$B-L$ Higgs Inflation in Supergravity with Several Consequences

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Based on:

- *C.P.*, *Universe* **4**, no.1, 13 (2018) [arXiv:1710.05759].

Outline

HEP 2018: Recent Developments in High Energy Physics and Cosmology.

SUGRA (i.e. Supergravity) Potential

- The General Einstein Frame Action For The Scalar Fields z^α Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{g}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-\widehat{V}\right)$$ Where We Use Units With $m_{\rm P}$=1.

Also K is The **Kähler Potential** With $K_{\alpha\bar\beta}=\dfrac{\partial^2 K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma}$; $D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta$, Where A^a_μ is The Vector Gauge Fields and T_a are the Generators of the Gauge Transformations Of z^α; Finally, $\widehat{V}=\widehat{V}_{\rm F}+\widehat{V}_{\rm D}$ With

$$\widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)$$ With W The **Superpotential** and ${\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$; $\widehat{V}_{\rm D}=\dfrac{1}{2}g^2{\rm D}_a^2$ with ${\rm D}_a=z_\alpha\left(T_a\right)^\alpha_\beta K_{,z^\beta}$.

- We Concentrate on **Higgs Inflation** (**HI**) Driven by $\widehat{V}_{\rm F}$ Since We Can Easily Assure $\widehat{V}_{\rm D}=0$ During HI.

> Therefore, HI Within SUGRA Requires The Appropriate Selection of the Functions W and K

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- **Difficulties And Possible Ways Out**
  - **The η Problem.** Coefficients of Order Unity in K May Spoil The Flatness of $\widehat{V}_{\rm F}$ Due To The Factor e^K. This Can Be Evaded If We Impose A **Shift Symmetry** so That $K=K(\Phi-\Phi^*)=K({\rm Im}(\Phi))$ and the Inflaton be $\phi=\sqrt{2}{\rm Re}(\Phi)$.

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- Complementarily, From Models of **non-Minimal Chaotic Inflation** (**nMI**) in SUGRA We know that $\widehat{V}_{\rm F}$ is Sufficiently Flat, If We Adopt $K=-{\rm N}\ln\left(1+c_{\mathcal{R}}(\Phi^{\rm p}+\Phi^{*\rm p})\right)+\cdots$ and Tune $N>0$ and n With The Exponent m of Φ in $W=\lambda S\Phi^{\rm q}$. E.g.,

If we Select $W=\lambda S\Phi^2$ and $K=-2\ln\left(1+2c_{\mathcal{R}}(\Phi^2+\Phi^{*2})\right)-(\Phi-\Phi^*)^2/2+|S|^2$

We Obtain $\widehat{V}_{\rm F}=e^KK^{SS^*}\left|W_{,S}\right|^2=\lambda^2\phi^4/4(1+c_{\mathcal{R}}\phi^2)^2\sim{\rm const}$ for $c_{\mathcal{R}}\gg1$.

How We can Apply These General Ideas to HI?

Selecting Conveniently the Superpotential and Kähler Potentials

- We Use 3 Superfields $z^1 = \Phi$, $z^2 = \bar{\Phi}$, **Charged** Under a Local Symmetry, e.g. $U(1)_{B-L}$, and $z^3 = S$ (**"Stabilizer" Field**).
  - **Superpotential** $W = \lambda S\left(\bar{\Phi}\Phi - M^2/4\right)$
  - W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry and Leads to a **Grand Unified Theory (GUT)** Phase Transition

Charge Assignments

Superfields:	S	Φ	$\bar{\Phi}$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	-1

At The SUSY Vacuum $\langle S\rangle = 0, |\langle\Phi\rangle| = |\langle\bar{\Phi}\rangle| = M/2$,
Since in The SUSY Limit, After HI, We Get

$$V_{\rm SUSY} = \lambda^2\left|\bar{\Phi}\Phi - M^2/4\right|^2 + \frac{1}{c_-(1-2r_\pm)}\lambda^2|S|^2\left(|\Phi|^2+|\bar{\Phi}|^2\right) + {\rm D-terms}$$

(c_- and $r_\pm$ Are Defined Below)

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- **Possible Kähler Potentials – Softly Broken Shift Symmetry For Higgs Fields**

- The Shift Symmetry Can Be Formulated By **The Functions** $F_\pm=\left|\Phi\pm\bar\Phi^*\right|^2$ With Coefficients c_+ and c_-, $c_+\leq c_-$.
- HI can be Obtained Selecting the Following K's Which Are **Quadratic and Invariant** Under $U(1)_{B-L}$ and R Symmetries:

$$K_1=-2\ln\left(1+c_+F_+\right)+c_-F_-+\ F_{1S}(|S|^2),\quad K_2=-2\ln\left(1+c_+F_+\right)+\ F_{2S}(F_-,|S|^2)$$ Where We Choose The Functions[1]

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Since the Simplest Kinetic Term for S, $|S|^2$, Leads to $m_S^2<0$ or $m_S^2<\widehat{H}_{\rm HI}^2$ Along the Inflationary Path.

