$$a \equiv -\frac{D-2}{2\pi}\sum_{n=1}^{\infty}\omega_n$$

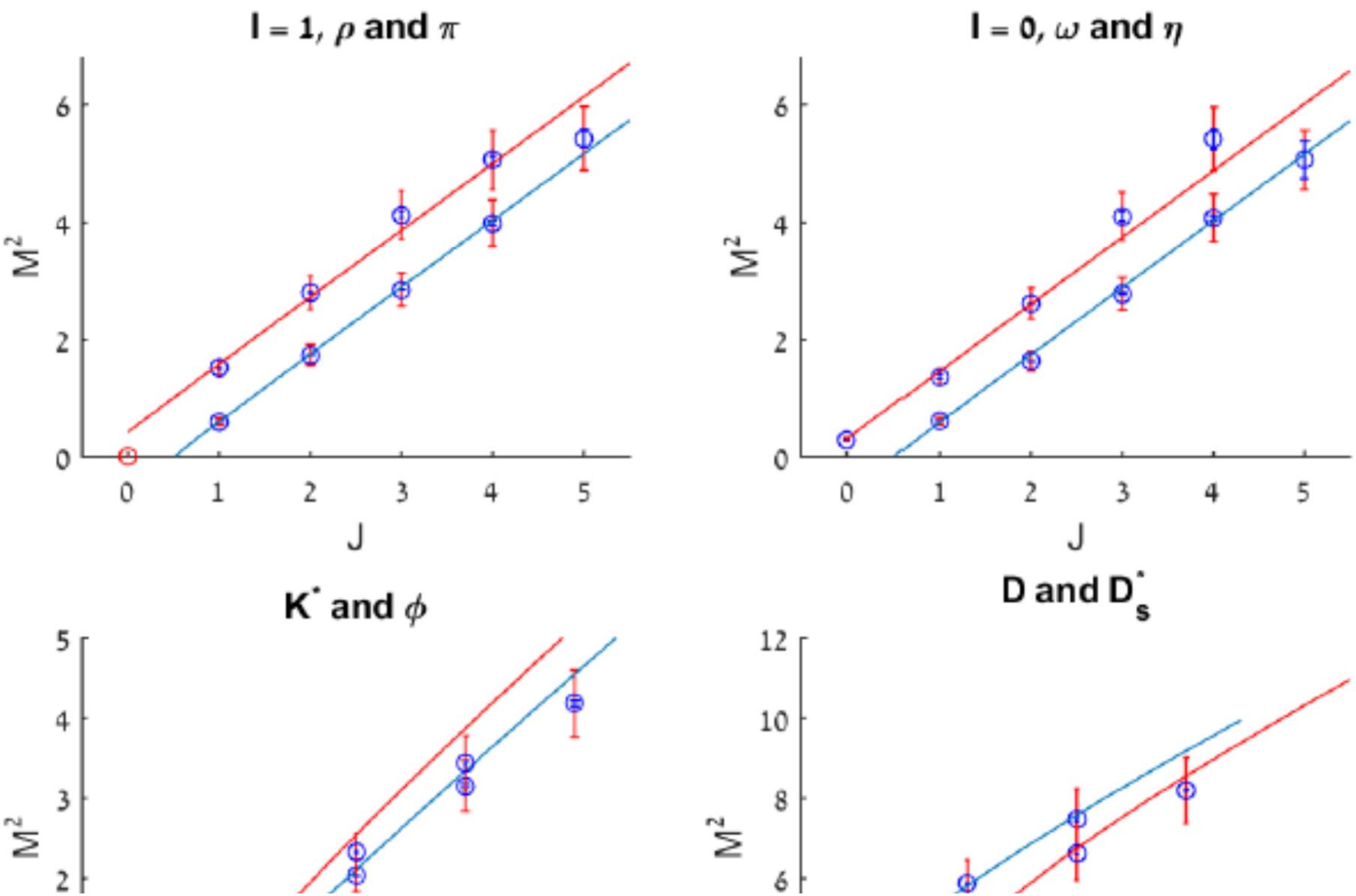
The intercept of a static str
as a function of the masse



$$\tan(\omega_n) = \frac{2q\omega_n}{q^2\omega_n^2 - 1}$$

The spectra fits

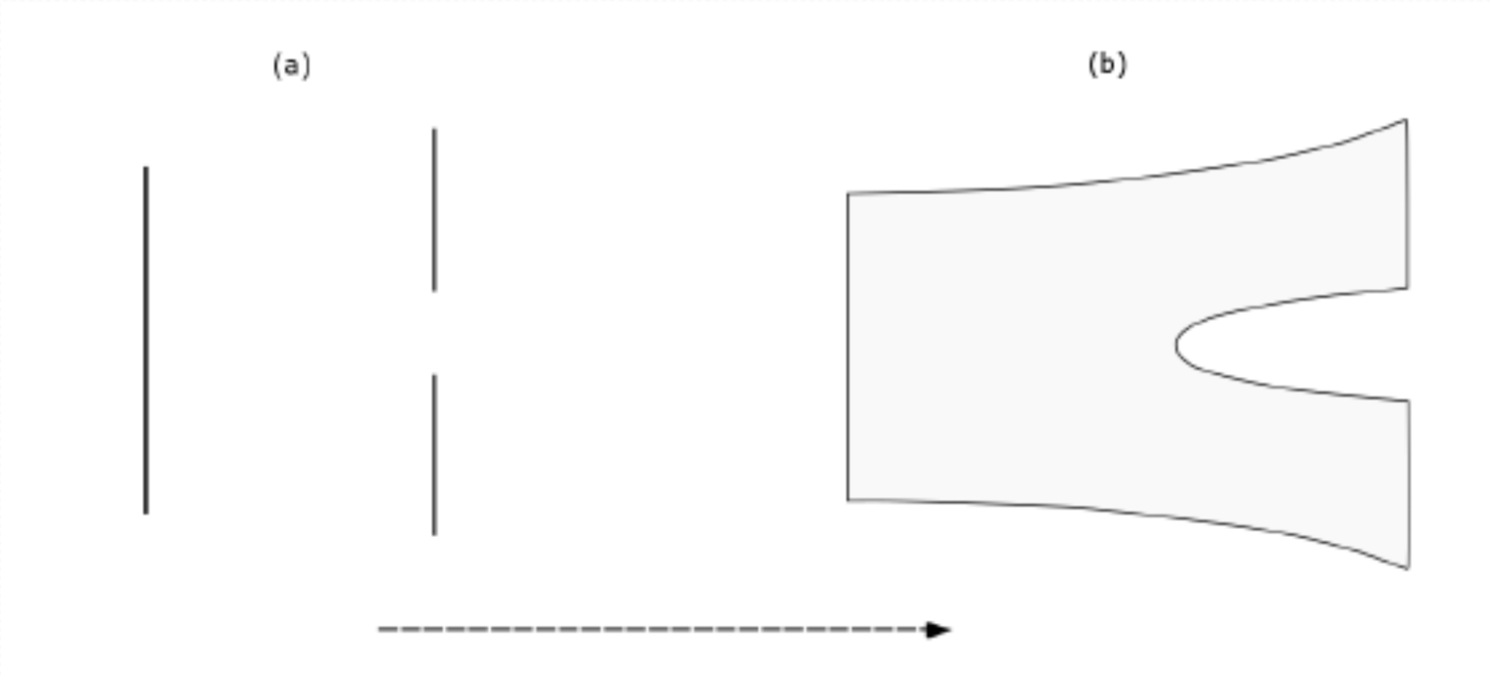
- The best fits of HISH to meson states



The decay of the hadronic string

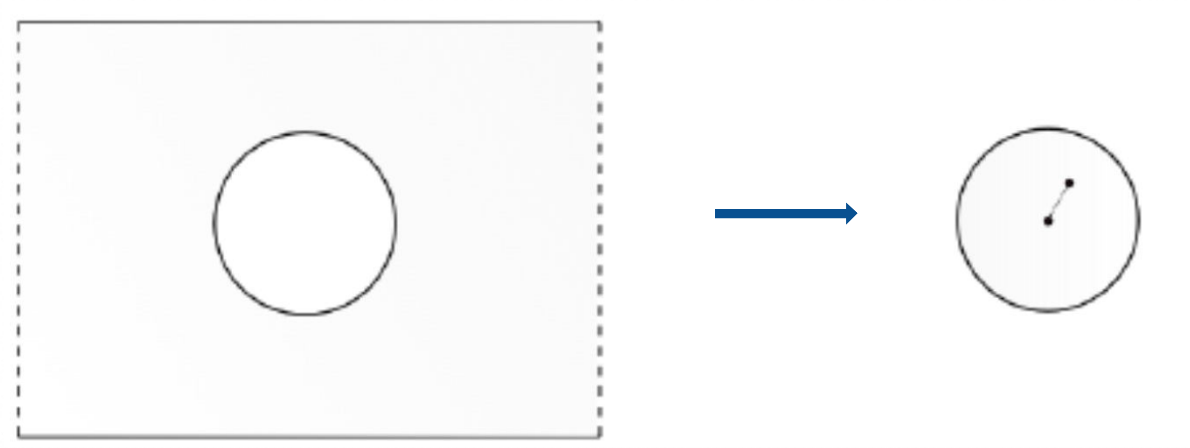
The decay of a long string in flat space-time

- The decay of a hadron is in fact the **breaking of a string into two strings**
- Obviously a type I open string can undergo such a split



The decay of a long string in flat space-time

- The total decay width is related by the optical theorem to the imaginary part of the self-energy diagram
- A trick that Polchinski et al used is to compactify one space coordinate and consider incoming and outgoing strings that wrap this coordinate so one can use the simple vertex operator of a closed string



The decay of a long string in flat space-time

- We would like to determine the dependence of the string amplitude on the string length L

$$\boxed{i\mathcal{A}_2 = \frac{iTN}{g^2} L \left[\frac{\kappa}{2\pi\sqrt{L}}\right]^2 \int_{|z|<1} \mathrm{d}^2 z \, \langle : e^{ip\cdot X(0)} :: e^{-ip\cdot X(z)} : \rangle}$$

open string coupling

Zero mode

Gravitational coupling

Normalization Of the vertex

Vertex operator

The decay of a long string in flat space-time

- A further dependence on L comes from the energy and momenta

$$P_L = (E, LT, 0) \qquad P_R = (E, -LT, 0) \qquad E = \sqrt{(TL)^2 - 8\pi T}$$

For open strings $\qquad \frac{a}{\alpha'} = 2\pi T\, a = \frac{D-2}{24} = 1$

For closed strings the tension and intercept are twice

The decay of a long string in flat space-time

- Using the vertex operator

$$\boxed{e^{iP\cdot X} = e^{i(P_L\cdot X_L + P_R\cdot X_R)}}$$

and the standard OPE

$$\boxed{\begin{aligned}\langle : e^{iP\cdot X(0)} :: e^{iP\cdot X(z)} : \rangle &= z^{-\frac{P_R^2}{4\pi T}} \bar{z}^{-\frac{P_L^2}{4\pi T}} (1 - z\bar{z})^{\frac{-P_R\cdot P_L}{4\pi T}} \\ &= |z\bar{z}|^{-2}(1 - z\bar{z})^{\tilde{J}}\end{aligned}}$$

$$\tilde{J} \equiv \frac{L^2 T}{2\pi} - 2$$

The decay of a long string in flat space-time

- After substituting the amplitude reads

$$\boxed{i\mathcal{A}_2 = \frac{iTN\kappa^2}{2\pi g^2}\lim_{t\to 0}\frac{\Gamma(t-1)\Gamma(1-\tilde{J})}{\Gamma(t-\tilde{J})}}$$

$$\boxed{= \frac{iTN\kappa^2}{2\pi g^2}\left(\tilde{J}\partial_{\tilde{J}}\ln[\Gamma(-\tilde{J})] + \lim_{t\to 0}\frac{\tilde{J}}{t}\right)}$$

regulator

- The imaginary part $\sum_k \pi k\delta(J-k)$ for $k=1,\ldots$

$$\boxed{\mathrm{Im}\mathcal{A}_2 = -\frac{iTN\kappa^2}{2g^2}\tilde{J}}$$

The decay of a long string in flat space-time

- Since A2 is the mass square shift the total decay width

$$\boxed{\Gamma = -\mathrm{Im}\delta(m) = -\mathrm{Im}\frac{1}{2m}\delta(m^2) = \frac{TN\kappa^2}{4g^2}\frac{\tilde{J}}{E}}$$

