Quest for Grand Unification

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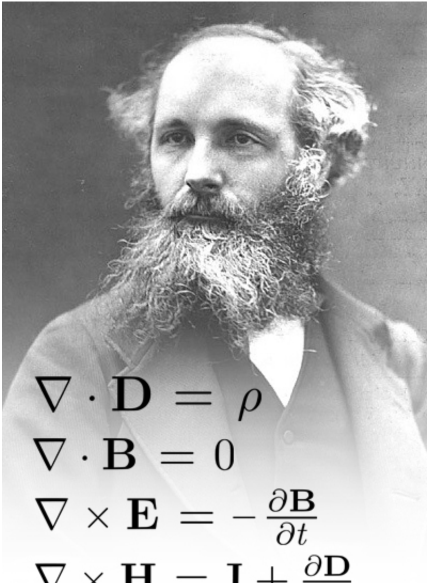
Outline

- Motivation
- Grand Unification
- Proton Decay
- Inflation, Axions and Magnetic Monopoles
- Supersymmetry



$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Electromagnetic Waves (Photon)   1865

A fifth dimension?

Theodor Kaluza
1885–1954

- Polish mathematician Kaluza showed in 1919 that gravity and electromagnetism could be unified in a single theory with 5 dimensions – using Einstein's theory of gravity

"The idea of achieving a unified theory by means of five-dimensional world would never have dawned on me…At first glance I like your idea tremendously"

Unifying EM and Gravity

Physics Beyond the Standard Model

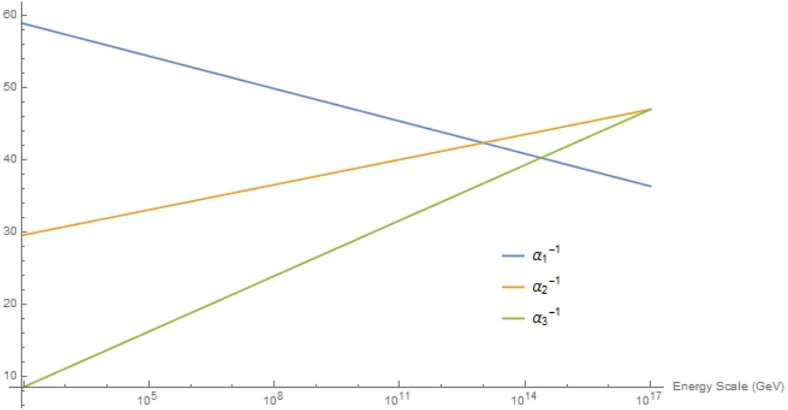
- Neutrino Physics: SM + Gravity suggests $m_\nu \lesssim 10^{-5}$ eV
- Electric charge quantization not explained in SM

  (Dirac requires monopoles)
- Dark Matter: SM offers no plausible DM candidate
- Origin of matter in the universe
- Inflation (resolve problems of standard big bang cosmology)

Grand Unified Theories (GUTs)

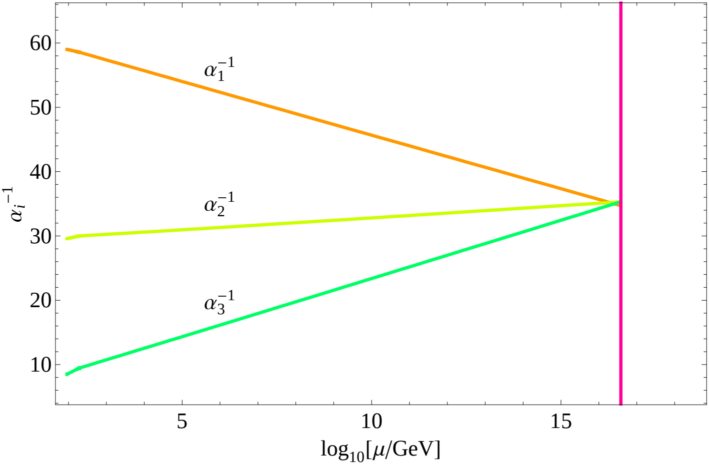
- Unification of SM/MSSM gauge couplings
- Unification of matter/quark-lepton multiplets
- Proton Decay
- Electric charge quantization, Magnetic monopoles predicted (as Dirac wanted)
- Seesaw physics, neutrino oscillations
- Baryogenesis/leptogenesis
- Inflation/gravity waves, $\delta\rho/\rho$

Running of Gauge Couplings in SM



Hint of unification ?

Gauge Coupling Unification in Non-SUSY SU(5)



Gauge Coupling Unification in the SU(5) model with additional fermions Q + $\bar{Q}$ + D + $\bar{D}$ at mass scale $\sim$ 1 TeV.

Quark-Lepton Unification

- Pati-Salam $\rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R$:
  $(4,2,1)+(\overline{4},1,2)$

  $$\begin{pmatrix} u & u & u & \nu_e \\ d & d & d & \ell \end{pmatrix}_{L,R} \Longrightarrow \text{16 chiral fields;}$$

  SM neutrinos can have tiny masses via seesaw mechanism(built in)

- Georgi-Glashow $\rightarrow SU(5)$: $10+\overline{5}$
  - 15 chiral fields;
  - Massless neutrinos

- Fritzsch-Minkowski, Georgi $\rightarrow SO(10)$:
  $16 \stackrel{SU(5)}{\longrightarrow} 10+\overline{5}+1(\nu_R)$

- $SU(5)\times U(1)_\chi$ (cf: 4-2-2)

SU(5) x $U(1)_{PQ}$ (Non-SUSY)

Model Contains axion (strong CP + DM), right-handed neutrinos, and a new scalar which drives inflation (& breaks $U(1)_{PQ}$)