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- For $c_+ \ll c_-$, Our Models are Completely **Natural**, Because The Theory Enjoys The Following **Enhanced Symmetries**:

$$\bar\Phi \to \bar\Phi + c^*,\ \Phi \to \Phi + c\ \ (c \in \mathbb{C})\ \ \text{and}\ \ S \to e^{i\alpha}S\,,\ \ \text{in the Limits}\ \ c_+ \to 0\ \&\ \lambda \to 0\,.$$

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- The **Free Parameters**, are **$r_\pm=c_+/c_-$ and λ/c_-** (not c_+, c_- and λ) Since If We Perform the Rescalings

$$\Phi\to\Phi/\sqrt{c_-},\ \ \bar\Phi\to\bar\Phi/\sqrt{c_-},\ \text{ and }\ S\to S,\ \text{ we see That } W \text{ Depends on } \lambda/c_- \text{ and } K \text{ on } r_\pm\,.$$

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The Relevant Super- & Kähler Potentials

• Promoting To Local The Already Existing $U(1)_{B-L}$ Global Symmetry of the MSSM, We Obtain a Superpotential Invariant under the $G_{\rm SM}\times U(1)_{B-L}$ Gauge Group:

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to Achieve HI & Break $U(1)_{B-L}$

$$+\ \lambda_\mu S H_u H_d$$

to Generate $\mu\sim 1$ TeV

$$+\ \lambda_{ij\nu}\bar\Phi N^c_i N^c_j$$

to Generate Majorana Masses for Neutrinos & Ensure The Inflaton Decay

$$+\ h_{ijN} N^c_i L_j H_u$$

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$$+\ W_{\rm MSSM}\ \text{with}\ \mu=0$$

(Note that 3 Right-Handed Neutrinos, N^c_i, Are Necessary To Cancel the $B-L$ **Gauge Anomaly**)

Super-fields	Representations under $G_{\rm SM}\times U(1)_{B-L}$	Global Symmetries R	B	L
Matter Fields				
e^c_i	$(\mathbf{1},\mathbf{1},1,1)$	0	0	-1
N^c_i	$(\mathbf{1},\mathbf{1},0,1)$	0	0	-1
L_i	$(\mathbf{1},\mathbf{1},-1/2,-1)$	2	0	1
u^c_i	$(\mathbf{3},\mathbf{2},-2/3,-1/3)$	1	$-1/3$	0
d^c_i	$(\mathbf{3},\mathbf{2},1/3,-1/3)$	1	$-1/3$	0
Q_i	$(\bar{\mathbf{3}},\mathbf{2},1/6,-1/3)$	1	$1/3$	0
Higgs Fields				
H_d	$(\mathbf{1},\mathbf{2},-1/2,0)$	0	0	0
H_u	$(\mathbf{1},\mathbf{2},1/2,0)$	0	0	0
S	$(\mathbf{1},\mathbf{1},0,0)$	4	0	0
$\bar\Phi$	$(\mathbf{1},\mathbf{1},0,2)$	0	0	-2
Φ	$(\mathbf{1},\mathbf{1},0,-2)$	0	0	2

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- The Above W May Cooperate With The Following Kähler Potential Potentials Which Respect the Imposed Symmetries

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$$F_{1S} = N_X\ln\left(1+X^\alpha X_\alpha/N_X\right)\quad \text{And}\quad F_{2S} = N_X\ln\left(1+X^\alpha X_\alpha/N_X + c_-F_-/N_X\right)\quad \text{With}\ N, N_X>0$$

and $X^\alpha = S, H_u, H_d, N^c_i$ – Placing $X^\alpha X_\alpha$ Inside the Argument of ln, We Obtain Similar Results.

Generating the μ-Term of MSSM

- The Origin of the μ Term Can be Explained If We Combine the Terms[2]

$$W_{\rm HI} + W_\mu = \lambda S\left(\bar\Phi\Phi - M^2/4\right) + \lambda_\mu S H_u H_d\,. \quad \textbf{(: I)}$$

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- The **Soft SUSY Breaking Terms** Corresponding to $W_{\rm HI} + W_{\mu}$ Are Included In

$$V_{\rm soft} = \left(\lambda A_\lambda S\bar\Phi\Phi + \lambda_\mu A_\mu S H_u H_d - {\rm a}_S S\lambda M^2/4 + {\rm h.c.}\right) + m_{\bar\alpha}^2\left|z^{\bar\alpha}\right|^2 \quad \text{with} \quad z^{\bar\alpha} = \Phi, \bar\Phi, S, H_u, H_d$$

where $m_\alpha, A_\lambda, A_\mu$ and ${\rm a}_S$ are Soft SUSY Breaking Mass Parameters.