- The leading behavior for string in d=26 is

$$\boxed{\frac{\Gamma}{L} = \frac{g^2T^{13}N}{4(4\pi)^{12}}}$$

$$\boxed{\Gamma = \frac{TN\kappa^2}{4g^2}\left[L_{tot} + \frac{4\pi}{T}\frac{1}{L_{tot}}\right]}$$

$$L_{tot} = \sqrt{L^2 - \frac{8\pi}{T}}$$

The decay of rotating and excited strings

- For a rotating string due to time dilation we get

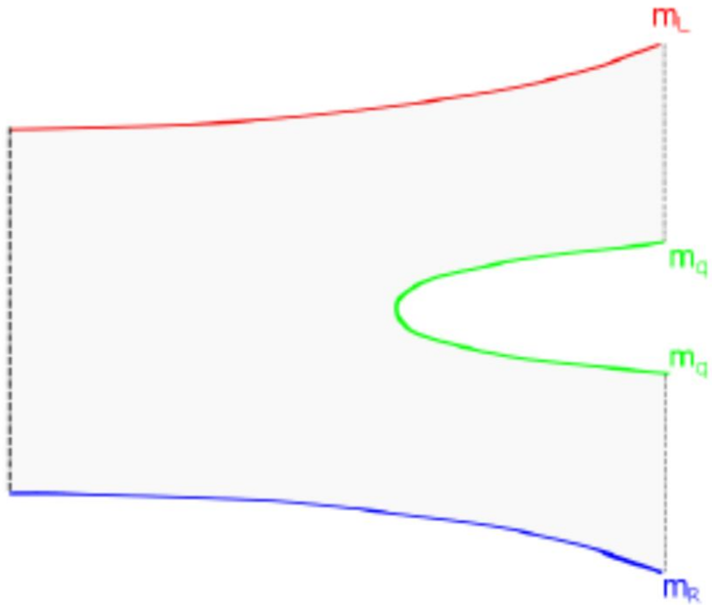
$$\boxed{\Gamma = \left(\frac{\Gamma}{L}\right)_{\text{stat}} \int_{-L/2}^{L/2} d\sigma \sqrt{1-(\sigma w)^2} = \frac{\pi}{4}\left(\frac{\Gamma}{L}\right)_{\text{stat}} L}$$

- For nth excited string

$$\boxed{\Gamma_n = \left(\frac{\Gamma}{L}\right)\sqrt{\frac{2\pi(n-a)}{T}}}$$

The decay width of a string with massive endpoints

- The decay of a string with **massive particles on its ends**



- The dependence on the masses:

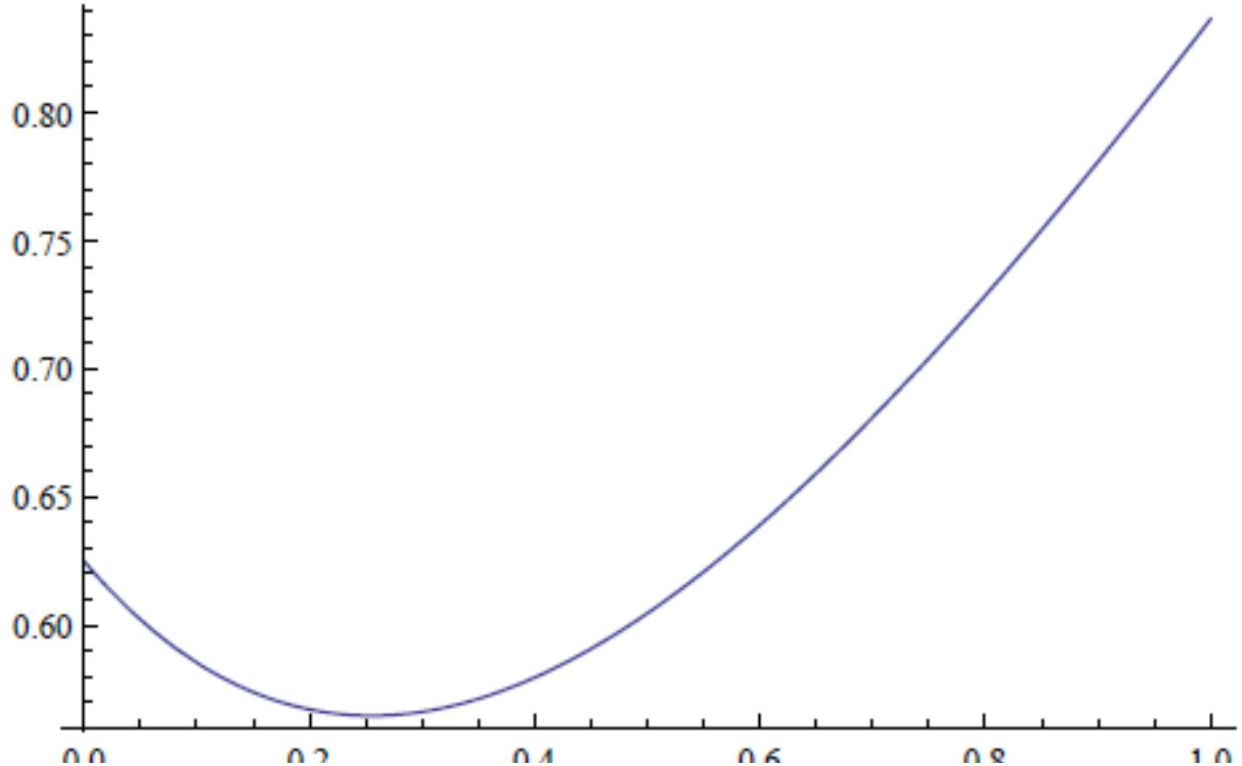
  (a) The length L(m1,m2)

  (b) The boundary conditions (not anymore Neuman)

The decay width of a string with massive endpoints

- For small endpoint masses we can expand *

$$\Gamma \propto \frac{\pi}{4}TL + \frac{\pi}{4}m - \frac{2\sqrt{2}}{3}m^{3/2}(TL)^{-1/2} + \mathcal{O}(L^{-3/2})$$



The decay width in non-critical dimensions

- N. Turok et all analyzed the decay width of **open string in d dimension**. They got

$$\boxed{\Gamma \sim L^{\frac{D-14}{12}} = L^{\frac{D-2}{12}-1}}$$

- Thus linearity for d=26 but $\Gamma \sim L^{-\frac{5}{6}}$ in D=4.
- But this analysis took only the transverse modes. Their result follows from $\boxed{\mathrm{Im}[\delta(m^2)] \sim t^{\frac{D-2}{24}} = t^a}$
- It was shown by Hellerman et al that the **intercept**

$$\boxed{a = a_{cr} + a_{PS} = \frac{(D-2)}{24} + \frac{(26-D)}{24} = 1}$$

- Thus for any d dimension $\boxed{\Gamma \sim \frac{t^a}{E} \sim L^1}$

The decay of a stringy hadron

- We just argued that the **intercept** of a string at D dim

  a=1

- In fact **experimental** value of the intercept a_{exp} is negati

$$a_{exp} = -|a_{exp}|$$

- Thus the leading order width of a string with no massive endpoints

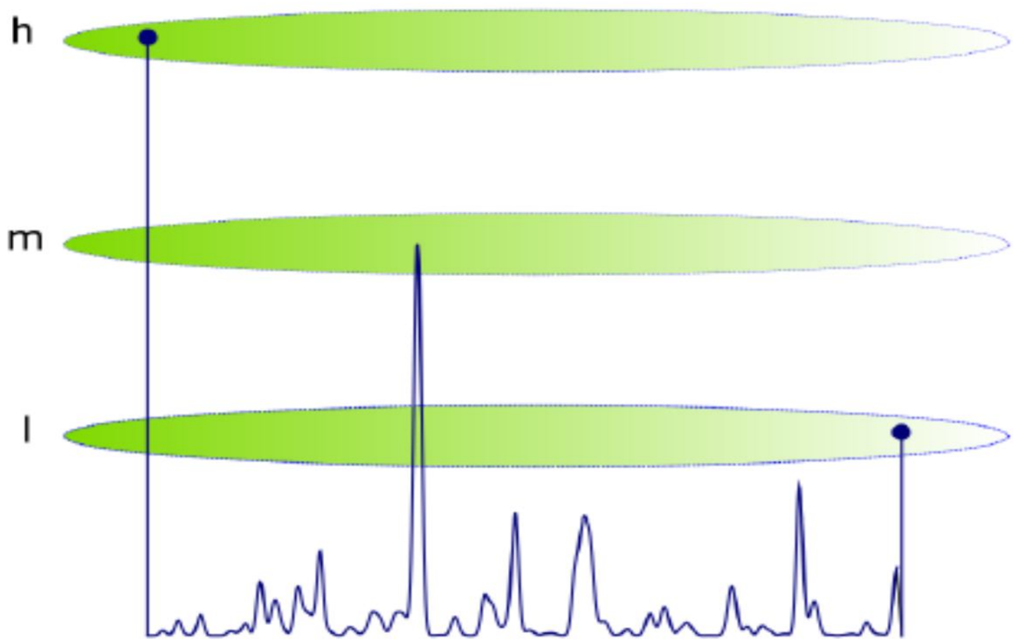
$$\boxed{\Gamma \sim \frac{N\kappa^2}{4g^2} TL \left[1 + \frac{4|a_{exp}|^2}{\alpha'^2 (TL)^4} + ...\right]}$$

- With massive endpoint we combine this with *

Exponential suppression of pair creation

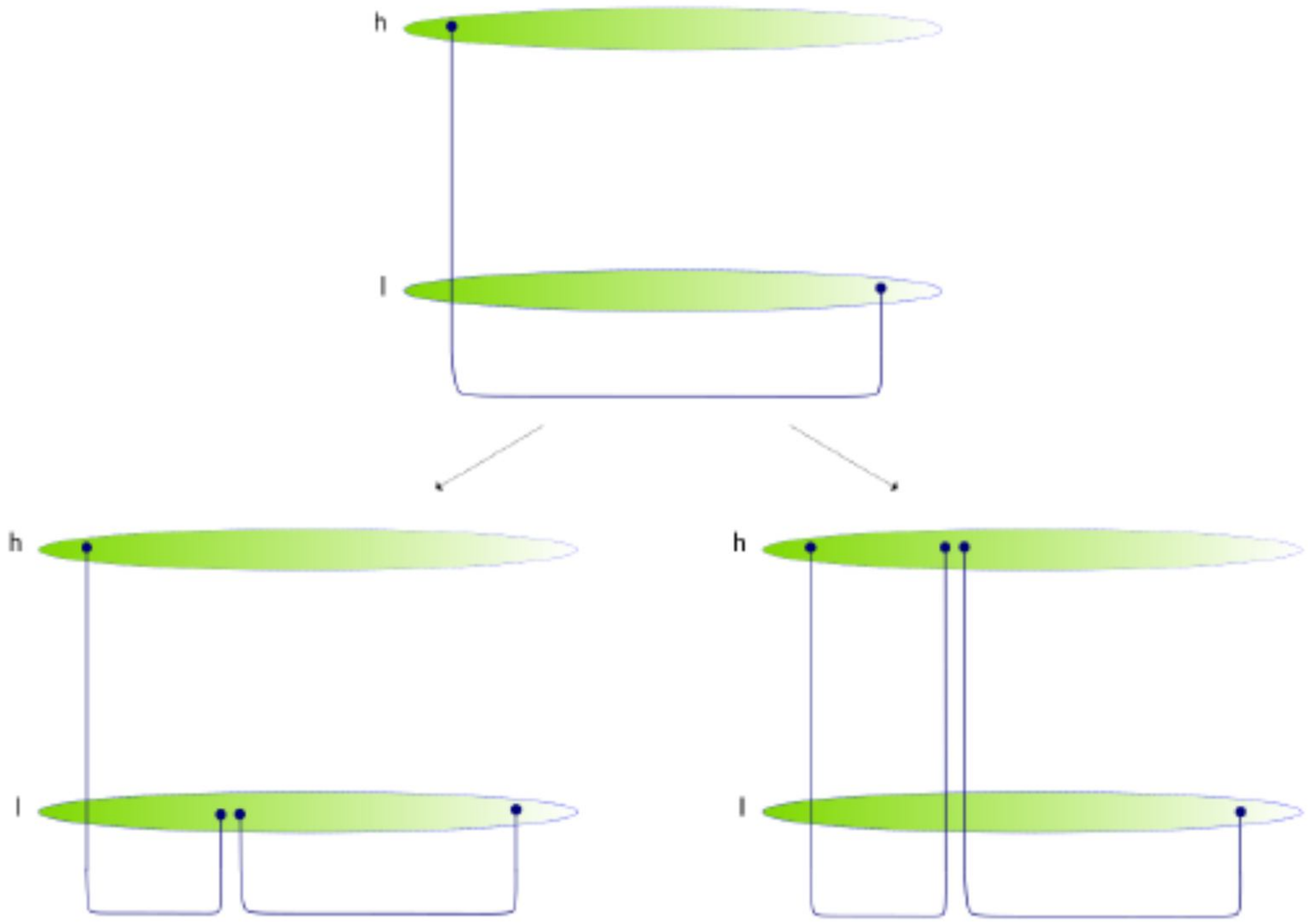
The suppression factor for stringy holographic hadrons

- The horizontal segment of the stringy hadron **fluctuates** and can reach flavor branes
- When this happens the string may **break up** , and the two new endpoints connect to a flavor brane



The suppression factor for stringy holographic hadrons

- There are in fact several possible **breakup patterns**



Determination of the suppression factor

- Assuming first that the string stretches in flat space-time we found (J.S, K. Peeters , M. Zamamklar) using both a string beads model and a continues one that

$$\Gamma = \text{const.} \cdot \exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right) \cdot T_{\text{eff}}\,\mathcal{P}_{\text{split}} \cdot L$$

$$\exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right) = \exp\left(-2\pi\,\frac{m_{sep}^2}{T_{\text{eff}}}\right)$$

- There are further corrections due to the curvature and due to the massive endpoints.