SU(5) x U(1)$_{PQ}$

Neutrino masses

Fixes bad mass relations

	T_L	F_L	ν_L^c	H_1	H_2	σ	Φ	χ
$SU(5)$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{45}^\star$
PQ symmetry (DFSZ) $U(1)_{PQ}$	$\alpha/2$	$\alpha/2$	$\alpha/2$	$-\alpha$	$-\alpha$	$-\alpha$	0	$-\alpha$

$$\mathcal{L}_{Yuk} = T_L.\mathbf{Y_{10}}.T_L.H_1 + T_L.\mathbf{Y_5}.F_L.H_2 + T_L.\mathbf{Y_{45}}.F_L.\chi$$
$$F_L.\mathbf{Y}_\nu.\nu_L^c.H_1 + \tfrac{1}{2}\mathbf{Y_N}\nu_L^c.\nu_L^c.\sigma + \text{h.c.}\,,$$

$$\longrightarrow$$

$$M_e = \mathbf{Y_5}\langle H_2\rangle + 2\mathbf{Y_{45}}\langle\chi\rangle$$
$$M_d = \mathbf{Y_5}^T\langle H_2\rangle - 6\mathbf{Y_{45}}^T\langle\chi\rangle$$
$$M_u = 4(\mathbf{Y_{10}} + \mathbf{Y_{10}}^T)\langle H_1\rangle$$
$$M_\nu \simeq \mathbf{Y}_\nu{}^T.\mathbf{Y_N}{}^{-1}.\mathbf{Y}_\nu\frac{\langle H_1\rangle^2}{\langle\sigma\rangle}$$

SU(5) x U(1)$_{PQ}$

Inflation with σ

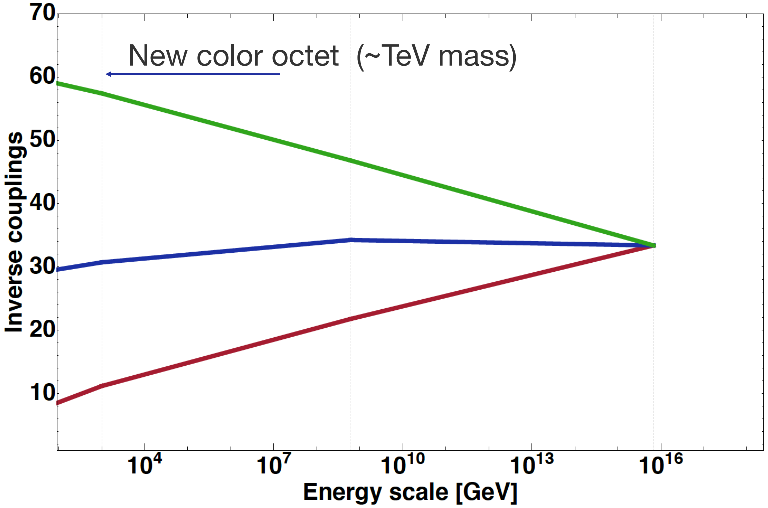
	T_L	F_L	ν_L^c	H_1	H_2	σ	Φ	χ
$SU(5)$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{45}^\star$
$U(1)_{PQ}$	$\alpha/2$	$\alpha/2$	$\alpha/2$	$-\alpha$	$-\alpha$	$-\alpha$	0	$-\alpha$

$$\sigma = \rho e^{ia/f_a}$$

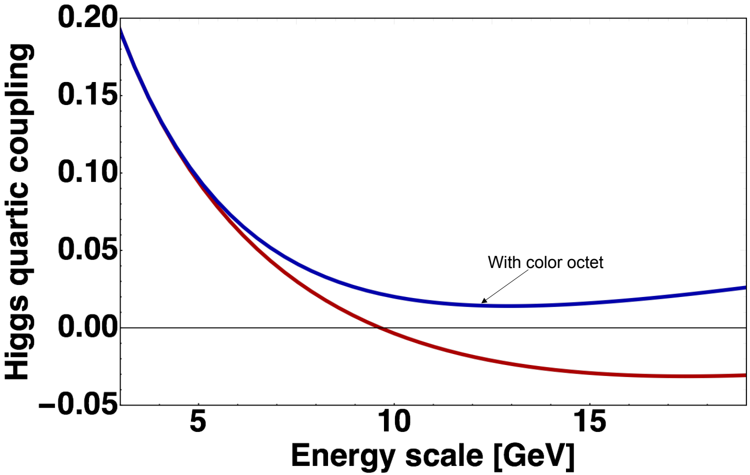
Inflaton

Axion

SU(5) x U(1)$_{PQ}$


New color octet (~TeV mass)
Inverse couplings
70
60
50
40
30
20
10
10^4
10^7
10^{10}
10^{13}
10^{16}
Energy scale [GeV]

Higgs vacuum is stabilized



SU(5) x U(1)$_{PQ}$

Baryon asymmetry arises via non-thermal leptogenesis

Baryogenesis

	T_L	F_L	ν_L^c	H_1	H_2	σ	Φ	χ
$SU(5)$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{45}^\star$
$U(1)_{PQ}$	$\alpha/2$	$\alpha/2$	$\alpha/2$	$-\alpha$	$-\alpha$	$-\alpha$	0	$-\alpha$

Right handed neutrinos from inflaton decay produces lepton asymmetry:

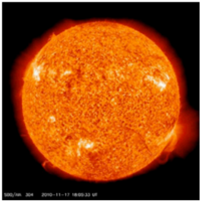
[Lazarides, Shafi '91
Asaka et al '92]

$$\eta_L \simeq -10^{-5}\left(\frac{T_{RH}}{10^9\,\mathrm{GeV}}\right)\left(\frac{M_N}{m_\rho}\right) \longrightarrow \boxed{M_N} \simeq 0.3\left(\frac{10^7\,\mathrm{GeV}}{\boxed{T_{RH}}}\right) m_\rho$$