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- **Minimizing** $V_{\rm tot} = V_{\rm SUSY} + V_{\rm soft}$ and Substituting in $V_{\rm soft}$ the SUSY v.e.vs of Φ and $\bar\Phi$ we get

$$\langle V_{\rm tot}(S)\rangle = \lambda^2 M^2 S^2/2c_-(1-2r_\pm) - \lambda {\rm a}_\mu M^2 S, \quad \text{where } m_S \ll M \text{ and } (|A_\lambda| + |{\rm a}_S|) = 2{\rm a}_\mu m_{3/2}$$

With $m_{3/2}$ being the Gravitino Mass. The Minimized $\langle V_{\rm tot}(S)\rangle$ w.r.t S leads to a **non-Vanishing $\langle S\rangle$** as Follows:

$$d\langle V_{\rm tot}(S)\rangle/dS = 0 \;\Rightarrow\; \langle S\rangle \simeq {\rm a}_\mu c_-(1-2r_\pm)m_{3/2}/\lambda \simeq 10^5 {\rm a}_\mu m_{3/2}\mathcal{F}(r_\pm).$$

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[2] *G. Dvali, G. Lazarides and Q. Shafi (1999).*

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Generating the μ-Term of MSSM

- The Origin of the μ Term Can be Explained If We Combine the Terms[2]

$$W_{\rm HI}+W_\mu=\lambda S\left(\bar\Phi\Phi-M^2/4\right)+\lambda_\mu S H_u H_d\,. \text{ (: I)}$$

- The **Soft SUSY Breaking Terms** Corresponding to $W_{\rm HI}+W_\mu$ Are Included In

$$V_{\rm soft}=\left(\lambda A_\lambda S\bar\Phi\Phi+\lambda_\mu A_\mu S H_u H_d-{\rm a}_S S\lambda M^2/4+{\rm h.c.}\right)+m_{\bar\alpha}^2\left|z^{\bar\alpha}\right|^2\quad \text{with } z^{\bar\alpha}=\Phi,\bar\Phi,S,H_u,H_d$$

where $m_\alpha, A_\lambda, A_\mu$ and ${\rm a}_S$ are Soft SUSY Breaking Mass Parameters.

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- The Allowed λ_μ Values Render Our Models Compatible With The **Best-Fit Points** in the CMSSM[3] E.g., Setting

$$m_0 = m_{3/2} \quad \text{and} \quad |A_\lambda| = |{\rm a}_S| = |A_0|.$$

CMSSM Region	$\|A_0\|$ (TeV)	m_0 (TeV)	$\|\mu\|$ (TeV)	${\rm a}_\mu$	λ_μ (10^{-6})
A/H Funnel	9.9244	9.136	1.409	1.086	**0.607**
$\tilde\tau_1-\chi$ Coannihilation	1.2271	1.476	2.62	0.831	**9.12**
$\tilde t_1-\chi$ Coannihilation	9.965	4.269	4.073	2.33	**1.75**
$\tilde\chi_1^\pm-\chi$ Coannihilation	9.2061	9.000	0.983	1.023	**0.456**

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The Inflationary Potential

- If We Use The Parametrization:

$$\Phi = \phi e^{i\theta}\cos\theta_\Phi/\sqrt{2} \quad \text{and} \quad \bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi/\sqrt{2} \quad \text{with} \quad 0\leq\theta_\Phi\leq\pi/2 \quad \text{and} \quad X^\beta = \left(x^\beta + i\bar{x}^\beta\right)/\sqrt{2},$$

Where $X^\beta = S, H_u, H_d, N^c_i$, We Can Show That A D-Flat Direction Is $\theta=\bar\theta=x^\beta=\bar{x}^\beta=0$, and $\theta_\Phi=\pi/4$ (: I)

- The Only **Surviving Term** of $\widehat{V}_{\rm F}$ Along the Path in Eq. (I) is

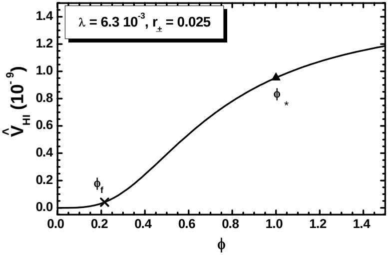
$$\widehat{V}_{\rm HI} = e^K K^{SS^*}\,|W_{,S}|^2 = \frac{\lambda^2(\phi^2-M^2)^2}{16 f_{\mathcal{R}}^2} \quad \text{With} \quad f_{\mathcal{R}} = 1 + c_+\phi^2$$

Playing The Role Of A **Non-Minimal Coupling to Gravity**.

- Along the Inflationary Path $K_{\alpha\bar\beta}$ Takes The Form

$$\left(K_{\alpha\bar\beta}\right) = \text{diag}\left(M_\pm, K_{SS^*}\right) \quad \text{with} \quad M_\pm = \frac{1}{f_{\mathcal{R}}^2}\begin{pmatrix}\kappa & \bar\kappa\\ \bar\kappa & \kappa\end{pmatrix},$$

and $K_{SS^*}=1$. Here $\kappa = c_- f_{\mathcal{R}}^2 - 2c_+$ and $\bar\kappa = 2c_+^2\phi^2$.