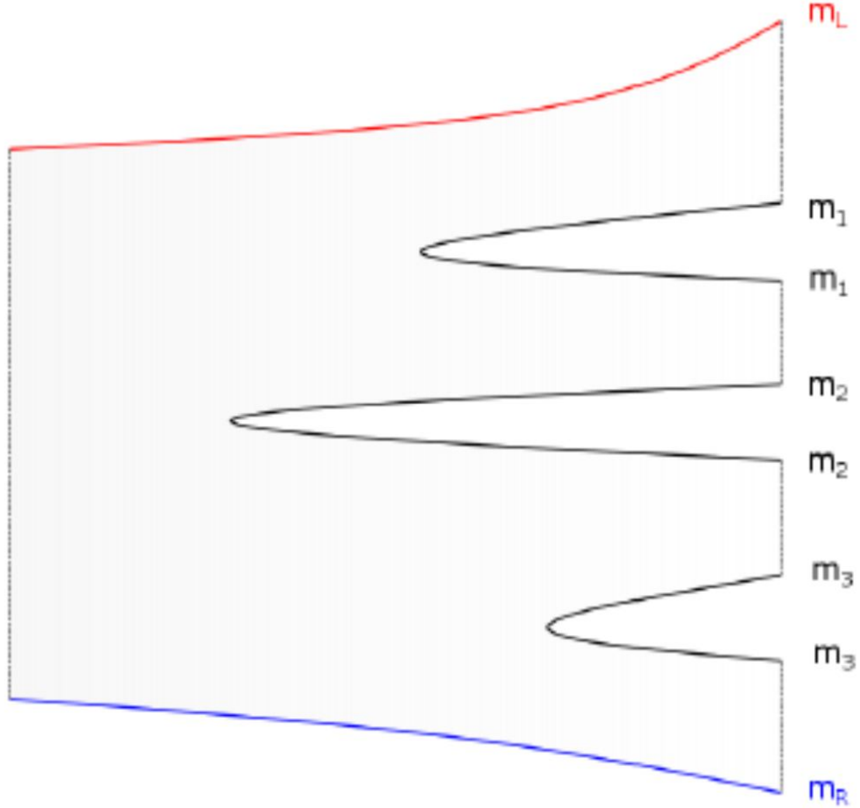
- $\Gamma = \exp\left(-2\pi C(T_{\text{eff}}, M, m_i)\frac{m_{sep}^2}{T_{\text{eff}}}\right)$

$$C(T_{\text{eff}}, M, m_i) \approx 1 + c_c\frac{M^2}{T_{\text{eff}}} + \sum_{i=1}^{2} c_{m_i}\frac{m_i}{M}$$

Multi string breaking and string fragmentation

- The basic process of a string splitting into two strings can of course repeat itself and thus eventually describe a decay of a single string into n strings
- The probability for a multi-decay

$$\boxed{\mathcal{P} = \frac{T_{\text{eff}}^2}{\pi^3} \sum_i \sum_{\omega_n=1}^{\infty} \frac{1}{\omega_n^2} \exp\left(-2\pi C \frac{m_{sep_i}^2 \omega_n}{T_{\text{eff}}}\right)} .$$

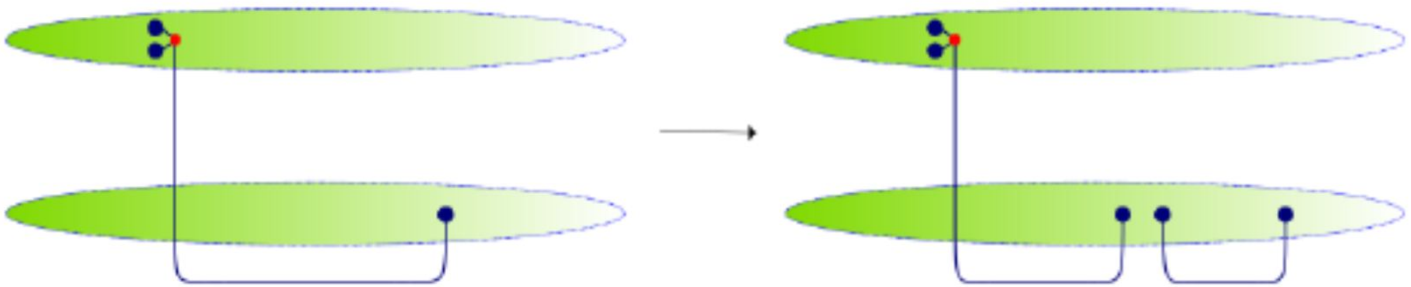


This mechanism is believed to be the **generator of jets**. It is incorporated in Pithya

The Decay process of the different types of hadrons

The decay process of Baryons

- A baryon in HISH is a string connected to a qurak and to a di-quark so its decay is also by a string splitting

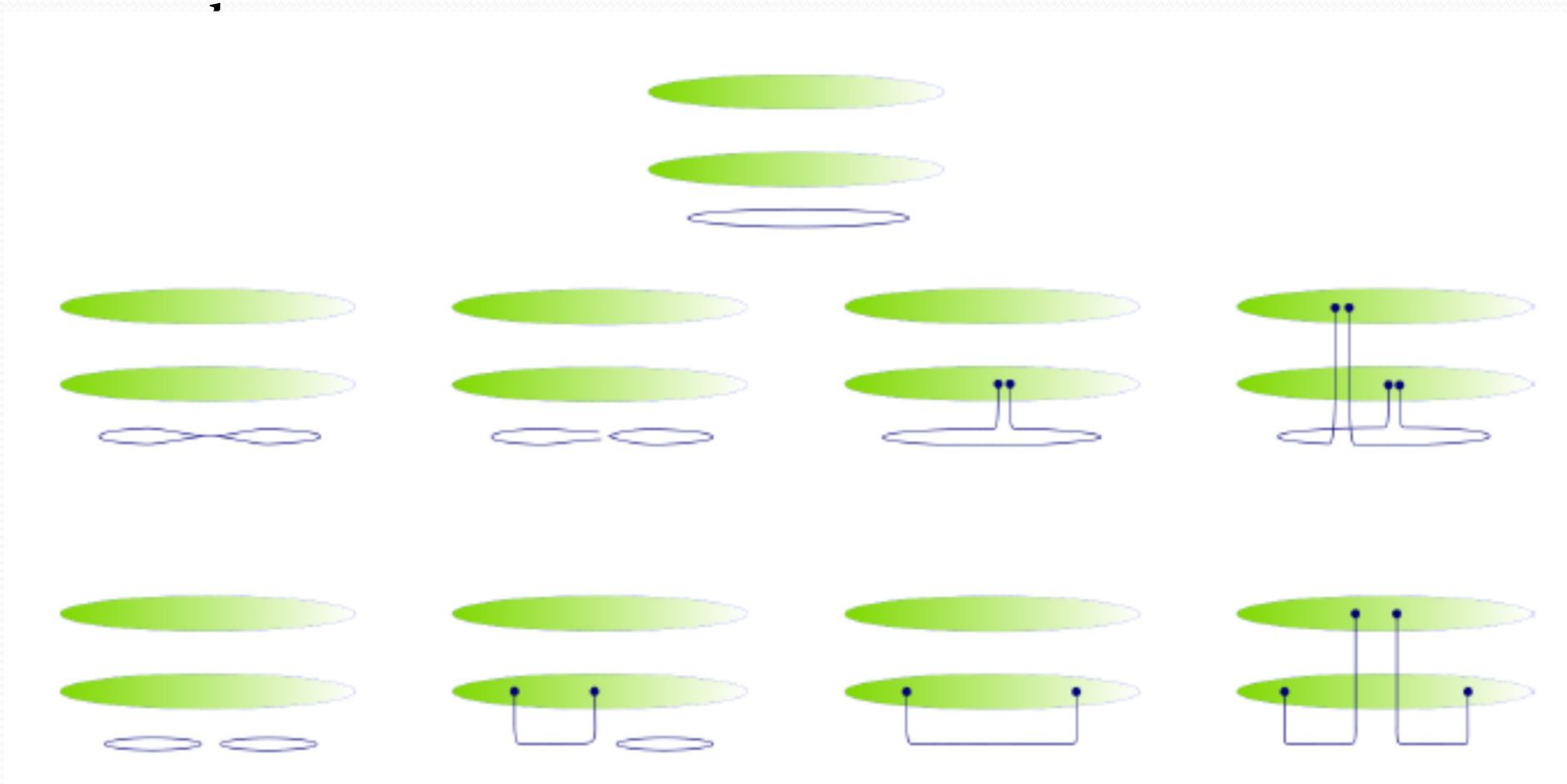


- A way to determine what is the diquark pair and which is the stand-alone quark is by identifying the decay products

$$
\begin{array}{ccc}
[(q_1 q_2) q_3] & [(q_1 q_3) q_2] & [(q_2 q_3) q_1] \\
\Downarrow & \Downarrow & \Downarrow \\
[(q_1 q_2) Q_i][\bar{Q}_i q_3] & [(q_1 q_3) Q_i][\bar{Q}_i q_2] & [(q_2 q_3) Q_i][\bar{Q}_i q_1]
\end{array}
$$

Decay of glueballs

- The glueball which is a folded rotating closed string



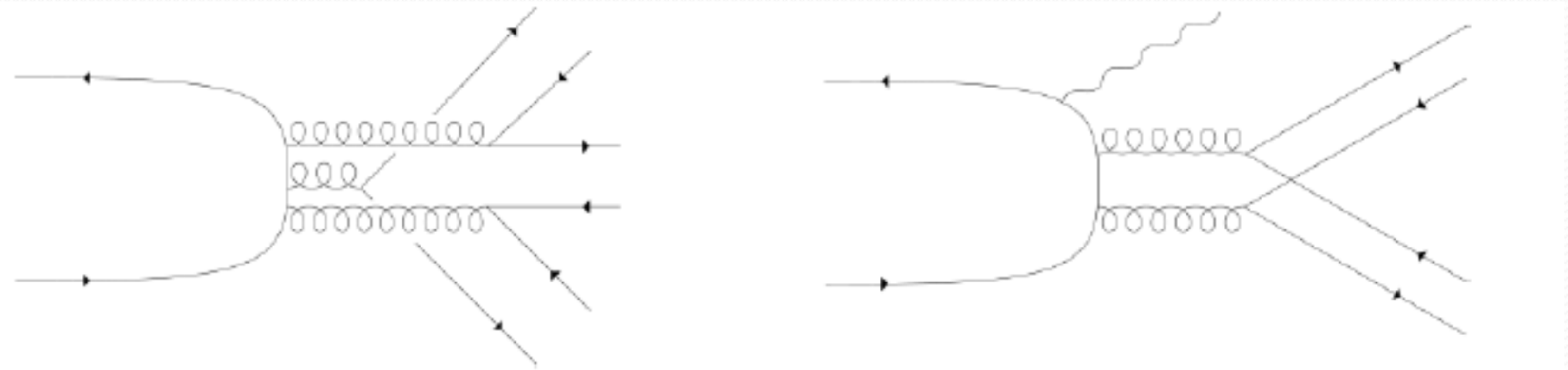
- The width

$$\Gamma_a \sim L\Gamma_{\text{cross}} \qquad \Gamma_b \sim L\Gamma_{\text{cross}} e^{-2\pi C m_{sep}^2/T}$$

