Complex Langevin analysis

of the spontaneous symmetry breaking in dimensionally reduced super Yang-Mills models

Stratos Kovalkov Papadoudis

National Technical University of Athens

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Acknowledgments

work done with

Konstantinos N. Anagnostopoulos
Takehiro Azuma
Yuta Ito
Jun Nishimura

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Outline

The IIB model

Matrix regularization of the Green-Schwarz action

$$S_{\text{Schild}} = \int d^2\sigma \left(\sqrt{\hat{g}}\alpha \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 - \frac{1}{2}\imath\bar{\psi}\Gamma^\mu\{X^\mu, \psi\} \right) + \beta\sqrt{\hat{g}} \right)$$

$$\{\_,\_\} \rightarrow \imath[\_,\_] \text{ and } \int d^2\sigma\sqrt{g}\,\_ \rightarrow \operatorname{tr}\_$$

Matrix IKKT (Eucleidean) model with fixed N

$$S = S_{\text{bozon}} + S_{\text{fermion}} = -N\operatorname{tr}\left(\frac{1}{4}[A_\mu, A_\nu]^2 + \frac{1}{2}(\bar{\psi}_\alpha(\Gamma_\mu)_{\alpha\beta}[A_\mu, \psi_\beta]) \right)$$

Non-perturbative definition candidate for Type IIB!

The IKKT model

Complex action

Phase produced from integrating fermions in partition function

$$Z = \int \mathcal{D}A \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) = \int \mathcal{D}A \, \mu(\mathcal{M}) \exp(-S)$$

responsible for spontaneous symmetry breaking.

$D = 4$ or 6: $\mu \equiv$ determinant and $D = 10$: $\mu \equiv$ pfaffian

SO(D) spontaneous breaking

Breakdown of original SO(10) to SO(4) would provide dynamical compactification of extra dimensions.

Gaussian Expansion Method

[arXiv:1007.0883] T. Aoyama, J. Nishimura and T. Okubo.
[arXiv:1108.1293] J. Nishimura, T. Okubo and F. Sugino.

$D = 6$

SO(6) breaks down to SO(3)...

$D = 10$

SO(10) breaks down to SO(3)!?

Outline

Monte Carlo

Calculate expectation value integrals,

$$\langle f \rangle_0 = \frac{\int_X f(x)\varrho(x)dx}{\int_X \varrho(x)dx}$$

by sampling configuration space via Monte Carlo:

- improve calculation time by following markovian chains
- maximize calculation efficiency by sampling integration space with appropriate probability
- ϱ while natural is *not* always the best! (overlap problem)

Complex weights

Expectation values now include a sign or phase,

$$w \longrightarrow w e^{\imath\gamma}$$

implying a signed or complex probability which makes no sense in either case and thus

$$\langle f \rangle = \frac{\int_X f(x) w(x) e^{\imath\gamma(x)} dx}{\int_X w(x) e^{\imath\gamma(x)} dx}$$

weight-sampling is not possible.

Re-weighting

Using the phase-quenched model...

A partial solution comes with re-weighting,

$$\langle f \rangle = \frac{\dfrac{\int_X f(x) w(x) e^{\imath\gamma(x)} dx}{\int_X w(x) dx}}{\dfrac{\int_X w(x) e^{\imath\gamma(x)} dx}{\int_X w(x) dx}} = \frac{\langle f e^{\imath\gamma} \rangle_0}{\langle e^{\imath\gamma} \rangle_0}$$

The problem is exponentially hard!

Outline

Langevin equation

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

Insert time evolution for x, $\tau : \mathbb{R}$

$$\frac{\partial}{\partial \tau} x(\tau) = v(x(\tau)) + \eta(\tau) \qquad x(\tau_0) = x_0 : \mathbb{R}$$

Drift $v : \mathbb{R} \to \mathbb{R}$

$$v(x) = -\frac{\partial}{\partial x} s(x) = w(x)^{-1} \frac{\partial}{\partial x} w(x) \qquad s(x) = -\log w(x)$$

Noise $\eta : \mathbb{R}$ with gaussian distribution noise : $\mathbb{R} \to \mathbb{R}_+$

$$\langle \eta(\tau) \eta(\tau') \rangle_{\text{noise}} = 2\delta(\tau - \tau') \qquad \varrho_{\text{noise}}(\eta) = \exp\left(-\frac{1}{4}\eta^2\right)$$