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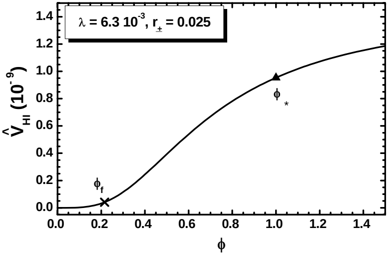

- The EF Canonically Normalized Fields, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{\kappa_+},\ \ \widehat\theta_+ = \frac{J\phi\theta_+}{\sqrt{2}},\ \ \widehat\theta_- = \sqrt{\frac{\kappa_-}{2}}\phi\theta_-, \quad\text{and}\quad \widehat\theta_\Phi = \phi\sqrt{\kappa_-}\left(\theta_\Phi - \frac{\pi}{4}\right),\ \ \left(\widehat{x}^\beta, \widehat{\bar{x}}^\beta\right) = \left(x^\beta, \bar{x}^\beta\right),$$

Where $\theta_\pm = (\theta\pm\bar\theta)/\sqrt{2},\ \kappa_+ = c_-\left(1 + 2r_\pm(c_+\phi^2-1)/f_{\mathcal R}^2\right) \simeq c_-$ and $\kappa_- = c_-\left(1 - 2r_\pm/f_{\mathcal R}\right) > 0 \quad\Rightarrow\quad \mathbf{r_\pm < 1/2}$.

- We Can Check the Stability of the Trajectory in Eq. (I) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial\widehat{z}^\alpha}\right|_{\rm Eq.\ (I)} = 0 \quad\text{and}\quad \widehat{m}^2_{z^\alpha} > 0 \quad\text{Where}\quad \widehat{m}^2_{z^\alpha} = {\rm Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \quad\text{With}\quad \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat{z}^\alpha\partial\widehat{z}^\beta}\right|_{\rm Eq.\ (I)} \quad\text{and}\quad z^\alpha = \theta_-, \theta_+, \theta_\Phi, x^\beta, \bar{x}^\beta.$$

Stability and Radiative Corrections

The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared		
			$K=K_1$	$K=K_2$
2 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}^2_{\theta+}$	$6\widehat{H}^2_{\rm HI}$	$6(1+1/N_X)\widehat{H}^2_{\rm HI}$
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta\Phi}$	$M^2_{BL}+6\widehat{H}^2_{\rm HI}$	$M^2_{BL}+6(1+1/N_X)\widehat{H}^2_{\rm HI}$
1 Complex Scalars	$\widehat{s},\widehat{\bar{s}}$	$\widehat{m}^2_s$	$6\widehat{H}^2_{\rm HI}/N_X$	
4 Complex Scalars	$h_\pm,\bar{h}_\pm$	$\widehat{m}^2_{h\pm}$	$3\widehat{H}^2_{\rm HI}(1+1/N_X\pm4\lambda_\mu/\lambda\phi^2)$	
3 Complex Scalars	$\tilde{\nu}^c_i,\bar{\tilde{\nu}}^c_i$	$m^2_{i\tilde{\nu}^c}$	$3\widehat{H}^2_{\rm HI}(1+1/N_X+16\lambda_{iN^c}/\lambda^2\phi^2)$	
1 Gauge boson	A_{BL}	M^2_{BL}	$g^2c_-(1-2r_\pm/f_{\mathcal{R}})\phi^2$	
4 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$24\widehat{H}^2_{\rm HI}/c_-\phi^2f^2_{\mathcal{R}}$	
	ψ_{iN^c}	$\widehat{m}^2_{\psi_{iN^c}}$	$48\lambda^2_{iN^c}\widehat{H}^2_{\rm HI}/\lambda^2\phi^2$	
	$\lambda_{BL},\widehat{\psi}_{\Phi-}$	M^2_{BL}	$g^2c_-(1-2r_\pm/f_{\mathcal{R}})\phi^2$	

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>0$. Especially

$$\widehat{m}^2_s>0\ \Leftrightarrow\ N_X<6\ \text{and}\ \widehat{m}^2_{H-}>0\ \Leftrightarrow\ \lambda_\mu\leq\lambda(1+1/N_X)\phi_{\rm f}/4\ \text{(E.g. }\lambda_\mu<9\cdot10^{-6}\ \text{for}\ r_\pm=0.03).$$

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha}>\widehat{H}^2_{\rm HI}$ and So Any Inflationary Perturbations Of The Fields Other Than ϕ Are Safely Eliminated;
- $M_{BL}\neq0$ Signals the Fact that That $U(1)_{B-L}$ Is Broken and so, **no Topological Defects** are Produced.
- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $\widehat{V}_{\rm HI}$ Can Be Kept Under Control Provided that
  - $M^2_{BL}>m^2_{\rm P}$ and $\widehat{m}^2_{\theta_\Phi}>m^2_{\rm P}$ Are not Taken Into Account.
  - The Renormalization Group Mass Scale Λ Is Determined By Requiring $\Delta\widehat{V}_{\rm HI}(\phi_\star)=0$ or $\Delta\widehat{V}_{\rm HI}(\phi_{\rm f})=0$.

