Cosmological Signatures of the SM Higgs Vacuum Instability

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The SM Electroweak Instability
The EW SM vacuum is metastable

Buttazzo et al., 2013
How can we *probe* the EW SM instability?

DM as Primordial Black Holes (no need of BSM)


Gravitational Waves

J.R. Espinosa, D. Racco and A.R., to appear
• During inflation, the SM Higgs is moved away from our current vacuum by quantum fluctuations and pushed over the barrier.

• The SM Higgs rolls down along the unstable region.

• Higgs fluctuations on super-Hubble scales are generated, they contribute to the curvature perturbation at small scales.

• At the end of inflation thermal effects modify the SM Higgs potential and make the Higgs roll down back to our current EW vacuum.

• The Higgs decays into radiation and the corresponding small scale curvature perturbation upon horizon re-entry generates PBHs and gravitational waves.
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The dynamics of the classical Higgs

\[ \ddot{h}_c + 3H \dot{h}_c + V'(h_c) = \xi \]

\[ \langle \xi(t)\xi(t') \rangle = \frac{H^3}{4\pi^2} \delta(t - t') \]

Gaussian random white noise induced by Higgs perturbations whose wavelength leaves the Hubble radius

First phase: quantum beats classical

$$\Delta h_q = \frac{H}{2\pi} \gg \Delta h_c \simeq \left| \frac{V'(h_c)}{3H^2} \right|$$
Second phase: classical beats quantum slow-roll

\[ \Delta h_q = \frac{H}{2\pi} \lesssim \Delta h_c \simeq \left| \frac{V'(h_c)}{3H^2} \right| \quad \text{or} \quad h_c^3 \gtrsim \frac{3H^3}{2\pi\lambda} \]
Second phase: rapid evolution

\[ h_c \simeq \sqrt{\frac{2}{\lambda}} \frac{1}{t_p - t} \]
Third phase: end of inflation and reheating

\[ V_T \simeq \frac{1}{2} m_T^2 h_c^2, \quad m_T^2 \simeq 0.12 T^2 e^{-h_c/(2\pi T)} \]

The Higgs oscillates around our current vacuum with EoS of radiation.
Last phase: the Higgs decays

\[ \gamma_h \sim \frac{3g_2^2 T^2}{(256\pi m_T)} \sim 10^{-3} T \]
The dynamics of the Higgs perturbations

\[ \ddot{\delta h}_k + 3H \dot{\delta h}_k + \frac{k^2}{a^2} \delta h_k + V''(h_c) \delta h_k = \frac{\delta h_k}{a^3 m_P^2} \frac{d}{dt} \left( a^3 \frac{H h_c^2}{H} \right) \]
During inflation

\[ \delta h_k(t) = C(k) \dot{h}_c(t) \]
\[ C(k) = \frac{H}{\dot{h}_c(t_k) \sqrt{2k^3}} \]

Gauge-invariant comoving curvature perturbation

\[ \zeta = \psi + H \frac{\delta \rho}{\dot{\rho}} = \frac{\dot{\rho}_{st}}{\dot{\rho}} \zeta_{st} + \frac{\dot{\rho}_h}{\dot{\rho}} \zeta_h \]
\[ \zeta_h = \psi + H \frac{\delta \rho_h}{\dot{\rho}_h} = H C(k) \]

A constant curvature perturbation is generated on small scales
The Higgs perturbations oscillate like its the classical value and upon decay the get transferred to curvature perturbation.

\[
\mathcal{P}_\zeta^{1/2}(t_{\text{dec}}) = \frac{\dot{\rho}_h(t_{\text{dec}})}{2\dot{\rho}(t_{\text{dec}})} \left( \frac{H}{2\pi} \right) \left( \frac{\dot{h}_c(t_e)}{h_c(t_e)\dot{h}_c(t_k)} \right)
\]
Radiation phase after inflation

The Higgs perturbations oscillate like the classical value and upon decay the get transferred to curvature perturbation.
DM as PBHs from the SM

phys. scale

$N = \ln a$

$\sim 60$ $\sim 40-17$ $t_{\text{form}}$ $t_{\text{end}}$

$H^{-1}$

$\kappa_{\text{PBH}}^{-1}$

CMB
Upon horizon re-entry, the small-scale curvature perturbations induced by the Higgs can form PBHs if they are sizeable enough.