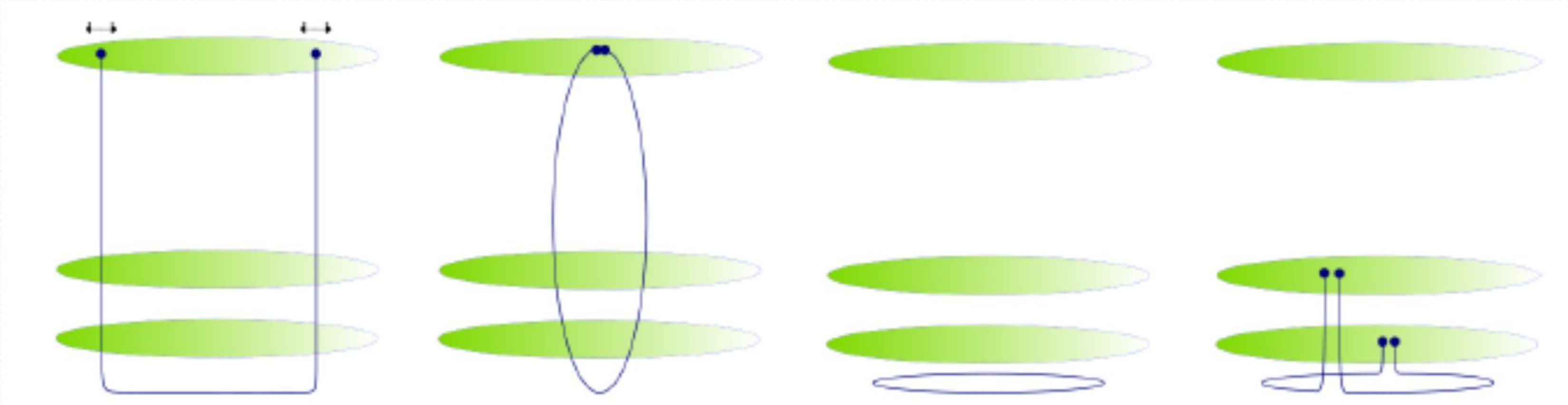
$$\Gamma_c \sim L e^{-2\pi C m_{sep}^2/T} \qquad \Gamma_d \sim L^2 e^{-2\pi C m_{sep}^2/T} e^{-2\pi C m'^2_{sep}/T}$$

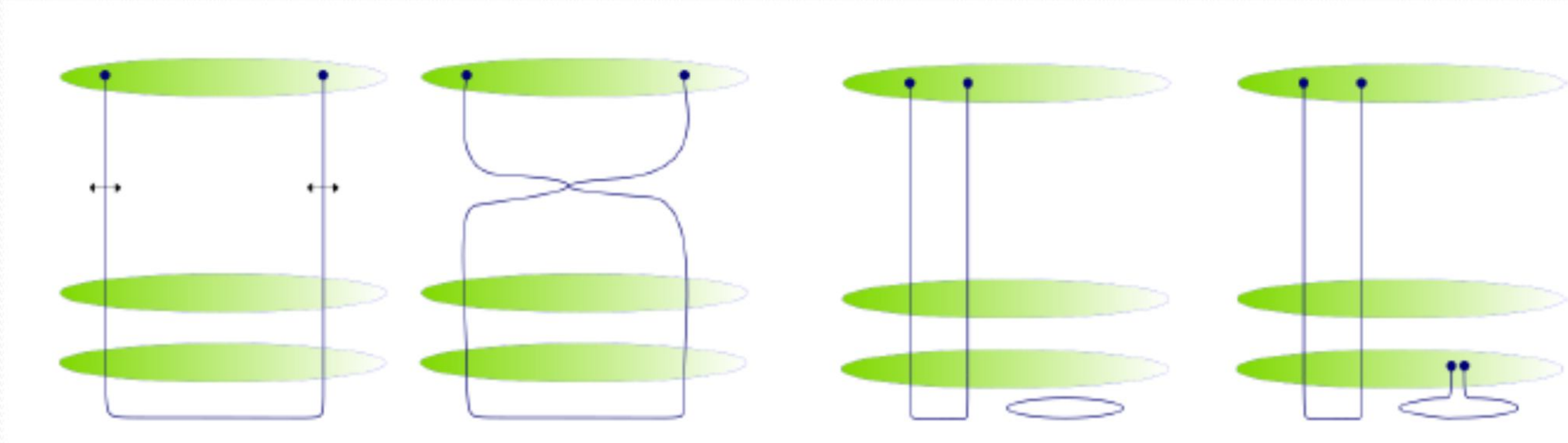
Zweig suppressed decay channels

- Certain heavy quarkonia mesons, build out of $c\bar{c}$ or $b\bar{b}$, cannot decay via the mechanism of breaking apart of the horizontal string
- In QCD the decay based of the annihilation of the pair into 3 gluons or e 2 gluons and a photon



Zweig suppressed decay channels



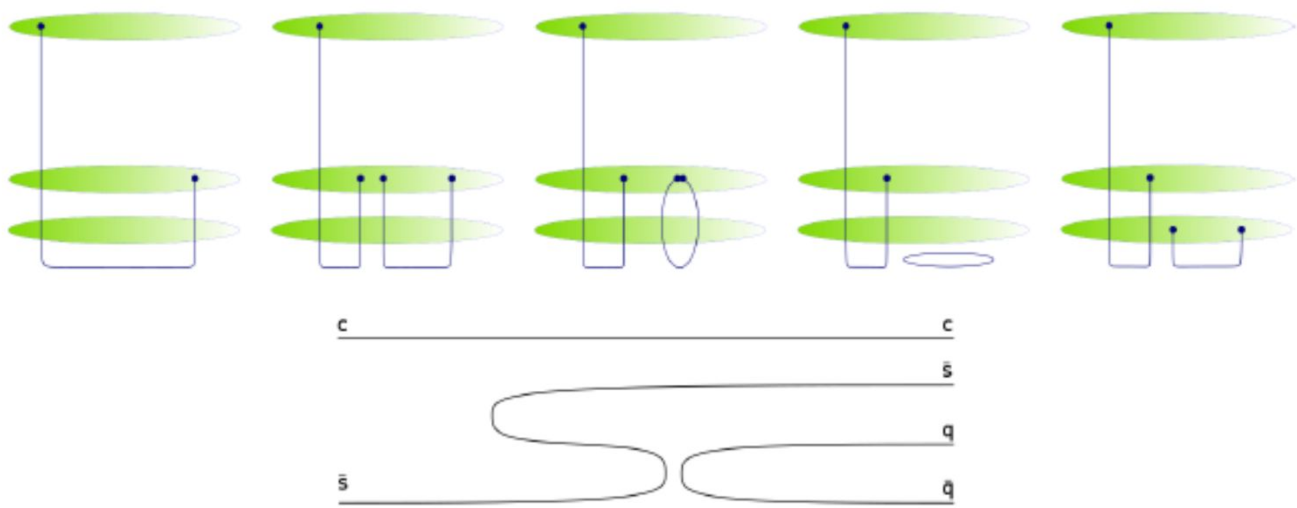


Zweig suppressed decay channels

- An approximation for probability of process a

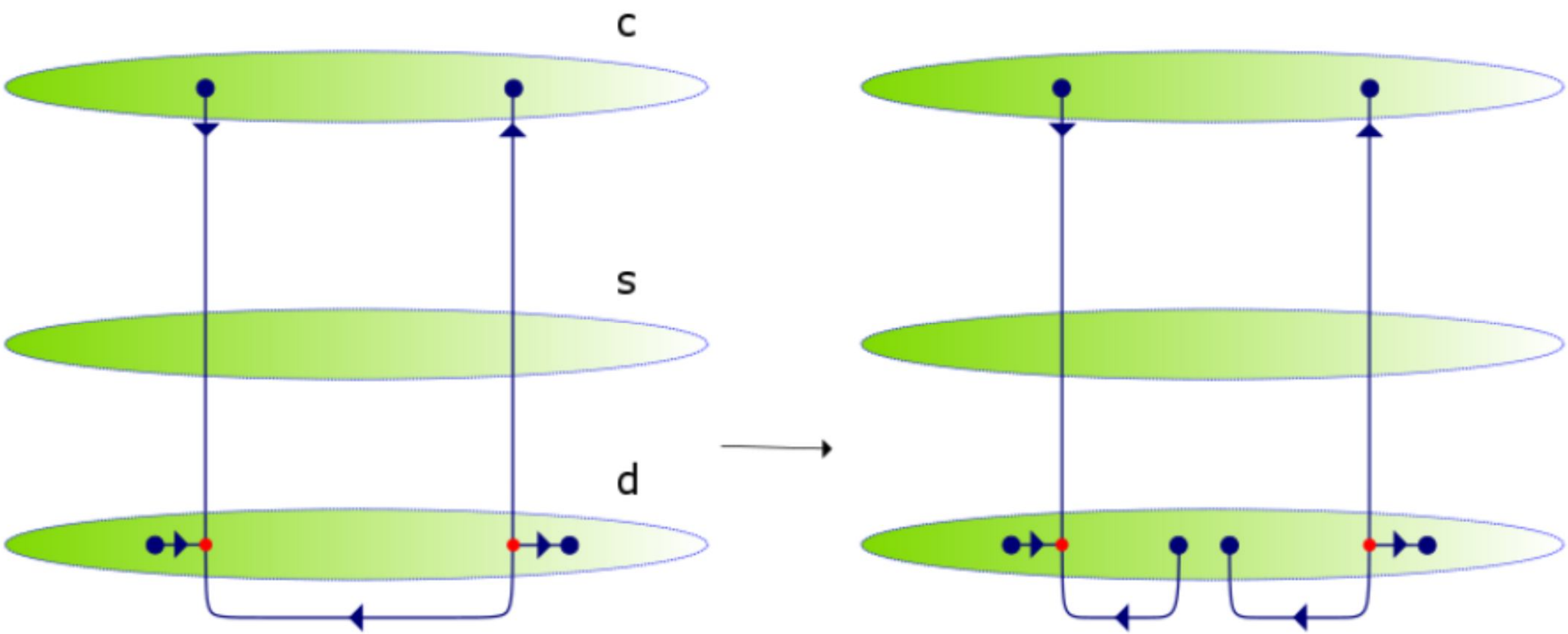
$$
\begin{aligned}
\mathcal{P} &= \int_{-\infty}^{\infty} dx\, \psi(x - L/2)\psi(x + L/2) = \\
&= \int_{-\infty}^{\infty} dx\, \exp[-T_{av}(x - L/2)^2]\exp[-T_{av}(x + L/2)^2] = \\
&= \sqrt{\frac{\pi}{2T_{av}}} e^{-T_{av}L^2/2} = \sqrt{\frac{\pi}{2T_{av}}} e^{\frac{4(M-2m)^2}{9T_{av}}}
\end{aligned}
$$

- **Virtual pair** combined with a **Zweig suppressed**



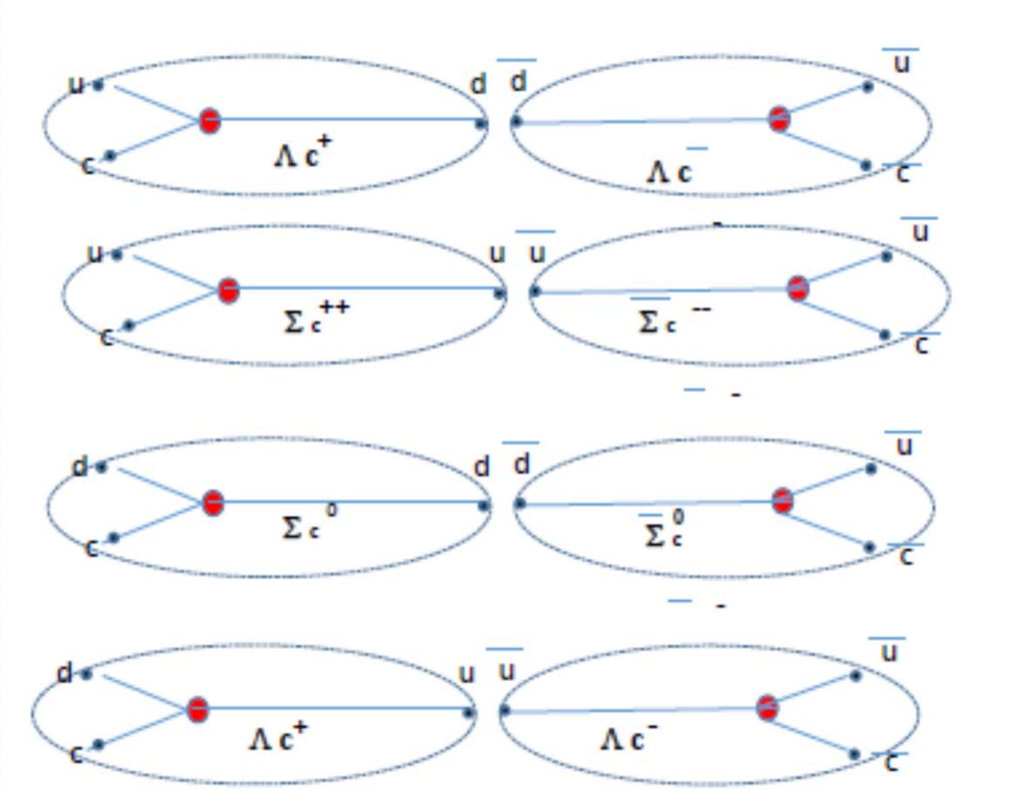
Decays of exotic hadrons

- An exotic tetraquark built from a string connecting a di-quark and and anti di-quark will decay predominantly to a baryon anti-baryon