Approximating the Inflationary Dynamics

- The **Slow-Roll Parameters** Are Determined Using the Standard Formulae Employing The Canonically Normalized $\widehat{\phi}$:

$$\widehat{\epsilon}=\frac{1}{2}\left(\frac{\widehat{V}_{\rm HI,\widehat{\phi}}}{\widehat{V}_{\rm HI}}\right)^2\simeq\frac{8}{c_-\phi^2 f_{\mathcal{R}}^2}\quad\text{AND}\quad\widehat{\eta}=\frac{\widehat{V}_{\rm HI,\widehat{\phi}\widehat{\phi}}}{\widehat{V}_{\rm HI}}=12\,\frac{1-c_+\phi^2}{c_-\phi^2 f_{\mathcal{R}}^2}\,.$$

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$$\widehat{N}_\star = \int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_\star} d\widehat{\phi}\,\frac{\widehat{V}_{\rm HI}}{\widehat{V}_{\rm HI,\widehat{\phi}}} \simeq ((1+c_+\phi_\star^2)^2-1)/16r_\pm$$

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$$\phi_\star \leq 1 \quad\Rightarrow\quad c_- \geq (f_{\mathcal{R}\star}-1)/r_\pm \simeq 100, \quad \text{with} \quad f_{\mathcal{R}\star} = \left(1+16r_\pm\widehat{N}_\star\right)^{1/2} \quad \text{and} \quad \widehat{N}_\star \simeq 58.$$

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- A Clear **Dependence of The Observables** (Spectral Index $n_{\rm s}$ and Tensor-To-Scalar Ratio, r) On $r_\pm$ and n Arises, I.e.,

$$n_{\rm s} = 1-6\widehat{\epsilon}_\star + 2\widehat{\eta}_\star \simeq 1-\frac{3}{2\widehat{N}_\star}-\frac{3}{8(\widehat{N}_\star^3 r_\pm)^{1/2}}\,,\quad r = 16\widehat{\epsilon}_\star \simeq +\frac{1}{2\widehat{N}_\star^2 r_\pm}+\frac{2}{(\widehat{N}_\star^3 r_\pm)^{1/2}}\,,$$

With Negligible $n_{\rm s}$ Running, $\alpha_{\rm s}$. The Variables With Subscript $\star$ Are Evaluated at $\widehat{\phi} = \widehat{\phi}_\star$.

Testing Against Observations

- The **Combined** Bicep2/*Keck Array* **and *Planck* Results**[4] Although Do Not Exclude Inflationary Models With Negligible r's, They **Seem to Favor** Those With r's of Order 0.01 Which Imply **Observable Gravitational Waves**.

Current Data: $r=0.028^{+0.026}_{-0.025} \;\Rightarrow\; 0.003\lesssim r\lesssim 0.054$ at 68% c.l. And $r\leq 0.07$ at 95% c.l.

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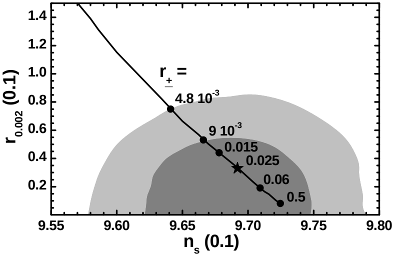

- For **Quite Natural $r_\pm$'s** We can Obtain Results Within the 1-σ Observationally Favored Range, I.e.,

$$9.63 \lesssim n_s/0.1 \lesssim 9.72 \text{ and } 0.7 \lesssim r/0.01 \lesssim 8.1\,.$$

Also, $3.46 \lesssim \widehat{m}_{\delta\phi}/10^{10}\text{GeV} \lesssim 420$.
- For $n_s = \mathbf{0.968}$ We Obtain $r = 0.043$;
- **Best-Fit Point:** $r_\pm = 0.025 \Rightarrow (n_s, r) = (\mathbf{0.969}, \mathbf{0.033})$ and $\widehat{m}_{\delta\phi} \simeq 8.63 \cdot 10^{10}\text{GeV}$.
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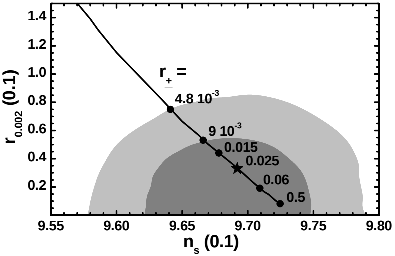

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- The Effective Theory is Valid Since $\widehat{V}_{\rm HI}^{1/4} \leq \Lambda_{\rm UV}$.

- **The Ultraviolet (UV) Cut-off Scale** is $\Lambda_{\rm UV} = m_{\rm P}$ Since The Expansions Abound $\langle\phi\rangle = 0$ Are Just ***$r_\pm$* Dependent**:

$$J^2\dot\phi^2 \simeq \left(1 + 6r_\pm^2\widehat{\phi}^2 - 10r_\pm^3\widehat{\phi}^4 + \cdots\right)\dot{\widehat{\phi}}^2 \quad\text{and}\quad \widehat{V}_{\rm HI} \simeq \frac{\lambda^2\widehat{\phi}^4}{16c_-^2}\left(1 - 2r_\pm\widehat{\phi}^2 + 3r_\pm^2\widehat{\phi}^4 - \cdots\right).$$

Consequently, **No Problem** With The Perturbative Unitarity Emerges for $r_\pm \leq 1$, Even If c_+ and c_- Are Large.