Proton Decay

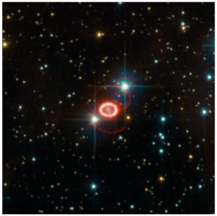
In this model $\tau_P \simeq 2.4 \times 10^{35}$ yr

Hyper-Kamiokande Physics Goals

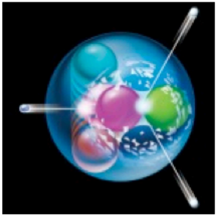


Solar neutrinos

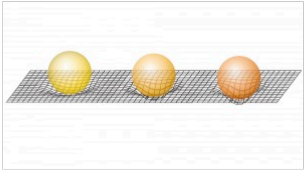
CP violation



Astrophysical neutrinos



Proton decay

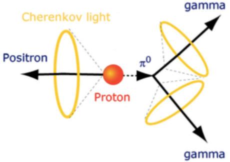


Mass hierarchy

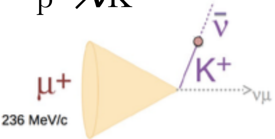
Nucleon decay

Flagship nucleon decay modes:

$p \rightarrow e^{+} \boldsymbol{\pi}^{0}$



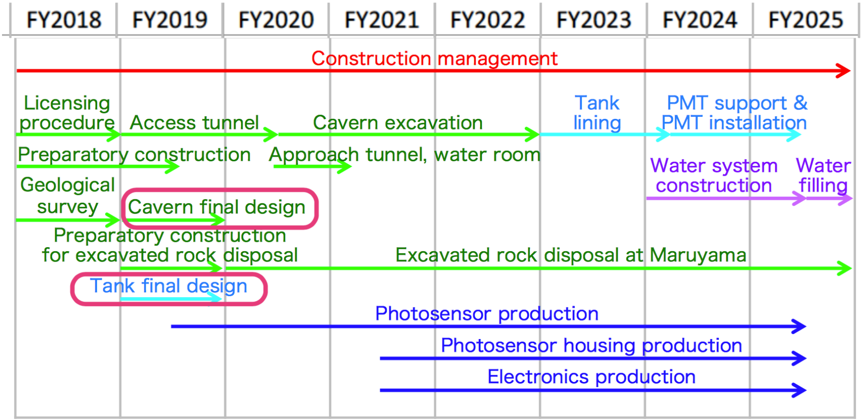
$p \rightarrow \bar{\boldsymbol{\nu}} K^{+}$



Mode	Sensitivity (90% CL) [years]	Current limit [years]
$p \rightarrow e^+\pi^0$	7.8×10^{34}	1.6×10^{34}
$p \rightarrow \bar{\nu}K^+$	3.2×10^{34}	0.7×10^{34}
$p \rightarrow \mu^+\pi^0$	7.7×10^{34}	0.77×10^{34}
$p \rightarrow e^+\eta^0$	4.3×10^{34}	1.0×10^{34}
$p \rightarrow \mu^+\eta^0$	4.9×10^{34}	0.47×10^{34}
$p \rightarrow e^+\rho^0$	0.63×10^{34}	0.07×10^{34}
$p \rightarrow \mu^+\rho^0$	0.22×10^{34}	0.06×10^{34}
$p \rightarrow e^+\omega^0$	0.86×10^{34}	0.16×10^{34}
$p \rightarrow \mu^+\omega^0$	1.3×10^{34}	0.28×10^{34}
$n \rightarrow e^+\pi^-$	2.0×10^{34}	0.53×10^{34}
$n \rightarrow \mu^+\pi^-$	1.8×10^{34}	0.35×10^{34}

Limits will be improved across all nucleon decay channels, some by an order of magnitude.

HK construction timeline



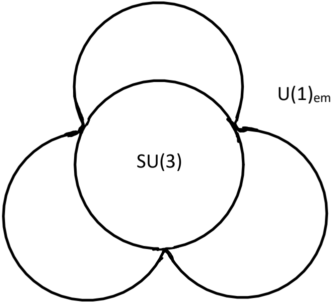
Data taking expected in 2026

Magnetic Monopoles in Unified Theories

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Examples:

1. SU(5) $\to$ SM (3-2-1)
   Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;



monopole mass $\sim \frac{M_G}{\alpha_G}$

3. $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

   Electric charge is quantized with the smallest permissible charge being $\pm(e/6)$;

   Lightest monopole carries two units of Dirac magnetic charge;

4. SO(10) $\rightarrow$ 4-2-2 $\rightarrow$ 3-2-1

   Two sets of monopoles:

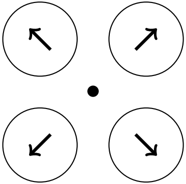
   First breaking produces monopoles with a single unit of Dirac charge.
   Second breaking yields monopoles with two Dirac units.

5. E_6 breaking to the SM can yield 'lighter' monopoles carrying three units of Dirac charge.

   The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics $\&$ cosmology.

Primordial Monopoles

They are produced via the Kibble Mechanism as $G \to H$:



Center of monopole has G symmetry $\langle \phi \rangle = 0$

Initial no. density $\propto T_c^{-3}$. With big bang cosmology such numbers are unacceptable.

$\mathrm{r}_{in} = \frac{N_m}{N_\gamma} \sim 10^{-2}$.