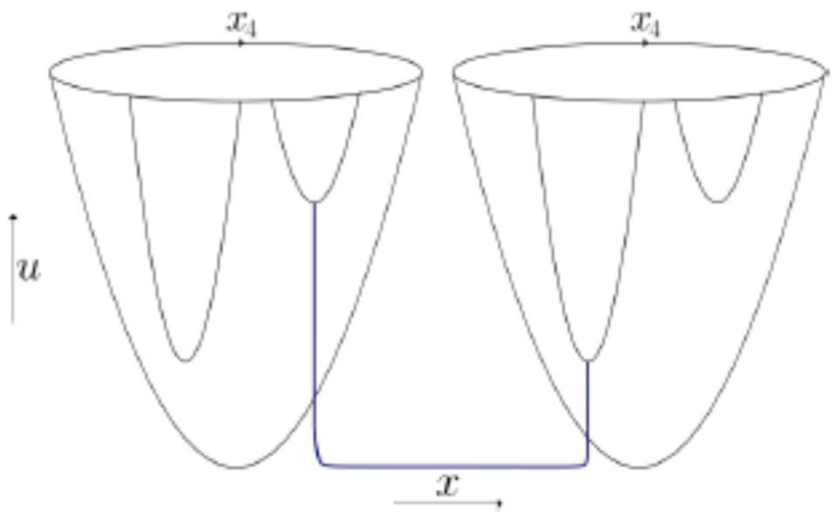
Decays of exotic hadrons

- The HISH picture of possible decays



Decays via breaking of the vertical segment

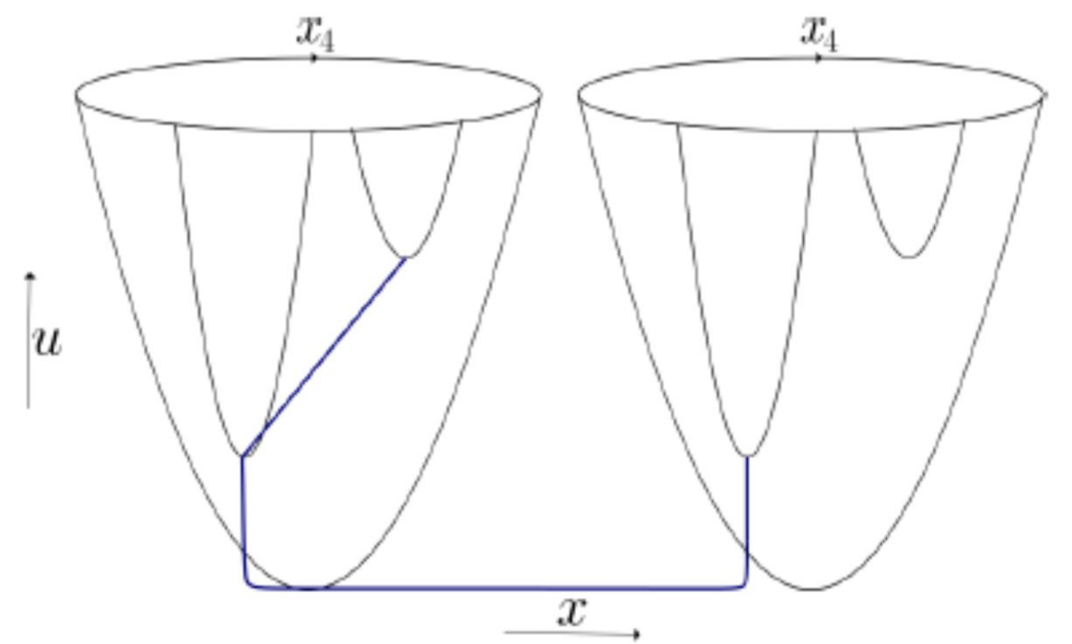
- Nothing prevents a breaking of the vertical segments. What is the hadronic interpretation of it?
- We first clarify the holographic set up of hadrons



- So the vertical segment of a heavy flavor does not cross that of a lighter flavor brane.

Decays via breaking of the vertical segment

- The vertical segment can split as follows



- We get a meson plus a string that stretches only in the radial and x4 but not is space-time coordinates

Decays via breaking of the vertical segment

- In a similar way to the computation of the width associated with the breaking of the horizontal segment, the width associated with the **breaking of a vertical segment** should be

$$\boxed{\Gamma_{\text{vertical}} \sim \int_{u_\Lambda}^{u_B} du \exp\left(-C_v \frac{(\Delta_{x_4}(u))^2}{\alpha'(u)}\right)}$$

- The interpretation of the decay processes associated with the vertical breaking is not well understood.
- One possibility is that the vertical string segments that are ``particles" from the space-time point of view are the **Goldstone boson mesons** pions and kaons.

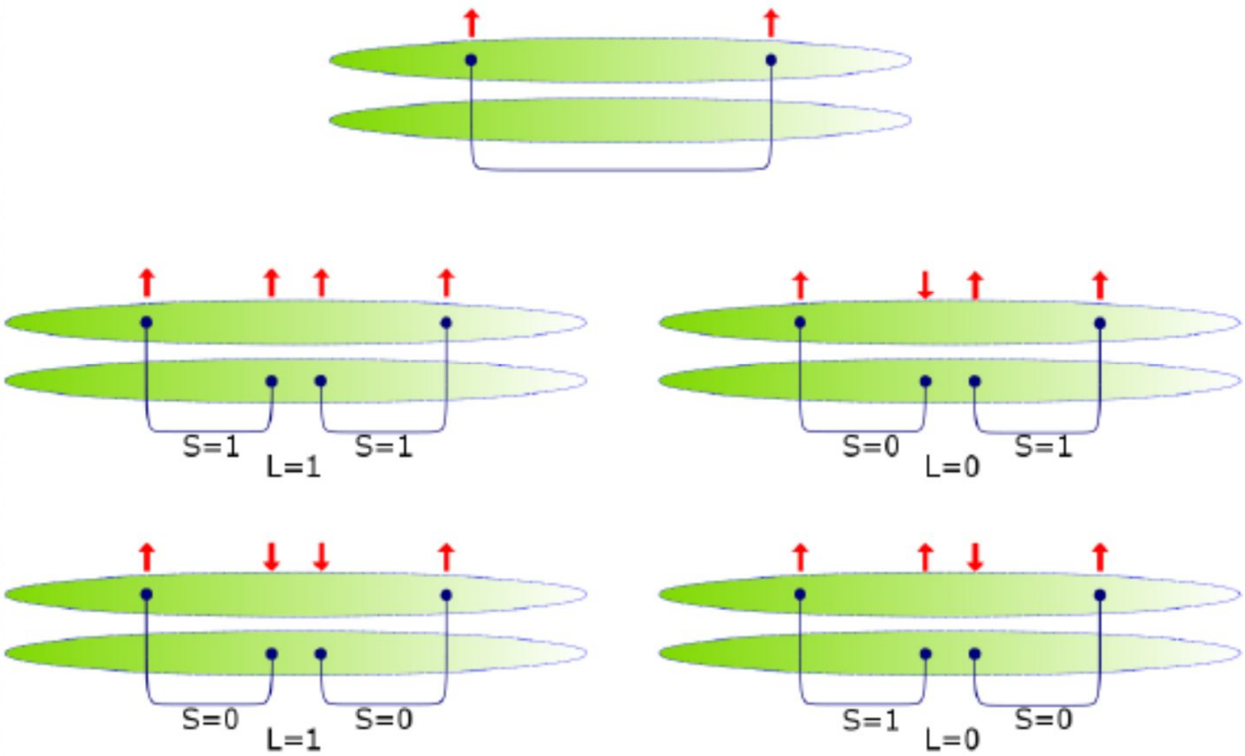
The Decay modes spin and flavor symmetry

Decay modes, spin, and avor symmetry

- Considerations of spin and isospin or more generally flavor symmetry of the initial and final states are very important in determining which decays are forbidden and the relative decay width of the allowed ones.
- How are such considerations been realized in the holographic decay mechanism of stringy hadrons.
- The spectra of hadrons is slightly affected by spin and isospin via the dependence of the intercepts .
- This issue has to be further studied

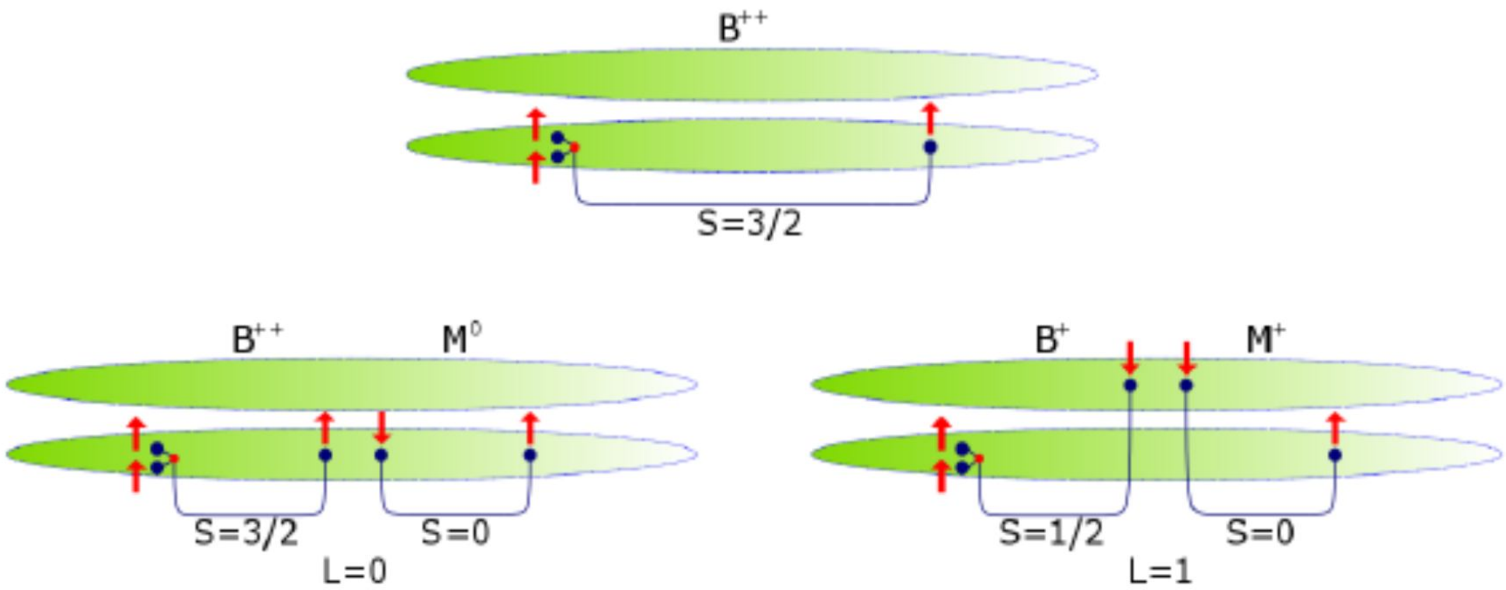
The spin structure of the stringy decays

- We assume that the **spin** degrees of freedom are carried by the **particles** that are on the string endpoints
- The spin structure of allowed decays of a neutral meson Mo with spin S=1 into M+ and M_ mesons. The arrows indicated the values of Sz



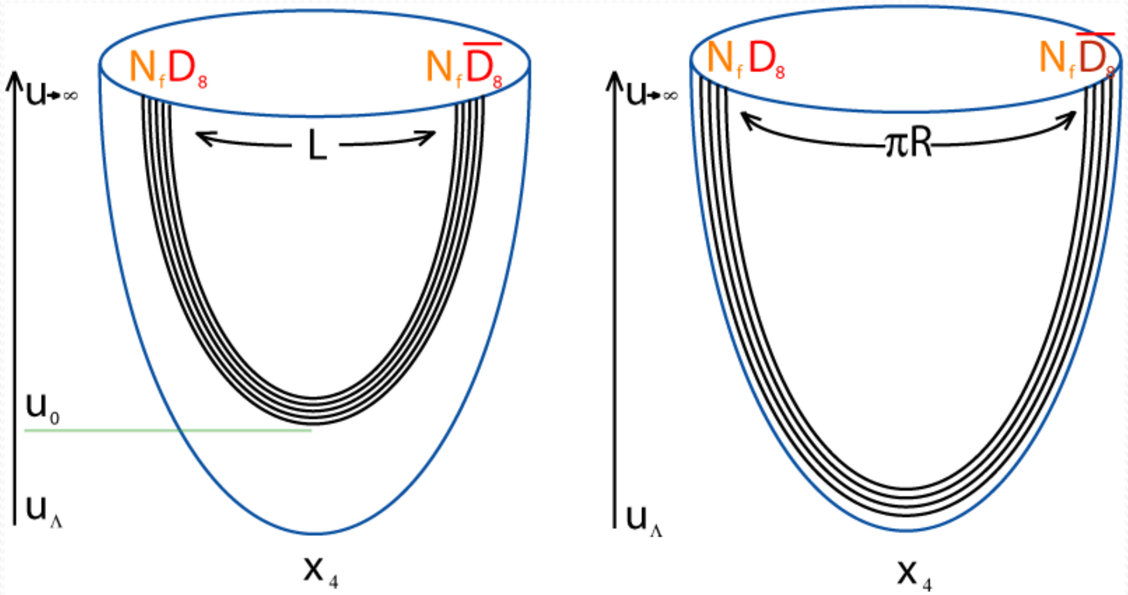
The spin structure of the stringy decays

- The spin structure of allowed decays of doubly charged baryon B++ with spin S = 3/2 into a baryon and a meson.