[4] *Planck Collaboration (2015);* Bicep2/*Keck Array and Planck Collaborations (2015)*

Perturbative Reheating

- At the SUSY Vacuum, The Inflaton And The RHNs, N_i^c, Acquire Masses $\widehat{m}_{\delta\phi}$ and M_{iN^c} Respectively Given by

$$\widehat{m}_{\delta\phi} \simeq \frac{\lambda M}{\sqrt{2c_-(1-2r_\pm)}} \;\; (\text{E.g. } 9\cdot 10^{10}\ \text{GeV for } r_\pm = 0.03) \;\; \text{and} \;\; M_{iN^c} = \lambda_{iN^c} M\,,$$

Where **We Restore $m_{\rm P}$** in the Formulas. $\widehat{m}_{\delta\phi}$ is only N and $r_\pm$ Dependent If We Impose a GUT Condition – See Below.

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- The Inflaton Can Decay Perturbatively Into:
  - **A Pair of RHNs** (N_j^c) With Majorana Masses M_{jN^c} Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to N_i^c}=\frac{\lambda_{iN^c}^2}{16\pi}\widehat{m}_{\delta\phi}\left(1-\frac{4M_{iN^c}^2}{\widehat{m}_{\delta\phi}^2}\right)^{3/2}\quad \text{With}\quad \lambda_{iN^c}=\frac{M_{iN^c}}{2\langle J\rangle M}\left(1-3c_+\frac{M^2}{m_{\rm P}^2}\right)\ \text{Arising from}\ \mathcal{L}_{\widehat{\delta\phi}\to N_i^c}=\lambda_{iN^c}\widehat{\delta\phi}\,N_i^cN_i^c\,.$$

  - **H_u and H_d** Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to H}=\frac{2}{8\pi}\lambda_H^2\widehat{m}_{\delta\phi}\quad \text{with}\quad \lambda_H=\frac{\lambda_\mu}{\sqrt{2}}\left(1-2c_+\frac{M^2}{m_{\rm P}^2}\right)\ \text{Arising from}\ \mathcal{L}_{\widehat{\delta\phi}\to H_uH_d}=-\lambda_H\widehat{m}_{\delta\phi}\widehat{\delta\phi}H_u^*H_d^*\,.$$

  - **MSSM (s)-Particles XYZ** Through The Following c_+-Dependent 3-Body Decay Width

$$\widehat{\Gamma}_{\delta\phi\to XYZ}=\lambda_y^2\frac{14}{512\pi^3}\frac{\widehat{m}_{\delta\phi}^3}{m_{\rm P}^2}\quad \text{With}\quad \lambda_y=Ny_3c_+\frac{M}{\langle J\rangle m_{\rm P}}\ \text{and}\ y_3=h_{t,b,\tau}(\widehat{m}_{\delta\phi})\simeq 0.5\,.$$

  This Decay Arises From $\mathcal{L}_{\widehat{\delta\phi}\to XYZ}=-\lambda_y(\widehat{\delta\phi}/m_{\rm P})\,(X\psi_Y\psi_Z+Y\psi_X\psi_Z+Z\psi_X\psi_Y)+\text{h.c.}$

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  This Decay Arises From $\mathcal{L}_{\widehat{\delta\phi}\to XYZ} = -\lambda_y(\widehat{\delta\phi}/m_{\rm P})\left(X\psi_Y\psi_Z + Y\psi_X\psi_Z + Z\psi_X\psi_Y\right) + \text{h.c.}$

- The Reheating Temperature, $T_{\rm rh}$, is given by

$$T_{\rm rh} = \left(72/5\pi^2 g_*\right)^{1/4}\widehat{\Gamma}_{\delta\phi}^{1/2} m_{\rm P}^{1/2} \ \text{with}\ \widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi\to N_i^c} + \widehat{\Gamma}_{\delta\phi\to H} + \widehat{\Gamma}_{\delta\phi\to XYZ}, \ \text{with}\ g_* \simeq 228.75\,.$$

Leptogenesis and $\widetilde{G}$ Abundance

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$$Y_B = -0.35\ 2\ \frac{5}{4}\ \frac{T_{\rm rh}}{\widehat{m}_{\delta\phi}}\ \frac{\widehat{\Gamma}_{\delta\phi\to N_i^c}}{\widehat{\Gamma}_{\delta\phi}}\varepsilon_i \quad \text{Where} \quad \varepsilon_i = \sum_{j\neq i}\frac{\mathrm{Im}\left[(m_{\rm D}^\dagger m_{\rm D})_{ij}^2\right]}{8\pi\langle H_u\rangle^2(m_{\rm D}^\dagger m_{\rm D})_{ii}}\Big(F_{\rm S}\left(x_{ij},y_i,y_j\right)+F_{\rm V}(x_{ij})\Big).$$

With $x_{ij} := M_{jN^c}/M_{iN^c}$ and $y_i := \Gamma_{iN^c}/M_{iN^c} = (m_{\rm D}^\dagger m_{\rm D})_{ii}/8\pi\langle H_u\rangle^2$ and $\widehat{m}_{\delta\phi} < 2M_{iN^c}$ For Some i with $i = 1,2,3$.