$\Rightarrow$ **Monopole Problem**

(Need Inflation)

Inflationary Cosmology [Starobinsky, Mukhanov, Chibisov, Guth, Linde, Hawking, $\cdots$]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for $n_s, r, dn_s/d\ \ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Slow-roll inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

  where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

Slow-roll inflation

- The tensor to scalar ratio r can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} \approx 3.0 \times 10^{16}\, r^{1/4} \text{ GeV}.$$

- The amplitude of the curvature perturbation is given by

$$\Delta_{\mathcal{R}}^2 = \frac{1}{24\pi^2}\left(\frac{V/m_p^4}{\epsilon}\right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$

- The spectrum of the tensor perturbation is given by

$$\Delta_h^2 = \frac{2}{3\,\pi^2}\left(\frac{V}{m_P^4}\right)_{\phi=\phi_0}.$$

- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2}\int_{\phi_e}^{\phi_0}\left(\frac{V}{V'}\right)d\phi.$$

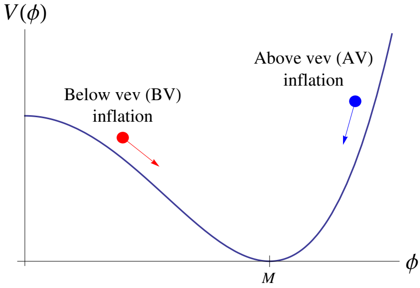
  Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

Inflation with a Higgs Potential [Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

- Consider the following Higgs Potential:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

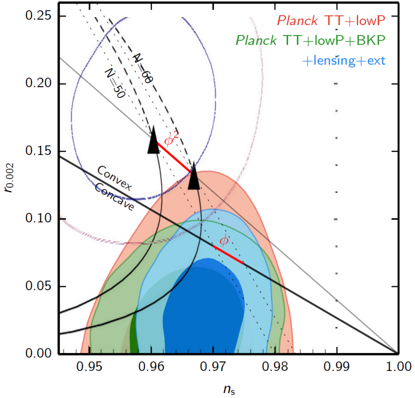
  Here ϕ is a gauge singlet field.



- WMAP/Planck data favors BV inflation ($r \lesssim 0.1$).

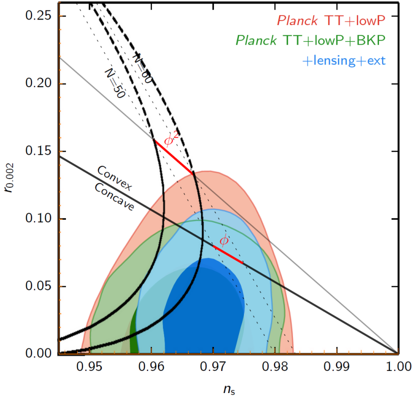
Note: This is for minimal coupling to gravity

Higgs Potential:

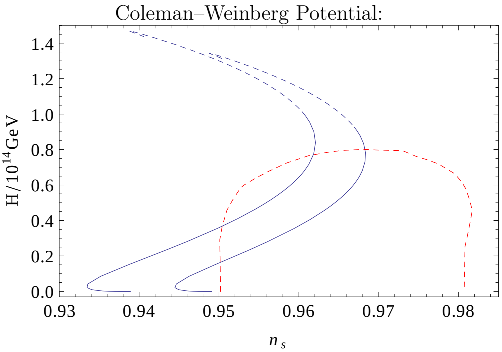


n_s vs. r for Higgs potential, superimposed on Planck and Planck+BKP 68% and 95% CL regions taken from arXiv:1502.01589. The dashed portions are for $\phi > v$. N is taken as 50 (left curves) and 60 (right curves).

Coleman–Weinberg Potential:

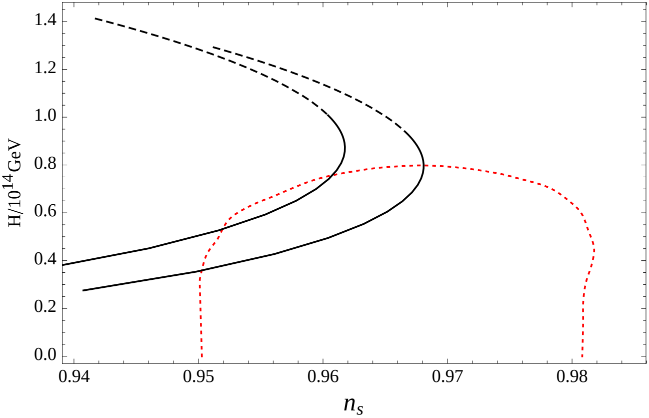


n_s vs. r for Coleman–Weinberg potential, superimposed on Planck and Planck+BKP 68% and 95% CL regions taken from arXiv:1502.01589. The dashed portions are for $\phi > v$. N is taken as 50 (left curves) and 60 (right curves).



n_s vs. H for Coleman–Weinberg potential, superimposed on Planck TT+lowP+BKP 95% CL region taken from arXiv:1502.02114. The dashed portions are for $\phi > v$. N is taken as 50 (left curves) and 60 (right curves).

Higgs Potential:



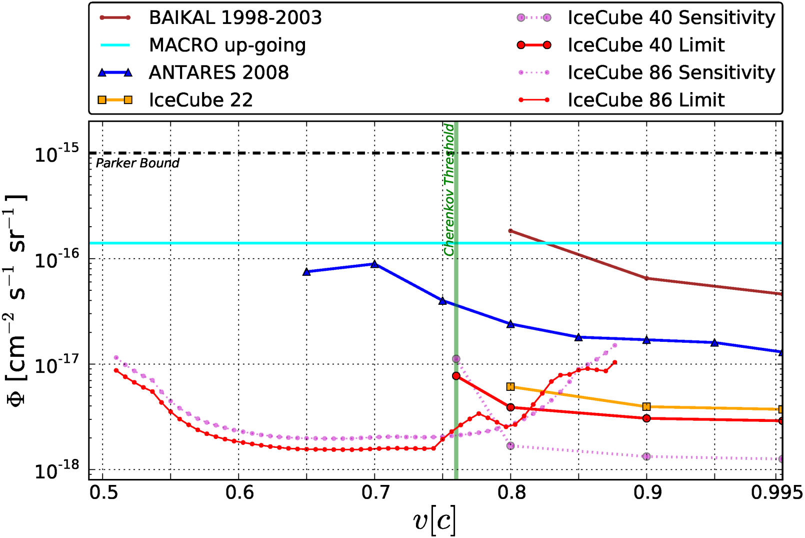
Primordial Monopoles

- Let's consider how much dilution of the monopoles is necessary. $M_I \sim 10^{13}$ GeV corresponds to monopole masses of order $M_M \sim 10^{14}$ GeV. For these intermediate mass monopoles the MACRO experiment has put an upper bound on the flux of $2.8 \times 10^{-16}\ \mathrm{cm}^{-2}$ s^{-1} sr^{-1}. For monopole mass $\sim 10^{14}$ GeV, this bound corresponds to a monopole number per comoving volume of $Y_M \equiv n_M/s \lesssim 10^{-27}$. There is also a stronger but indirect bound on the flux of $(M_M/10^{17}$ GeV$)10^{-16}\mathrm{cm}^{-2}$ s^{-1} sr^{-1} obtained by considering the evolution of the seed Galactic magnetic field.
- At production, the monopole number density n_M is of order H_x^3, which gets diluted to $H_x^3 e^{-3N_x}$, where N_x is the number of e-folds after $\phi = \phi_x$. Using

$$Y_M \sim \frac{H_x^3 e^{-3N_x}}{s}\,,$$

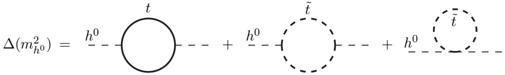
where $s = (2\pi^2 g_S/45)T_r^3$, we find that sufficient dilution requires $N_x \gtrsim \ln(H_x/T_r) + 20$. Thus, for $T_r \sim 10^9$ GeV, $N_x \gtrsim 30$ yields a monopole flux close to the observable level.

Relativistic Monopoles at IceCube



Source: IceCube Collaboration, Eur. Phys. J. C (2016) 76:133

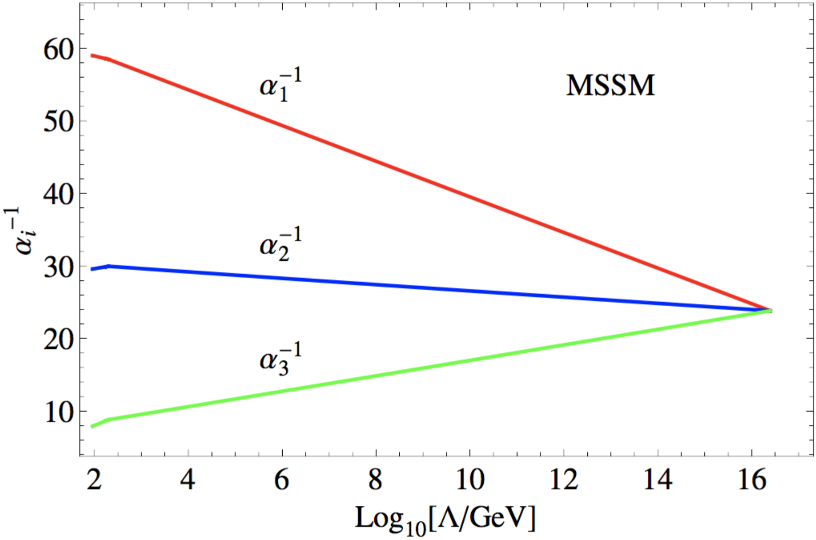
Supersymmetry



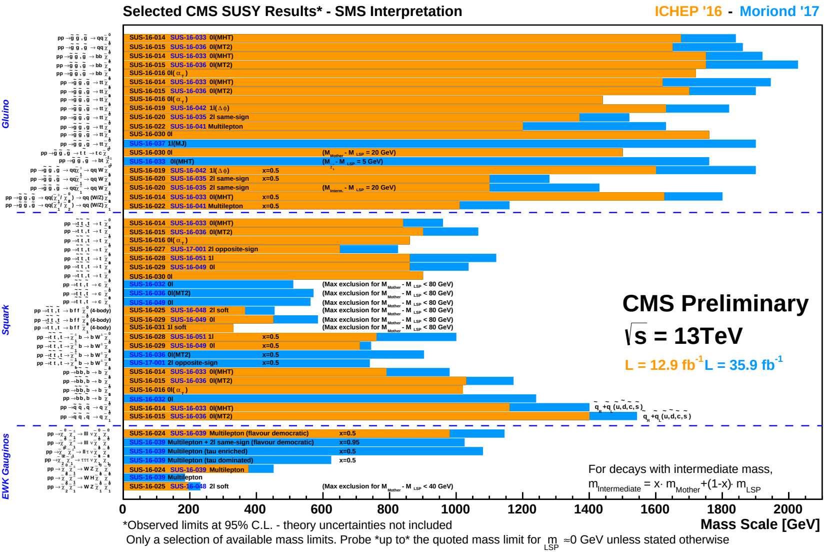
Source: Martin, Adv.Ser.Direct.High Energy Phys. 21 (2010) 1-153

- Resolution of the gauge hierarchy problem
- Predicts new particles, some maybe found at LHC ?
- Unification of the SM gauge couplings at $M_{GUT} \sim 2 \times 10^{16}$ GeV
- Cold dark matter candidate (LSP)
- Compelling inflation models

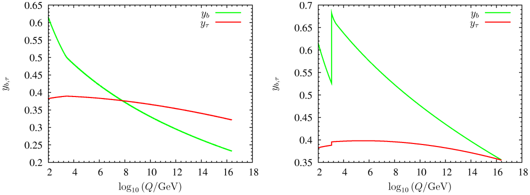
Why Supersymmetry ?