This happens if the density contrast is above a critical value

$$\Delta(\vec{x}) = \frac{4}{9a^2 H^2} \nabla^2 \zeta(\vec{x}) \gtrsim \Delta_c \simeq 0.45$$

The mass of the PBH is roughly the mass contained inside the Hubble volume and the mass fraction at formation time is

$$\beta(M) = \int_{\Delta_c}^{\infty} \frac{d\Delta}{\sqrt{2\pi} \sigma_\Delta} e^{-\Delta^2 / 2\sigma_\Delta^2}$$

$$\sigma^2_\Delta(M) = \int_0^{\infty} d \ln k W^2(k, R) \mathcal{P}_\Delta(k)$$
The variance

\[ \sigma_\Delta(M(k)) = \sqrt{\frac{16}{81} \int \left( \frac{q^3}{2\pi^2} |\zeta_q|^2 \right) \left( \frac{q}{k_c} \right)^4 e^{-(q/k_c)^2} \, d\ln q} \]
The mass fraction

\[ \beta = 1.6 \cdot 10^{-16} \]
The mass fraction vs the variance

Fine-tuning

\[ \beta(\sigma_{\Delta}) \]

\[ \beta = 1.6 \cdot 10^{-16} \]

\[ \Delta_c = 0.45 \]
\[ \frac{\Omega_{\text{PBH}}(M)}{\Omega_{\text{CDM}}} = \frac{\beta(M)}{1.6 \cdot 10^{-16}} \left( \frac{\gamma}{0.2} \right)^{3/2} \left( \frac{g_*(T_f)}{106.75} \right)^{-1/4} \left( \frac{M}{10^{-15} M_\odot} \right)^{-1/2} \]
The non-Gaussianity

\[ \beta(M) = \frac{1}{\sqrt{2\pi} \nu} \exp \left[ -\frac{1}{2} \nu^2 \left( 1 - S_3 \frac{\sigma_\Delta}{3} \left( \nu - 2 - \frac{1}{\nu^2} \right) \right) \right] \]
Gravitational waves from the SM instability

Meanwhile, the same small-scale curvature perturbation induced by the Higgs can source, at second-order, gravitational waves

$$\Box h^s_{ij} = S_{ij}$$

$$S_{ij} = \mathcal{O}(\partial_i \zeta \partial_j \zeta)$$

$$\Omega_{GW}(f) \approx 3 \cdot 10^{-8} \left( \frac{f}{f_*} \right)^{n_T}$$

$$n_T = \begin{cases} 
3 & \text{for } f < f_*, \\
-0.6 & \text{for } f > f_*.
\end{cases}$$
Consistency relation

Three-point correlator

\[ \langle h^r(k_1) h^s(k_2) h^t(k_3) \rangle \equiv (2\pi)^3 \delta(k_1 + k_2 + k_3) B_{h}^{rst}(k_1, k_2, k_3) \]

Shape (peaked at equilateral configurations)

\[ S_h^{rst}(k_1, k_2, k_3) = k_1^2 k_2^2 k_3^2 \frac{B_{h}^{rst}(k_1, k_2, k_3)}{\sqrt{\mathcal{P}_h(k_1) \mathcal{P}_h(k_2) \mathcal{P}_h(k_3)}} \]

\[ \sum_{\text{pol}} S_h = \mathcal{O}(1500) \]
Conclusions

1. The SM Higgs may be responsible of the dark matter we see in the Universe as the result of the EW instability

2. Future interferometers may detect gravitational waves ascribable to the presence of the EW instability: measuring the spectral index as well as the three-point correlator may assess the origin of the signal

Argument: the mechanism needs a fine tuning of one per mille on the initial value of the Higgs, not compatible with dS fluctuations leading to an exponentially small probability.

1. Anthropic principle in action, have to count how many universes, number much larger than small probability.

2. The small probability does not apply to PBH abundance.

3. AdS bubbles may shrink.

4. Increase the initial value of the Higgs, increase number of e-folds plus merging and accretion (PBHs are extremely exponentially clustered).

5. Can avoid AdS by having the quartic positive at large field values, this already relaxes the fine-tuning by ten.

6. Relax the fine-tuning by account for the velocities (KM equation not FP equation).