- Here $F_{\rm V}$ and $F_{\rm S}$ Represent, Respectively, The Contributions From **Vertex And Self-Energy** Diagrams

$$F_{\rm V}(x) = -x\ln\left(1+x^{-2}\right) \quad \text{and} \quad F_{\rm S}(x,y,z) = -2x(x^2-1)/\left(x^2-1\right)^2+\left(x^2z-y\right)^2$$

[5] *M. Kawasaki, K. Kohri, and T. Moroi (2005); J.R. Ellis, K.A. Olive, and E. Vangioni (2005).*

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- The **Thermally Produced $\widetilde{G}$ Yield** At The Onset of BBN Is Estimated To Be: $Y_{\widetilde{G}} \simeq 1.9\cdot 10^{-22}T_{\rm rh}/{\rm GeV}$.

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(iii) The **Achievement Of Baryogenesis** via non-Thermal Leptogenesis Dictates at 95% c.l. $Y_B = \left(8.64^{+0.15}_{-0.16}\right)\cdot 10^{-11}$.

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(iv) $\widetilde{G}$ Constraints. Assuming Unstable $\widetilde{G}$, We Impose an Upper Bound[5] on $Y_{\widetilde{G}}$ In Order to Avoid Problems With the SBB Nucleosynthesis:

$$Y_{\widetilde{G}}\lesssim\begin{cases}10^{-14}\\10^{-13}\end{cases}\Rightarrow\ T_{\rm rh}\lesssim\begin{cases}5.3\cdot10^{7}\ \mathrm{GeV}\\5.3\cdot10^{8}\ \mathrm{GeV}\end{cases}\ \text{for}\ \widetilde{G}\ \text{Mass}\ m_{\widetilde{G}}\simeq\begin{cases}0.69\ \mathrm{TeV}\\10.6\ \mathrm{TeV}.\end{cases}$$

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Lepton-Number Asymmetry And Light Neutrino Data

- $m_{i\mathrm{D}}$ are the Dirac Masses In a Basis (Called N_i^c-Basis) Where N_i^c Are Mass Eigenstates. In the Weak (primed) Basis

$$U^\dagger m_\mathrm{D} U^{c\dagger} = d_\mathrm{D} = \mathrm{diag}\,(m_{1\mathrm{D}}, m_{2\mathrm{D}}, m_{3\mathrm{D}}) \text{ Where } L' = LU \text{ and } N^{c\prime} = U^c N^c \quad (: \mathrm{I}).$$

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- Working in the N_i^c-Basis, the Type I Seesaw Formula Reads

$$m_\nu = -m_{\mathrm{D}}\, d_{N^c}^{-1}\, m_{\mathrm{D}}^{\mathsf{T}}, \quad \text{Where } d_{N^c} = \mathrm{diag}\,(M_{1N^c}, M_{2N^c}, M_{3N^c}) \ \text{with } M_{1N^c} \leq M_{2N^c} \leq M_{3N^c} \ \text{Real and Positive.}$$

- Replacing m_{D} from Eq. (I) in the Above Equation and We Extract The Mass Matrix of Light Neutrinos In The Weak Basis

$$\bar{m}_\nu = U^\dagger m_\nu U^* = -d_{\mathrm{D}} U^c d_{N^c}^{-1} U^{c\mathsf{T}} d_{\mathrm{D}},$$

Which Can Be Diagonalized by the Unitary PMNS Matrix U_ν Parameterized As Follows:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\varphi_1/2} & & \\ & e^{-i\varphi_2/2} & \\ & & 1 \end{pmatrix},$$

with $c_{ij} := \cos\theta_{ij}$, $s_{ij} := \sin\theta_{ij}$, δ the CP-Violating Dirac Phase and φ_1 and φ_2 the two CP-violating Majorana Phases.

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$$U^\dagger m_{\mathrm{D}} U^{c\dagger} = d_{\mathrm{D}} = \mathrm{diag}\,(m_{1\mathrm{D}}, m_{2\mathrm{D}}, m_{3\mathrm{D}}) \;\text{ Where }\; L' = LU \;\text{ and }\; N^{c\prime} = U^c N^c \quad (:\mathrm{I}).$$

- Working in the N_i^c-Basis, the Type I Seesaw Formula Reads

$$m_\nu = -m_{\mathrm{D}}\, d_{N^c}^{-1}\, m_{\mathrm{D}}^{\mathsf{T}}, \;\text{ Where }\; d_{N^c} = \mathrm{diag}\,(M_{1N^c}, M_{2N^c}, M_{3N^c}) \;\text{ with }\; M_{1N^c} \leq M_{2N^c} \leq M_{3N^c} \;\text{ Real and Positive.}$$