Stochastic behaviour

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

- The solution x to Langevin equation depends on noise η.
- Even the initial condition can be random: $x_0 = \eta_0$
- x has a probability distribution $\varrho : (\mathbb{R} \to \mathbb{R}) \to \mathbb{R}$

Fokker-Planck equation

$$\frac{\partial}{\partial \tau} \varrho(x, \tau) = \mathcal{L}^*(x) \varrho(x, \tau) \qquad \varrho(x, \tau_0) = \delta(x - x_0)$$

Expectation values equation

$$\frac{\partial}{\partial \tau} \langle \mathcal{O}(\tau) \rangle_{\text{noise}} = \langle \mathcal{L} \mathcal{O}(\tau) \rangle_{\text{noise}}$$

Criterion of convergence

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

EV leading contributions

$$\langle \mathcal{O}(\tau) \rangle_{\text{noise}} \sim \int_0^\infty \exp((\tau - \tau_0)u) p(u,\tau) du$$

Drift norm probability

$$p(u,\tau) = \int \delta(u(x) - u) \varrho(x,\tau) dx \qquad u = |v|$$

Criterion for convergence

$$p(u,\tau) \lesssim exp(-\kappa u) \qquad \kappa > 0$$

Stochastic quantization assertion

$$\lim_{\tau \to \infty} \langle \mathcal{O}(\tau) \rangle_{\text{noise}} = \langle \mathcal{O} \rangle_w$$

Outline

Complex Langevin equation

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

Insert time evolution for z, $\tau : \mathbb{R}$

$$\frac{\partial}{\partial \tau} z(\tau) = v(z(\tau)) + \eta(\tau) \qquad z(\tau_0) = z_0 : \mathbb{R}$$

Drift $v : \mathbb{C} \to \mathbb{C}$

$$v(z) = w(x)^{-1} \frac{\partial}{\partial x} w(x) \Big|_{x \to x + \imath y = z}$$

Noise $\eta : \mathbb{R}$ with gaussian distribution noise : $\mathbb{R} \to \mathbb{R}_+$

$$\langle \eta(\tau)\eta(\tau')\rangle_{\text{noise}} = 2\delta(\tau - \tau') \qquad \text{noise}(\eta) = \exp\left(-\frac{1}{4}\eta^2\right)$$

Discretized Langevin equation

[arXiv:1802.01876] Keitaro Nagata, Jun Nishimura and Shinji Shimasaki.

Discretized time t with n time-steps fixed or variable

$$\tau = n\bar{\epsilon}$$

Insert time evolution for $z : \mathbb{C}$, $n : \mathbb{N}$

$$z_{n+1} = z_n + \epsilon v(z_n) + \sqrt{\epsilon}\eta_n \qquad z_0 : \mathbb{C}$$

Drift $v : \mathbb{C} \to \mathbb{C}$

$$v(z) = w(x)^{-1} \frac{\partial}{\partial x} w(x) \Big|_{x \to z}$$

Noise $\eta : \mathbb{R}$ with gaussian distribution noise : $\mathbb{R} \to \mathbb{R}_+$

$$\langle \eta_n \eta_{n'} \rangle_{\text{noise}} = 2\delta_{nn'}$$

Outline

IKKT Langevin dynamics

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT Langevin equation

$$\frac{d(A_\mu)_{ij}}{d\tau} = -\frac{\delta}{\delta(A_\mu)_{ji}} S + (\eta_\mu)_{ij} \quad A_\mu \text{ are hermitian originally}$$

IKKT drift term

$$\frac{\delta}{\delta(A_\mu)_{ji}} S_{\text{bozon}} = N[[A_\mu A_\nu] A_\nu]_{ij}$$

$$\frac{\delta}{\delta(A_\mu)_{ji}} S_{\text{fermion}} = -\text{tr}\left(\left(\frac{\delta}{\delta(A_\mu)_{ji}} \mathcal{M}\right) \mathcal{M}^{-1}\right)$$

IKKT complexification

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

- A_μ no longer hermitian, just traceless!
  - Still, the closer it is to hermitian the better.
  - SU(N) matrix gauge symmetry becomes SL(N)
- Noise remains hermitian (and traceless!).
- Observables $\mathcal{O}$ must be holomorphic extensions of the real counter-part.