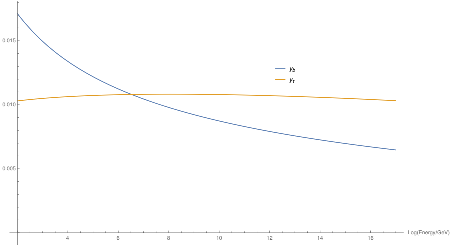
Where is SUSY ?



b$-\tau$ Yukawa coupling unification



Without Supersymmetry

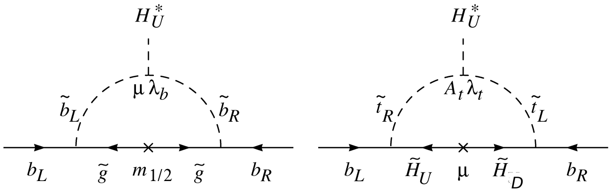


b-τ YU and finite threshold corrections [1]

Dominant contributions to the bottom quark mass from the gluino and chargino loop

$$\delta y_b \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}} \tan\beta}{m_1^2} + \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_2^2} + \dots$$

where $m_1 \approx (m_{\tilde{b}_1} + m_{\tilde{b}_2})/2$ and $m_2 \approx (m_{\tilde{t}_2} + \mu)/2$



where $\lambda_b = y_b$ and $\lambda_t = y_t$

[1] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev.D 50, 7048 (1994)

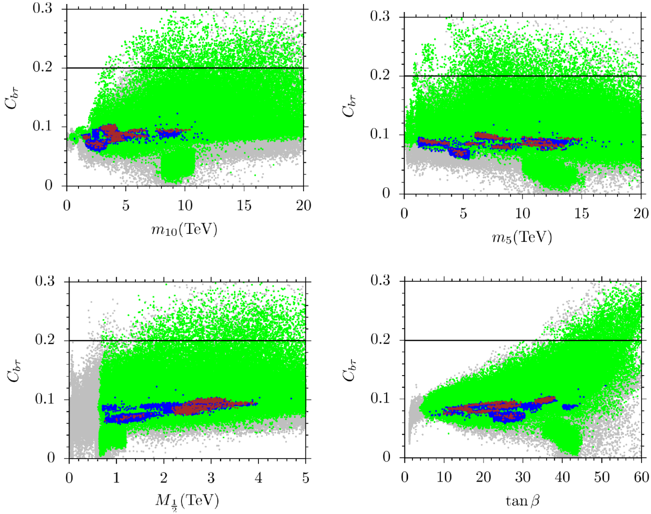
$b - \tau$ YU in SU(5)

$$
\begin{aligned}
0 \leq & \quad m_{10} & \leq 20 \text{ TeV} \\
0 \leq & \quad m_5 & \leq 20 \text{ TeV} \\
0 \leq & \quad M_{1/2} & \leq 5 \text{ TeV} \\
-3 \leq & \quad A_t/m_{10} & \leq 3 \\
-20 \leq & \quad A_{b\tau}/m_5 & \leq 20 \\
1.2 \leq & \quad \tan\beta & \leq 60 \\
0 \leq & \quad m_{H_d} & \leq 20 \text{ TeV} \\
0 \leq & \quad m_{H_u} & \leq 20 \text{ TeV}
\end{aligned}
$$

$$
R \equiv \frac{\mathrm{Max}(y_b, y_\tau)}{\mathrm{Min}(y_b, y_\tau)} \leq 1.1 \quad b - \tau \text{ YU Condition}
$$

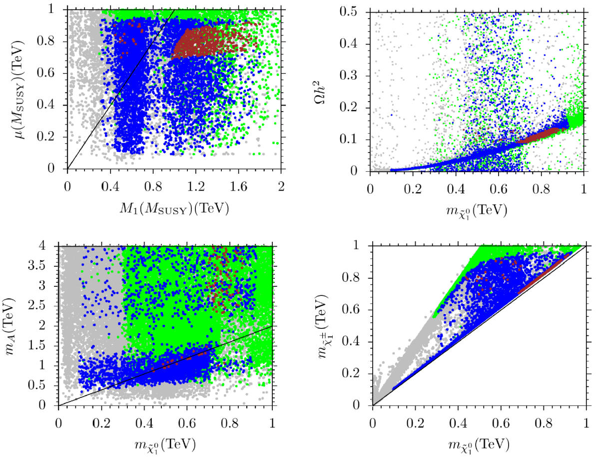
$$
y_b : y_\tau = (1 - C) : (1 + 3C), \quad |C| \leq 0.2 \quad b - \tau \text{ QYU Condition}
$$

$b - \tau$ QYU in SU(5)



All points are consistent with REWSB and neutralino LSP. The green points satisfy the LHC constraints. Blue points form a subset of green and they are compatible with the QYU and Fine-tuning conditions. Brown points are a subset of blue and they are consistent with the WMAP bound on the relic abundance of neutralino LSP within 5σ.

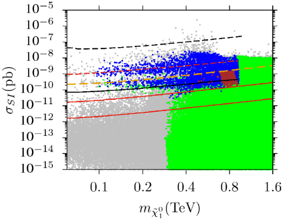
Higgsino-like dark matter, $A-$resonance and chargino-neutralino coannihilation scenarios.