- Replacing m_{D} from Eq. (I) in the Above Equation and We Extract The Mass Matrix of Light Neutrinos In The Weak Basis

$$\bar{m}_\nu = U^\dagger m_\nu U^* = -d_{\mathrm{D}} U^c d_{N^c}^{-1} U^{c\mathsf{T}} d_{\mathrm{D}},$$

Which Can Be Diagonalized by the Unitary PMNS Matrix U_ν Parameterized As Follows:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\varphi_1/2} & & \\ & e^{-i\varphi_2/2} & \\ & & 1 \end{pmatrix},$$

with $c_{ij} := \cos\theta_{ij}$, $s_{ij} := \sin\theta_{ij}$, δ the CP-Violating Dirac Phase and φ_1 and φ_2 the two CP-violating Majorana Phases.

Parameter	Best Fit Value	
	Normal	Inverted
	Hierarchy	
$\Delta m^2_{21}/10^{-3}\mathrm{eV}^2$	7.56	
$\Delta m^2_{31}/10^{-3}\mathrm{eV}^2$	2.55	2.49
$\sin^2\theta_{12}/0.1$	3.21	
$\sin^2\theta_{13}/0.01$	2.155	2.14
$\sin^2\theta_{23}/0.1$	4.3	5.96
δ/π	1.40	1.44

- The Masses, $m_{i\nu}$, of ν_i Are Calculated as Follows:

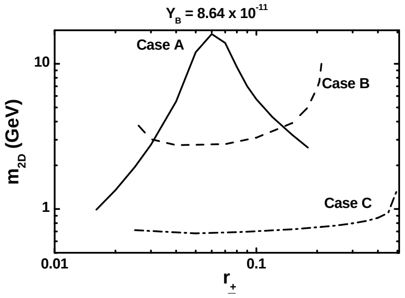
$$m_{2\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{21}} \;\text{ and }$$

$$\begin{cases} m_{3\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{31}}, & \text{for NO } m_\nu\text{'s} \\ \quad\text{or} & \\ m_{1\nu} = \sqrt{m^2_{3\nu} + \left|\Delta m^2_{31}\right|}, & \text{for IO } m_\nu\text{'s} \end{cases}$$

- $\sum_i m_{i\nu} \leq 0.23$ eV at 95% c.l. From *Planck* Data.

Combining Inflationary and Post-Inflationary Requirements

- To Verify The Compatibility of the Post-Inflationary Constraints, We Can **Further Constrain $r_\pm$** In Conjunction With The Low Energy Neutrino Physics Parameter
- All the Requirements can be Met Along the lines Presented in the $r_\pm - m_{2D}$ Plane for $\lambda_\mu = 10^{-6}$.



CASES : Hierarchy :	A NO	B NO	C IO
$m_{r\nu}$ / eV	0.01	0.05	0.01
$\Sigma_i m_{i\nu}$ / eV	0.074	0.17	0.11
m_{1D} / GeV	0.5	1	0.13
m_{3D} / GeV	150	170	100
φ_1	$-\pi/2$	π	$-3\pi/4$
φ_2	0	$\pi/3$	$5\pi/4$
$M_{1N^c}/10^{10}$ GeV	3	3	0.45
$M_{2N^c}/10^{11}$ GeV	0.4 - 8.7	1 - 16	0.3 - 1
$M_{3N^c}/10^{14}$ GeV	4.9	2.2	2.7

- We take $m_{r\nu} = m_{1\nu}$ for NO ν_i's and $m_{r\nu} = m_{3\nu}$ for IO ν_i's .
- **The Inflaton Decays into the Lightest and Next-to-Lightest of RHN** Since $2M_{iN^c} > \widehat{m}_{\delta\phi}$ for $i = 3$.
- Y_B Is Equal to its Central Value and **the $\widetilde{G}$ Constraint is Under Control** Even for $m_{3/2} \sim 1$ TeV Since We Obtain

$$0.7 \lesssim Y_{\widetilde{G}}/10^{-15} \lesssim 3 \ \text{ and } \ 0.4 \lesssim T_{\rm rh}/10^7\text{GeV} \lesssim 1.8\,.$$

Conclusions

• We Proposed A Variant of non-Minimal Higgs Inflation (named **Kinetically Modified**) Which can be Elegantly Implemented Within a $B-L$ Extension of MSSM, Adopting A Superpotential Determined by an R-Symmetry and Several **SemiLogarithmic Kähler Potentials** Which Respect a Softly Broken **Shift Symmetry**.

[6] *E.g., Core+, LiteBird, Bicep3/Keck Array and PRISM*

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  - It Offers a Nice Solution to **the μ Problem of MSSM**, Provided that λ_μ is Somehow Small;
  - It allows for **Baryogenesis** via non-TL Compatible With $\widetilde{G}$ **Constraints and Neutrino Data**. In particular We may have $m_{3/2} \sim 1$ TeV, With The Inflaton Decaying Mainly to N_1^c and N_2^c – We Obtain M_{iN^c} in the Range $(10^9 - 10^{15})$ GeV.

Thank You!

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