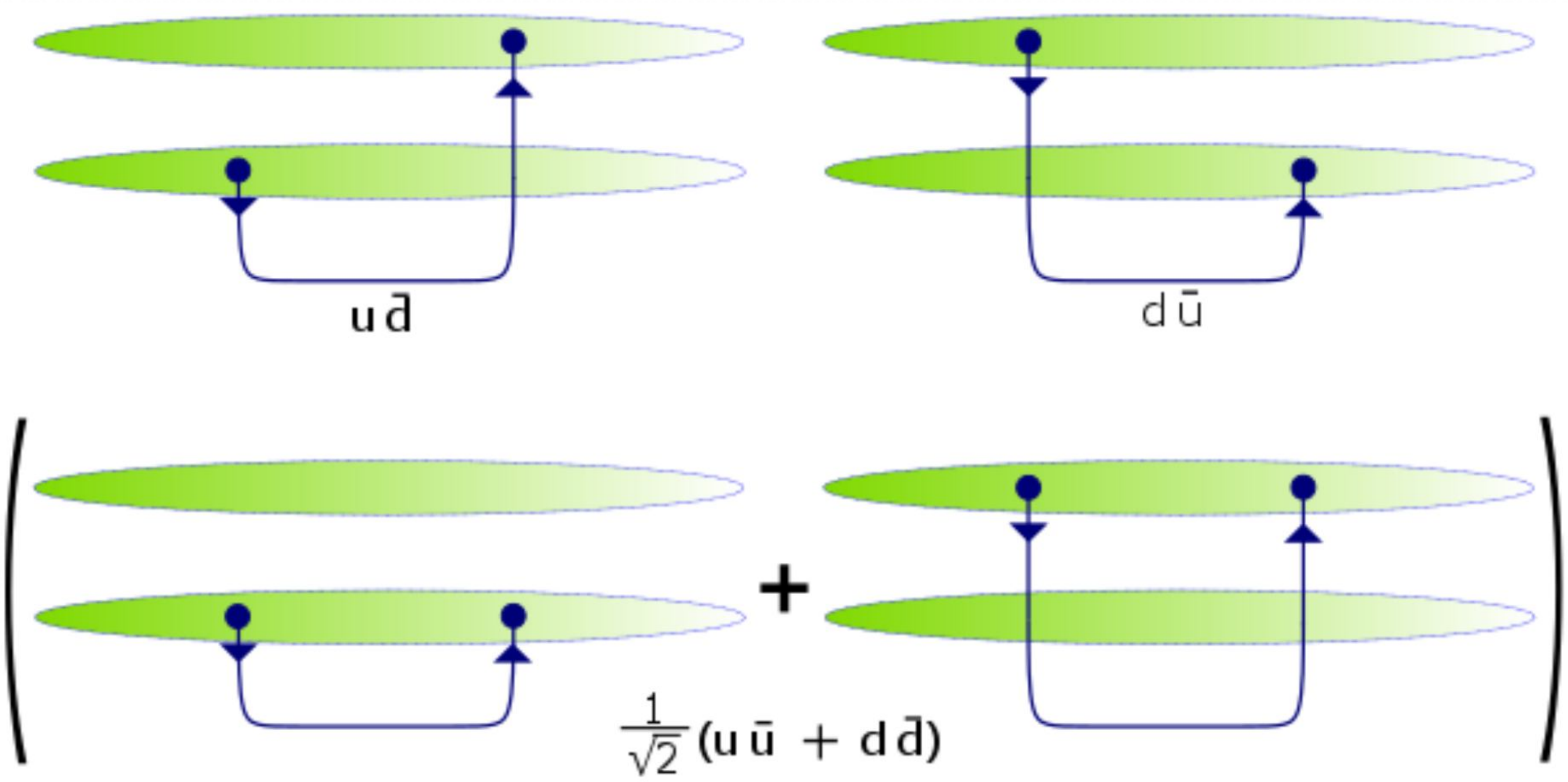
Isospin constraints on decays of stringy mesons

- Isospin approximate symmetry is realized in holography by the fact that the u and d flavor branes are located at roughly the same holographic radial coordinate.
- The world volume of a stack of N_f coincident flavor branes is characterized by a $U(N_f)$ flavor gauge symmetry.
- In fact we can have $U_L(N_f)U_R(N_f)$ that is geometrically spontaneously broken in the IR to $U_D(N_f)$



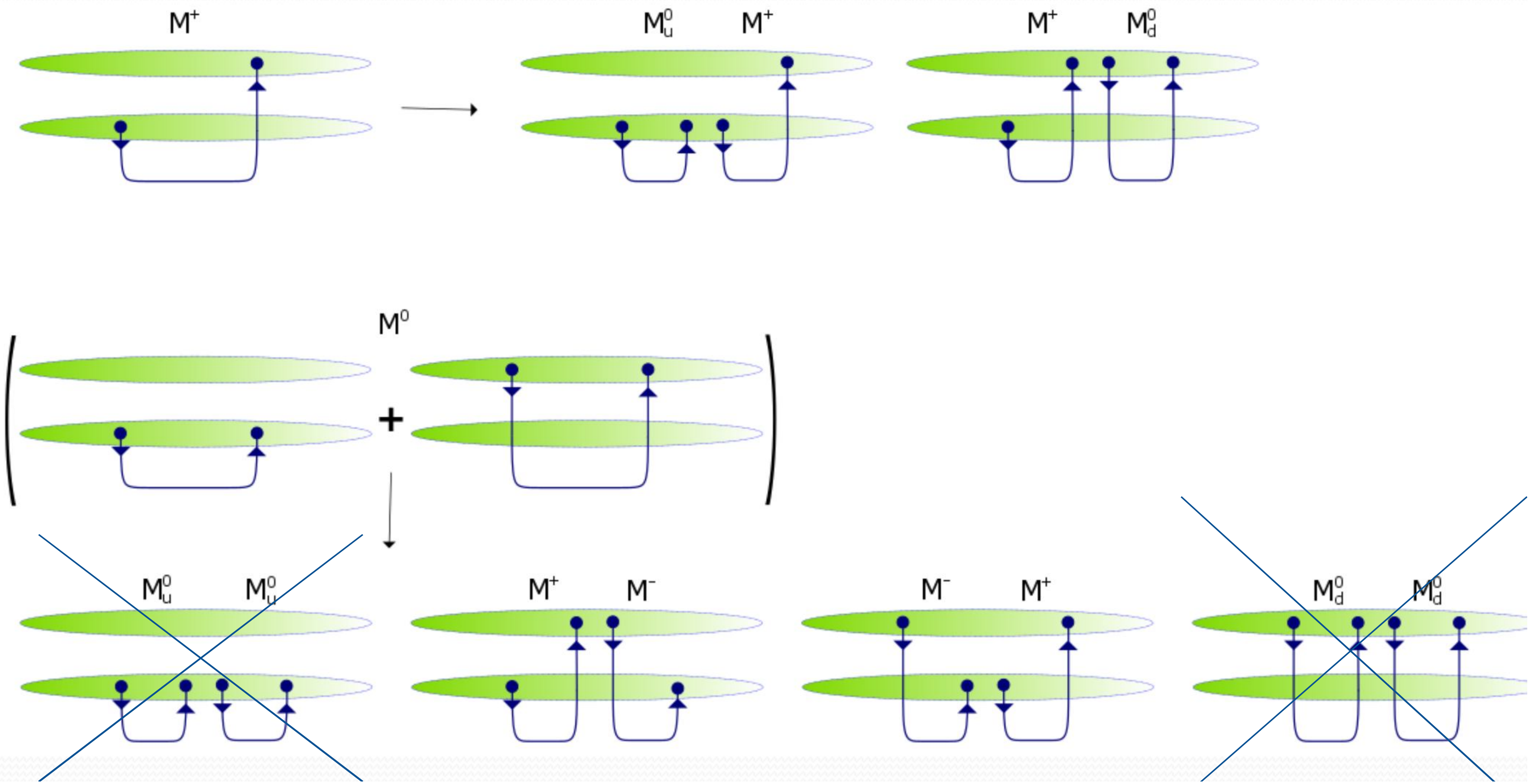
Isospin constraints on decays of stringy mesons

- The stringy mesons of the isospin triplet.



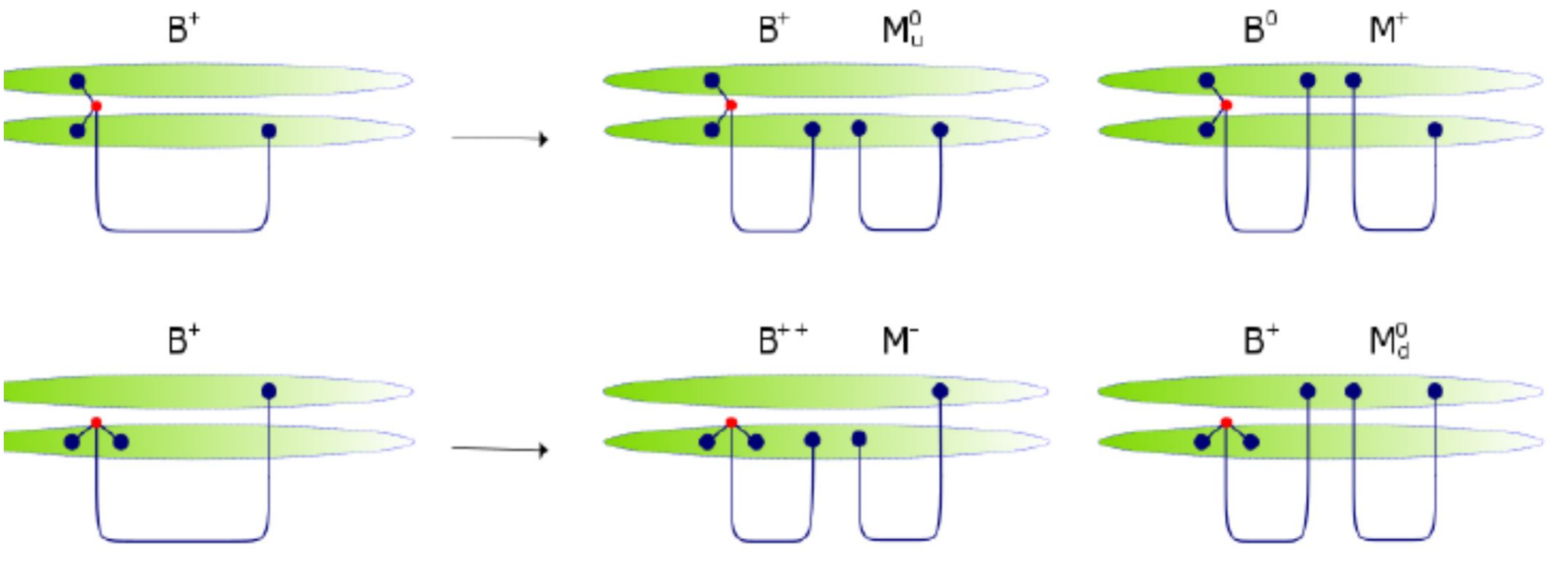
Isospin constraints on decays of stringy mesons

- The decay processes of mesons involve the breaking apart of the horizontal string and the attachment of its endpoints to either the u or the d flavor branes.