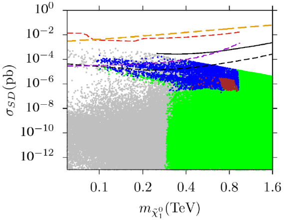


The color coding is the same as previous figure. In addition, the blue points satisfy the QYU condition and brown

	Point 1	Point 2	Point 3
m_{10}	2325	5805	3299
M_5	4334	5756	4813
$M_{1/2}$	1317	2478	1002
m_{H_d}	1574	6740	1592
m_{H_u}	3698	8052	4206
$\tan\beta$	22.6	13.8	26.7
A_t/m_{10}	-1.73	-1.65	-1.46
$A_{b,\tau}/m_5$	0.29	-2.46	0.05
μ	107.8	714.9	835.4
Δ_{EW}	35.3	117	163
m_h	124.5	126.4	124.1
m_H	1334	6513	946.3
m_A	1326	6471	940.1
$m_{H^\pm}$	1336	6514	950
$m_{\tilde{\chi}^0_{1,2}}$	102.8, 111.4	701.3, 716.4	441.7, 783.3
$m_{\tilde{\chi}^0_{3,4}}$	579.9, 1104	1128, 2110	831, 899
$m_{\tilde{\chi}^\pm_{1,2}}$	110.8, 1093	732.5, 2088	792, 894
$m_{\tilde{g}}$	2954	5361	2369
$m_{\tilde{u}_{L,R}}$	3420, 3424	7354, 7302	3780, 3839
$m_{\tilde{t}_{1,2}}$	1403, 2569	2797, 5473	1548, 2747
$m_{\tilde{d}_{L,R}}$	3421, 4957	7355, 7187	3781, 5140
$m_{\tilde{b}_{1,2}}$	2572, 4831	5539, 6868	2751, 4958
$m_{\tilde{\nu}_1}$	4457	6007	4902
$m_{\tilde{\nu}_3}$	4411	5852	4822
$m_{\tilde{e}_{L,R}}$	4455, 2246	6002, 5791	4899, 3196
$m_{\tilde{\tau}_{1,2}}$	2053, 4404	5464, 5851	2947, 4816
$\sigma_{SI}(\mathrm{pb})$	0.10×10^{-8}	0.72×10^{-9}	0.20×10^{-8}
$\sigma_{SD}(\mathrm{pb})$	0.82×10^{-4}	0.15×10^{-5}	0.59×10^{-6}
$\Omega_{CDM}h^2$	0.05	0.097	0.098
$y_{t,b,\tau}$	0.50, 0.13, 0.17	0.51, 0.07, 0.1	0.52, 0.16, 0.21
C	0.08	0.08	0.07

Direct detection!





The color coding is the same as previous figure.

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]

[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\,(\Phi\,\overline{\Phi} - M^2)$$

S = gauge singlet superfield, $(\Phi\,,\overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,,\overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\,\overline{\Phi} \to \Phi\,\overline{\Phi},\;\; S \to e^{i\alpha}\,S,\;\; W \to e^{i\alpha}\,W$$

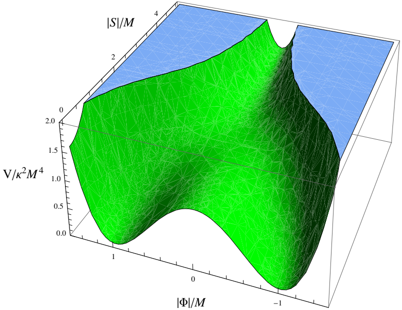
$\Rightarrow \quad W$ is a unique renormalizable superpotential

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle \overline{\Phi} \rangle| = |\langle \Phi \rangle| = M, \; \langle S \rangle = 0$$



Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_\pm^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\mathsf{1loop}} = \frac{1}{64\pi^2}\mathsf{Str}[\mathcal{M}^4(S)(\ln\frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley $(\Phi = 0)$

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right)\ln\frac{\left(x^4-1\right)}{x^4} + 2x^2\ln\frac{x^2+1}{x^2-1} + 2\ln\frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

Tree level + radiative corrections + minimal Kähler potential yield:

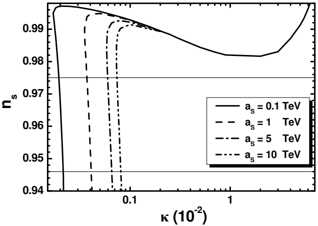
$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

$\delta T/T$ proportional to M^2/M_p^2, where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model.
Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

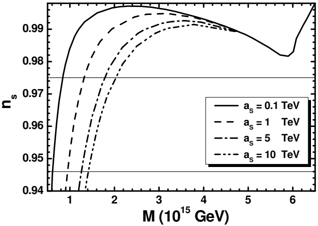
- include soft SUSY breaking terms, especially a linear term in S;
- employ non-minimal Kähler potential.

Results

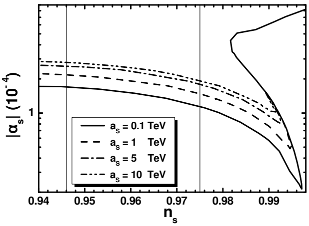
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



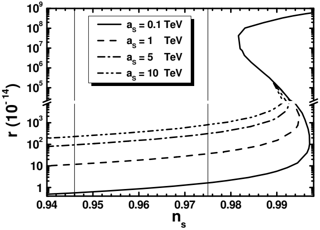
(a)



(b)



(a)



(b)

MSSM μ-Problem and Inflation

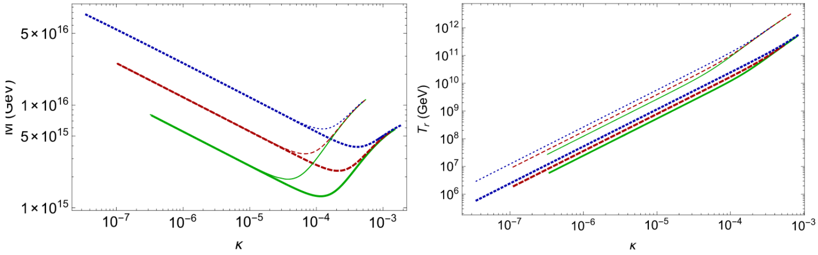
$U(1)_R$ symmetry prevents a direct μ term but allows the superpotential coupling

$$\lambda H_u H_d S$$

Since $\langle S \rangle$ acquires a non-zero VEV $\propto m_{3/2}$ from supersymmetry breaking, the MSSM μ term of the desired magnitude is realized.