Outline

1. The IKKT model
2. Complex Action
   - The problem...
   - Langevin dynamics
   - Complex Langevin dynamics
3. IKKT Langevin dynamics
   - The theory...
   - The study...
   - The results...

IKKT Langevin issues

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Imaginary part escaping

Counter with gauge cooling?

Singular drift

Counter by shifting fermion matrix:

$$\Delta S_{\text{fermion}} = N m_{\text{fermion}} \mathrm{tr}(\bar{\psi}_\alpha (\Gamma_6)_{\alpha\beta} \psi_\beta) \qquad \Gamma_6 = 1_2 \otimes 1_2 = 1_4$$

Probe SSB by mass breaking-term

$$\Delta S_{\text{boson}} = \frac{1}{2} N \varepsilon \sum_\mu m_\mu \mathrm{tr} A_\mu A_\mu$$

IKKT observables

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

SSB order parameter

$$\lambda_\mu = N^{-1}\,\mathrm{tr}(A_\mu A_\mu) \qquad \Delta S_{\mathrm{boson}} = \frac{1}{2}N^2\varepsilon\sum_\mu m_\mu\lambda_\mu$$

SO(6) breaking to SO(2)

$$m_\mu = (0.5, 0.5, 1.0, 2.0, 4.0, 8.0)$$
$$\lambda_{12} = \frac{1}{2}(\lambda_1 + \lambda_2)$$

IKKT order parameters

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Order parameter expectation values

$$\rho_\mu(N, \varepsilon, m_{\text{fermion}}) = R^{-1}\langle\lambda_\mu\rangle \qquad R = \sum_\nu \langle\lambda_\nu\rangle$$

Large N limit

$\lim_{N\to\infty} \rho_\mu(N, \varepsilon, m_{\text{fermion}}) = \rho_\mu(\varepsilon, m_{\text{fermion}})$

SSB limit

$\lim_{\varepsilon\to\infty} \rho_\mu(\varepsilon, m_{\text{fermion}}) = \rho_\mu(m_{\text{fermion}})$

Original model limit

$\lim_{m_{\text{fermion}}\to\infty} \rho_\mu(m_{\text{fermion}}) = \rho_\mu$

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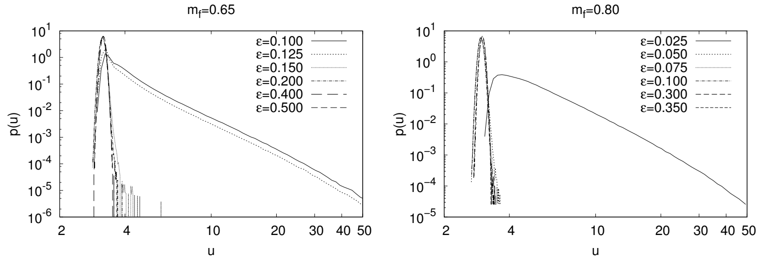
IKKT drift norm u

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT drift histogram

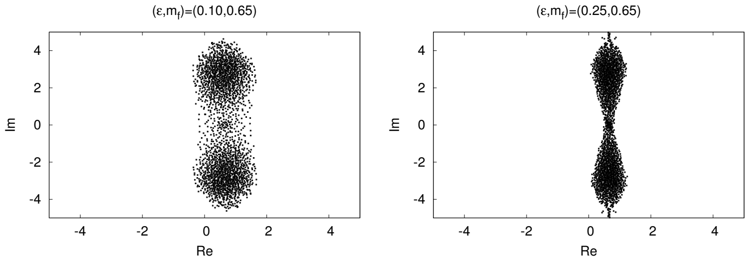


$$u^2 = \frac{1}{6N^3} \sum_{\mu} \sum_{ij} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2$$

IKKT fermion matrix $\mathcal{M}$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

IKKT fermion matrix eigenvalue scatter-plot

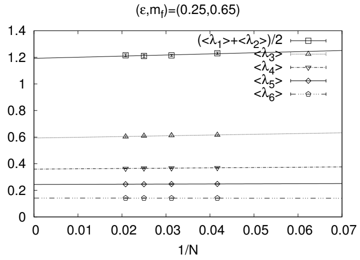


$$\Delta S_{\text{fermion}} = N m_{\text{fermion}} \text{tr}(\bar{\psi}_\alpha (\Gamma_6)_{\alpha\beta} \psi_\beta) \qquad \Gamma_6 = 1_2 \otimes 1_2 = 1_4$$