Isospin constraints on decays of stringy baryon

- Possible decays of B+



Facing experimental data

String length and the phenomenological intercept

- Hadrons admit **modified Regge trajectories** even for no orbital angular momentum!!
- This is due to the fact that there is a **quantum length** caused by a repulsive Casimir force.

$$\boxed{F_C = -2a/L^2}$$

$$\boxed{L^2 = L_{cl}^2 + L_0^2 \,.}$$

- The quantum length is related to the intercept

$$\boxed{L_0^2 = -C\frac{a}{T}}$$

- There are different ways to determine C.
- Our approach is to extract it from the fits.

Check of the linear dependence on L

- Is the **experimental data** admit the **linear dependence on L**

$$\Gamma = \frac{\pi}{2} A T L(M, m_1, m_2, T)\,.$$

- For short strings with important role of the massive endpoints we add a **phase space factor**

$$\Gamma = \frac{\pi}{2} A \times \Phi(M) \times T L(M, m_1, m_2, T)\,.$$

- The phase space factor

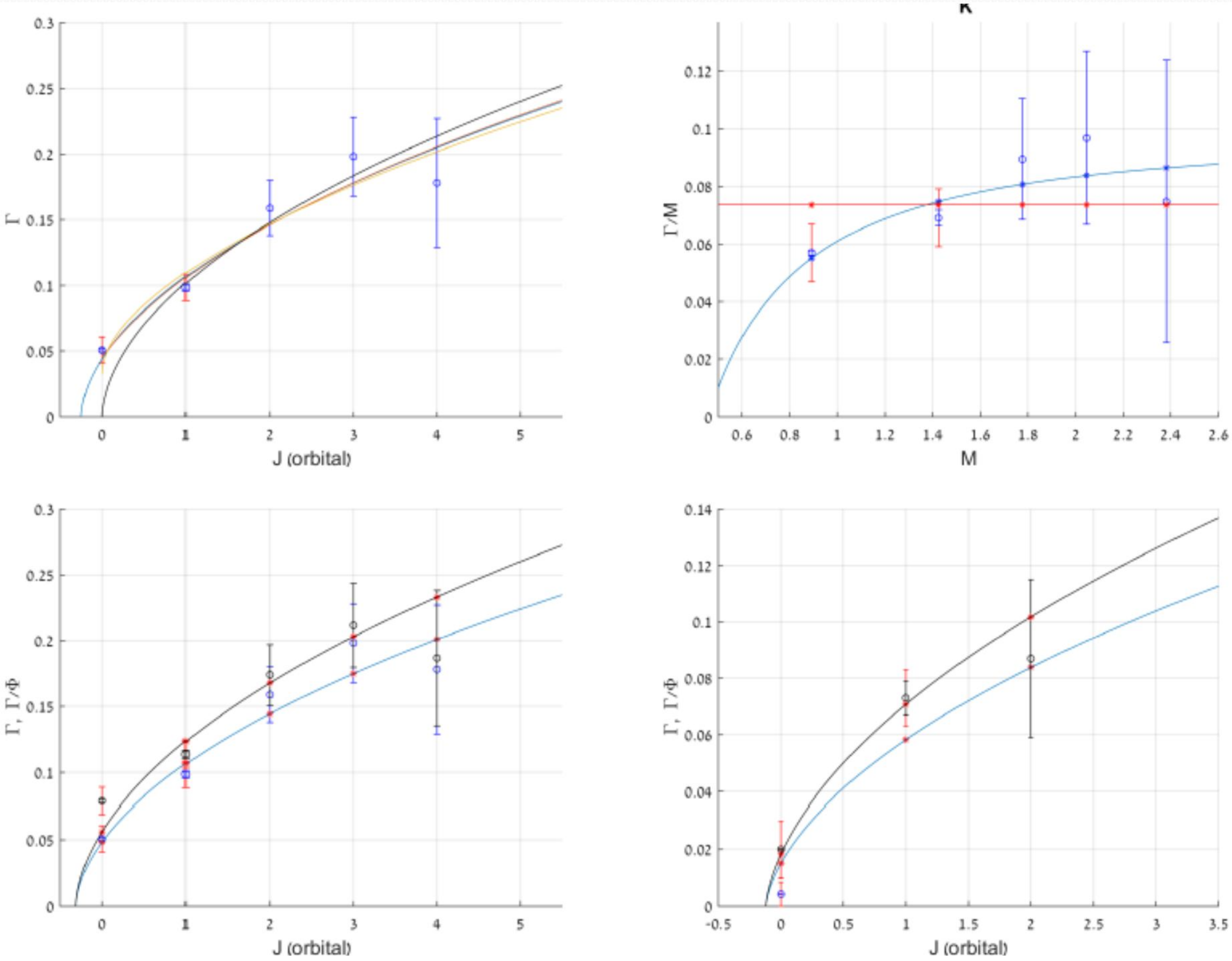
$$\Phi(M, M_1, M_2) \equiv 2\frac{|p_f|}{M} = \sqrt{\left(1 - (\frac{M_1 + M_2}{M})^2\right)\left(1 - (\frac{M_1 - M_2}{M})^2\right)}$$

A test case: The K

- We compare our model to the decays of K* trajectory

State	J^P	Mass	Width	Γ/M	Decay modes[4]
$K^*(892)$	1^-	891.66 ± 0.26	50.8 ± 0.9	$(5.7\pm0.1)\%$	$K\pi$ (100%)
$K_2^*(1430)$	2^+	1425.6 ± 1.5	98.5 ± 2.7	$(6.9\pm0.2)\%$	$K\pi$ (50%), $K^*\pi$ (25%), $K^*\pi\pi$ (13%), $K\rho$ (9%), ...
$K_3^*(1780)$	3^-	1776 ± 7	159 ± 21	$(9.0\pm1.1)\%$	$K\rho$ (31%), $K^*\pi$ (20%), $K\pi$ (19%), $K\eta$ (∼30%),...
$K_4^*(2045)$	4^+	2045 ± 9	198 ± 30	$(9.7\pm1.5)\%$	$K\pi$ (10%), $K^*\pi\pi$ (9%), 5 more modes (7% or less), ...
$K_5^*(2380)$	5^-	2382 ± 24	178 ± 50	$(7.5\pm2.1)\%$	$K\pi$ (6%), no other measured modes.

test case: The K

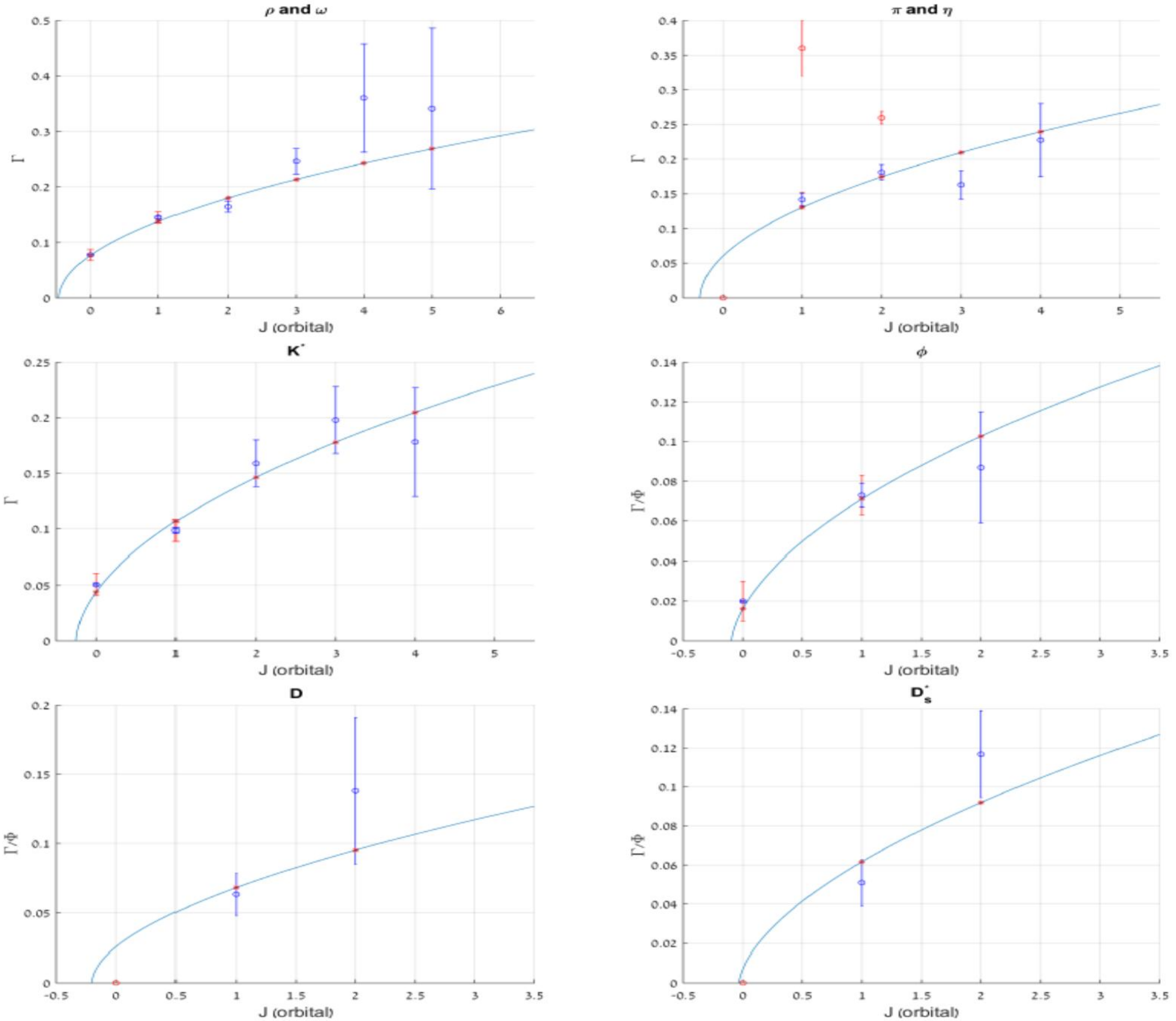


Fit results: the meson trajectories

- Meson fits

Trajectory (No. of states)		a (from spectrum)	A (fitted value)	$\sqrt{\chi^2/DOF}$
ρ	5[a]	-0.46	0.097	1.76
ω	5[a]	-0.40	0.120	2.31
ρ and ω (avg.)	6	-0.46	0.108	1.14
π	3[a]	-0.34	0.100	1.66
η	3[a]	-0.29	0.108	1.56
π and η (avg.)	4	-0.29	0.109	1.52
K^*	5	-0.25	0.098	0.77
ϕ	3	-0.10	0.074	0.50
D	2	-0.20	0.072	0.87
D_s^*	2	-0.03	0.076	1.44

Fit results: the meson trajectories



The decay width of baryons

- For baryons the linearity with L is somewhat modified.

Trajectory (No. of states)		a (from spectrum)	A (fitted value)	$\sqrt{\chi^2/DC}$
N (even)	2	-0.77	0.080	3.33
N (odd)	3	-1.11	0.082	2.43
Δ (even)	3	-1.37	0.101	1.90
Λ	4	-0.46	0.041	2.33
Σ $(S = 1/2)$	2	-0.95	0.052	0.96
Σ $(S = 3/2)$	3	-1.22	0.100	1.57

Exponential suppression of pair creation

- The **ratio of the decay width** to a strange pair versus to a light quark pair is

$$\boxed{\lambda_s = \exp\left(-2\pi C(m_s^2 - m_{u/d}^2)/T_{\rm eff}\right) \approx 0.3}$$

Hadron	J^P	Light channel		$s\bar{s}$ channel		Ratio	λ_s
$\rho_3(1690)$	3^-	$\omega\pi$	16±6%	$K\bar{K}\pi$	3.8±1.2%	0.24±0.12	0.30±0.15
$K_4^*(2045)$	4^+	$K^*\pi\pi\pi$	7±5%	$\phi\bar{K}^*$	1.4±0.7%	0.20±0.17	0.32±0.28

- In **radiative decays**

$$\frac{\Gamma(J/\Psi \to \gamma f_2'(1525))}{\Gamma(J/\Psi \to \gamma f_2(1270))} = 0.31 \pm 0.06\,. \quad \frac{\Gamma(\Upsilon \to \gamma f_2'(1525))}{\Gamma(\Upsilon \to \gamma f_2(1270))} = 0.38 \pm 0.10$$

Zweig suppressed decays and the string length

- The probability of a meson to decay via **annihilation of the quark and antiquark**

$$\boxed{\Gamma = \Gamma_Z \exp(-T_Z L^2/2)}$$

- The decays of upsilon

State	Full width [keV]	$B(ggg)$	$B(\gamma gg)$	Partial width [keV]	Best fit [keV]
$\Upsilon(1S)$	54.02±1.25	81.7±0.7%	2.2±0.6%	45.3±1.3	45.2
$\Upsilon(2S)$	31.98±2.63	58.8±1.2%	1.87±0.28%	19.4±1.7	20.6
$\Upsilon(3S)$	20.32±1.85	35.7±2.6%	0.97±0.18%	7.5±0.9	7.1

Summary

- In spite of five decades of research, the story of the strong decays of mesons and baryons has yet not been fully deciphered. **One does not know who to determine the decay width from QCD.**
- We believe, though not in the same strength as for the spectrum, that the decays of hadronic states tell us that indeed hadrons are strings.
- This is based on three ingredients:

(i) The linearity relation between the decay width and the length of the string

(ii) The exponential suppression factor associated with

the creation of a pair that accompanies the breaking of the string into two strings.