μ-Term Inflation with Non-Minimal Kähler: [Rehman, Shafi, Vardag, 2017]

- $W = S(\kappa\overline{\Phi}\Phi - \kappa M^2 + \lambda H_u H_d)$
- $K = K_{min} + \kappa_s \frac{|S|^4}{4m_p^2} + \kappa_{ss}\frac{|S|^6}{6m_p^4}$
- $V = \kappa^2 M^4 \left(1 + \frac{\gamma_S}{2}\left(\frac{M}{m_P}\right)^4 x^4 - \kappa_S \left(\frac{M}{m_P}\right)^2 x^2 + a\frac{m_{3/2}}{\kappa M}x\right)$
  where $\gamma_S = 1 + 2\kappa_S^2 - \frac{7\kappa_S}{2} - 3\kappa_{SS}$ and $x = |S|/M$



Reheat Temperature vs κ for $m_{3/2} = 1$ TeV (solid-green), 10 TeV (dashed-red), and 100 TeV(dotted-blue), $n_s = 0.9655$, $\kappa_S = 0.02$, $\kappa_{SS} = 0$ and $\gamma = 2(10)$ for thick (thin) curves.

- $W = S\left[\kappa M^2 - \kappa Tr(\Phi^2) - \frac{\beta}{M_*}Tr(\Phi^3)\right] + \gamma\bar{H}\Phi H + \delta\bar{H}H$
$+y^u_{ij}10_i 10_j H + y^{d,e}_{ij}10_i\bar{5}_j\bar{H} + y^(_{ij}\nu)1_i\bar{5}_j H + m_{\nu_{ij}}1_i 1_j$
- $K = K_{min} + \kappa_S\frac{|S|^4}{4m_P^2} + \kappa_{SS}\frac{|S|^6}{6m_P^4} + \cdots$
- $V \supset \kappa^2\left|M^2 - \frac{1}{2}\sum_i \phi_i^2 - \frac{\beta}{4\kappa M_*}d_{ijk}\phi_i\phi_j\phi_k\right|^2 +$
$\sum_i\left|\kappa S\phi_i + \frac{3\beta}{4M_*}d_{ijk}S\phi_j\phi_k - \gamma T^i\bar{H}_a H_b\right|^2 + \sum_b|\gamma T^i\phi^i\bar{H} + \delta\bar{H}_b|^2$
$+\sum_b|\gamma T^i\phi^i H + \delta H_b|^2 + D - terms + V_{soft}$

$$n_S \simeq 1 - 2\kappa_S + \left(\frac{8(1-\kappa_S)}{9(4/27-\xi^2)} + 6\gamma_S x_0^2 \right) \left(\frac{M_\xi}{m_P} \right)^2$$

$$-\frac{275\kappa^2}{16\pi^2} |\partial_{x_0}^2 F(5x_0^2)| \left(\frac{m_P}{M_\xi} \right)^2 ,$$

where $x_0 = |S_0|/M_\xi$ and $M_\xi^2 = M^2(4/27\xi^2 - 1)$

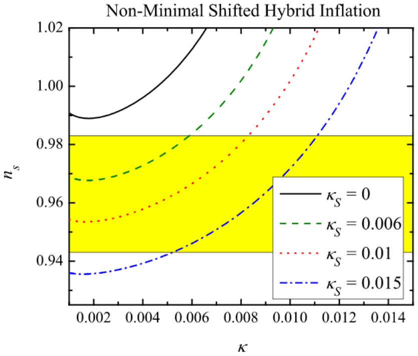
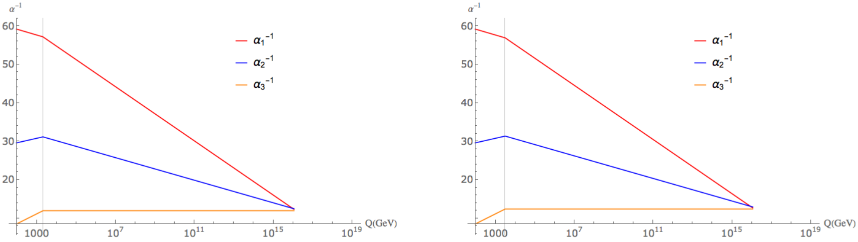


Figure: n_S vs κ for shifted hybrid inflation with $\xi = 0.3$, $T_r = 10^9$ GeV. $1 - \sigma$ bounds from WMAP7 are shown in yellow.

MSSM with Vector Like Particles

Inflation in SU(5) introduces light particles G(1,8,0) and T(1,3,0). Gauge coupling unification is restored by introduction of vector like particles L(1,2,1/2),$\bar{L}$(1,2,-1/2) and 2(E(1,1,1)+$\bar{E}$(1,1,-1)) at scale M_{SUSY}($\sim$TeV)



From Left to Right : Columns showing Gauge Coupling Unification and $b - \tau$ Yukawa Unification at $M_{SUSY} = 2$ TeV, 3 TeV

Summary

- Unification of all forces remains a compelling idea.
- Grand unification explains charge quantization, predicts monopoles and proton decay.
- Also explains tiny neutrino masses via seesaw mechanism.
- Non-SUSY gauge coupling unification require new particles/new physics below M_{GUT}.
- In non-SUSY inflation with Higgs potential, r $\gtrsim$ 0.02 (minimal coupling to gravity).
- SUSY models offer plausible dark matter candidates such as TeV mass higgsino.
- Class of SUSY inflation models predict $\frac{\delta T}{T} \propto (\frac{M}{M_P})^2$, with M $\sim 10^{16}$ GeV; $r \leq 10^{-4}$.
- $b - \tau$ Yukawa Unification can be implemented in SUSY models with heavy particle masses; **Find Them.**

Thank You!