IKKT order parameters $N \to \infty$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Eliminating finite-size effects at the large-N limit $N \to 0$

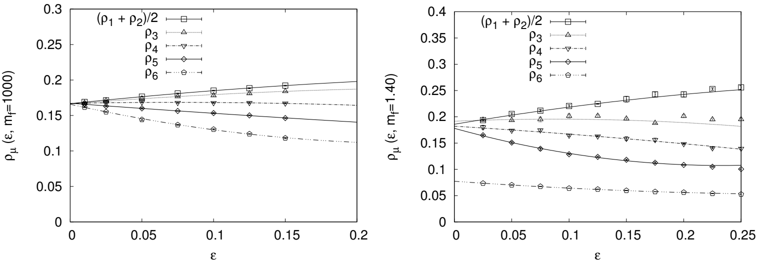


$$\lim_{N\to\infty} \rho_\mu(N, \varepsilon, m_{\text{fermion}}) = \rho_\mu(\varepsilon, m_{\text{fermion}})$$

IKKT order parameters $\varepsilon \to 0$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Quadratic extrapolation to SSB at $\varepsilon \to 0$

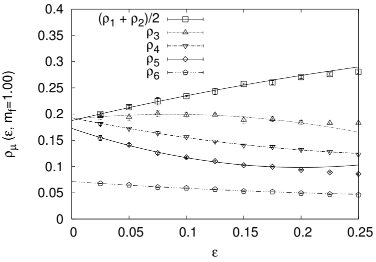


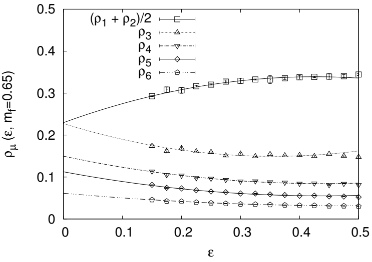
$$\lim_{\varepsilon \to \infty} \rho_\mu(\varepsilon, m_{\text{fermion}}) = \rho_\mu(m_{\text{fermion}})$$

IKKT order parameters $\varepsilon \to 0$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Quadratic extrapolation to SSB at $\varepsilon \to 0$



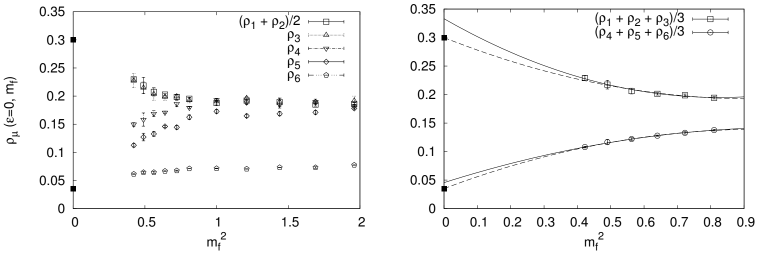


$$\lim_{\varepsilon\to\infty} \rho_\mu(\varepsilon, m_{\text{fermion}}) = \rho_\mu(m_{\text{fermion}})$$

IKKT order parameters $m_{\text{fermion}} \to 0$

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Quadratic extrapolation to original model at $m_{\text{fermion}} \to 0$



$$\lim_{m_{\text{fermion}} \to \infty} \rho_\mu(m_{\text{fermion}}) = \rho_\mu$$

Summary

[arXiv:1712.07562] Konstantinos N. Anagnostopoulos, Takehiro Azuma, Yuta Ito, Jun Nishimura and Stratos Kovalkov Papadoudis.

Conclusions:

- Spontaneous Symmetry Breaking of the reduced super Yang-Mills models (matrix) was observed for $D = 6$.
- At $N \to \infty$, $\varepsilon \to 0$, as m_{fermion} decreases, symmetry starts to break down to SO(3) as expected by the GEM result.

Outlook:

- Study of the real model $D = 10$ (work in progress)
- Looking for SO(10) $\to$ SO(4) for meaningful interpretation of dynamical compactification to 4-dimensional space-time.

~ The End ~