(iii) The constraints due to approximated symmetries like isospin baryon number and flavor SU(3) which are realized in the stringy description

Summay

- In this work we have used two string frameworks (i)Strings of a holographic confining background in critical dimensions (ii) **HISH model** of strings in at four space-time dimensions.
- We saw that the effect of the intercept can be thought of as a repulsive Casimir force, giving it non-zero length, and mass and width, even when it is not rotating.
- We found that the decay coefficient is universal

$$A = 0.095 \pm 0.015\,.$$

- Open questions: creation mechanisms of the hadronic states, Jet formation, scattering amplitudes, weak interactions, incorporating leptons.

Open questions

- Our model assumes chargeless massive endpoint particles. The endpoint of a string on a flavor

brane carries a charge associated with the symmetry group of the flavor branes. Thus it is natural to add an interaction, for instance EM interaction, between the two string endpoints.

- It is easy to check that this change will introduce a classical modification of the intercept. One can use it to determine the difference between md and mu
- Magnetic moment and other EM properties can be computed.

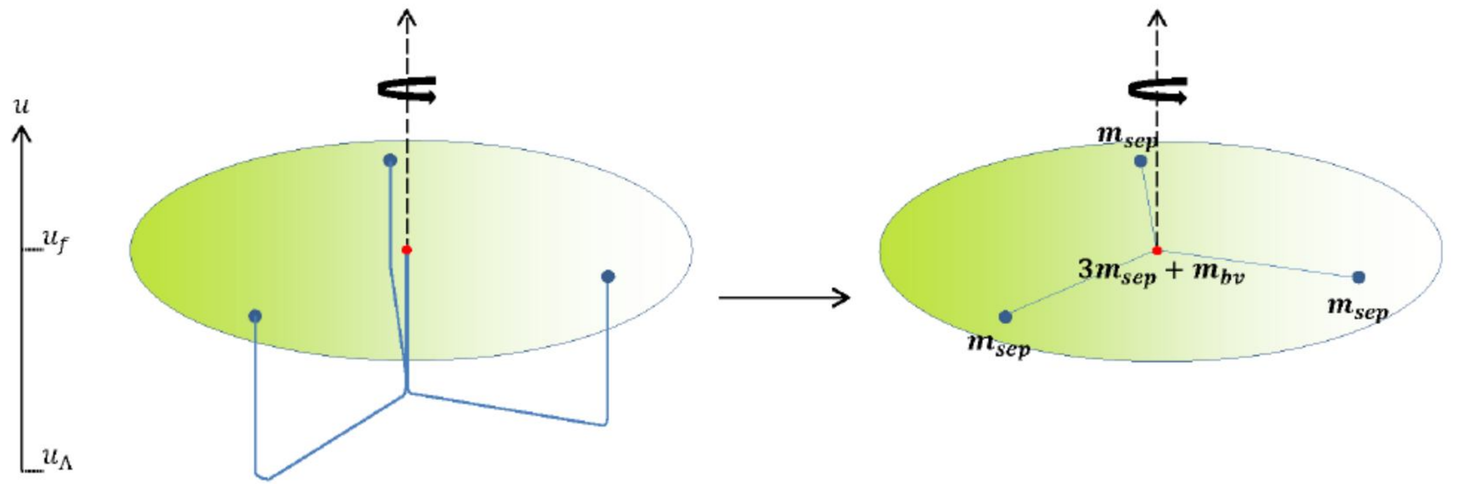
Open Questions

- As was discussed in the introduction, the models we are using are not the outcome of a full quantization of the system.
- The quantization of the rotating string without massive endpoints was analyzed . The quantum Regge trajectories associated with strings with massive endpoints require determining the contributions to the intercept to order J^0 from both the “Casimir" term and the Polchinski-Strominger term.
- Once a determination of the intercept as a function of m^2/T is made, an improved fit and a re-examination of the deviations from a universal model should be made.

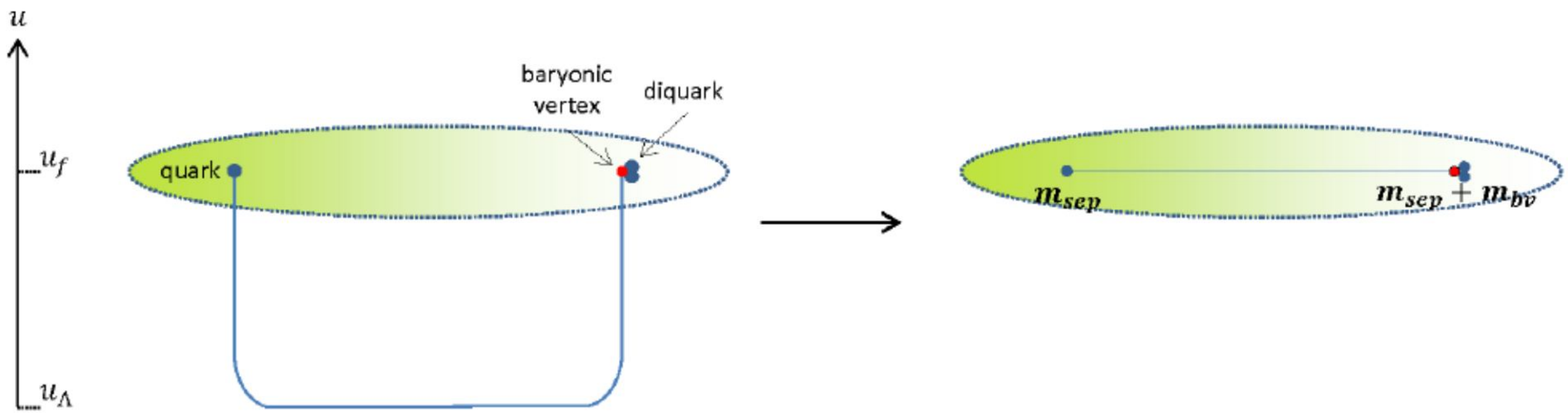
(c) The HISH baryon and its stability

From holographic to HISH baryons

- The symmetric configuration

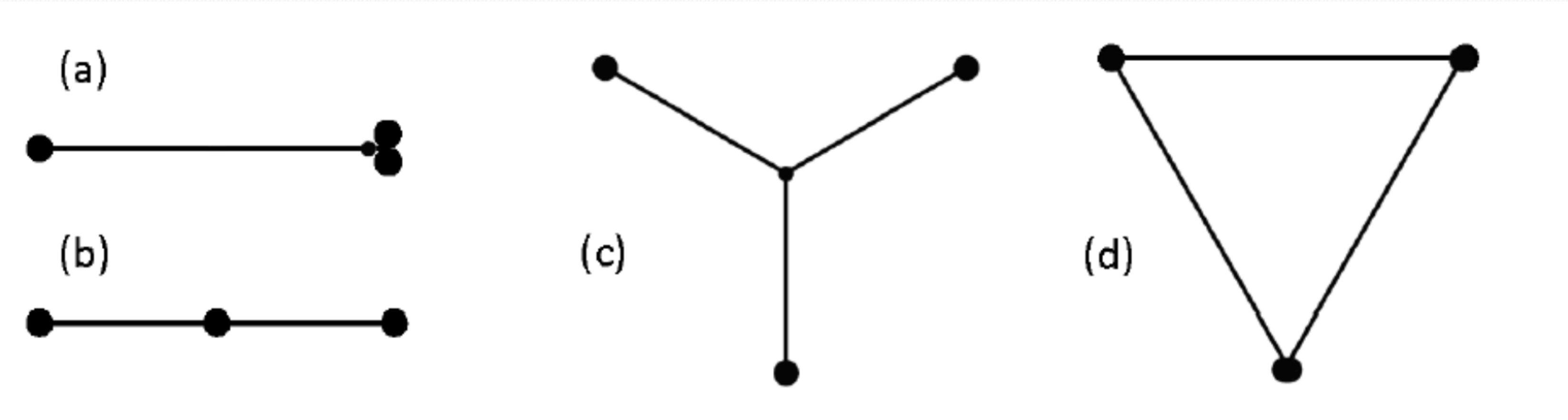


- The asymmetric configuration



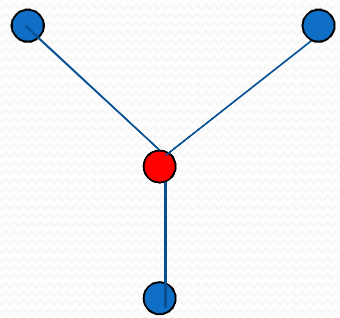
Possible baryon c=3onfigurations

- A priori for Nc=3 there are several possible configurations



From large Nc to three colors

- Naturally the analog at $N_c=3$ of the symmetric configuration with a central baryonic vertex is the old Y shape baryon

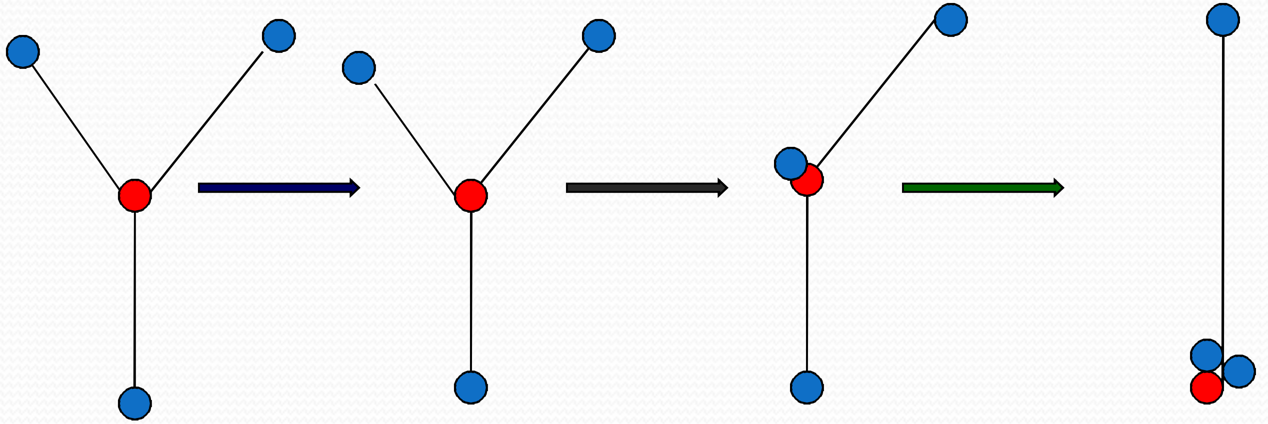


- The analog of the asymmetric setup with one quarks on one end and N_c-1 on the other is a straight string with quark and a di-quark on its ends.

Stability of an excited baryon

- It was shown that the classical Y shape three string configuration is unstable. An arm that is slightly shortened will eventually shrink to zero size.
- We have examined Y shape strings with massive endpoints and with a massive baryonic vertex in the middle.
- The analysis included numerical simulations of the motions of mesons and Y shape baryons under the influence of symmetric and asymmetric disturbance.
- We indeed detected the instability
- We also performed a perturbative analysis where the instability does not show up.

Baryonic instability



The conclusion from both the simulations and the qualitative analysis is that indeed the Y shape string configuration is unstable to asymmetric deformations.

Thus an excited baryon is an unbalanced single string with a quark on one side and a di-quark and the baryonic vertex on the other side.

(d) The HISH Glueball

The HISH Glueball

- The map of the classical folded rotating closed string in holographic background to a similar string in four dimensions is simple.
- Unlike the case of the open string here there are no vertical segments involved and correspondingly no msep.
- It is just the string tension dependence on the holographic background
- However, as will be seen in later, the form of the quantum string yields another significant difference
- The relation between the energy and angular momentum is modied from the linear Regge trajectory

$$J = \alpha'_{\ closed}(E - m_0)^2$$

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