

Sudden Future Singularities in Quintessence and Scalar-Tensor Quintessence Models

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Introduction

- Theoretical challenges of Λ CDM model \longrightarrow alternative dark energy models which predict the existence of exotic cosmological singularities.
- These singularities can be either geodesically complete or incomplete
- Classification of Singularities
 - Behaviour of the scale factor $\alpha(t)$ and/or its derivatives at the time t_s
 - Energy density and pressure of the content of the universe at t_s .

- John D. Barrow ("Sudden future singularities" (2004), arXiv:gr-qc/0403084) [gr-qc]

Scale Factor:

$$a(t) = \left(\frac{t}{t_s}\right)^m (a_s - 1) + 1 - \left(1 - \frac{t}{t_s}\right)^q$$

$$1 < q < 2$$

- Big-Bang singularity at $t = 0$
- New type of singularity at $t = t_s$ - a Sudden Future Singularity (SFS)

- Divergence of the scalar curvature $R = 6 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) \rightarrow \infty$
- Manifests as a singularity of pressure p (or \ddot{a}) only
- Leads to the dominant energy condition (DEC) $\rho \geq |p|$ violation only

$$\begin{aligned} \Rightarrow \alpha = \text{finite} & \quad \Rightarrow \dot{\alpha} = \text{finite} & \quad \Rightarrow \ddot{a} \rightarrow -\infty \\ \Rightarrow \rho = \text{finite} & \quad \Rightarrow p \rightarrow \infty & \quad \text{for } t = t_s \end{aligned}$$

Quintessence Models

• Action:

$$S = \int \left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} - V(\phi) \right] \sqrt{-g} d^4x$$

• Potential:

$$V(\phi) = A|\phi|^n, \quad A > 0, \quad 0 < n < 1$$

• Dynamical equations:

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\ddot{\phi} = -3H\dot{\phi} - An|\phi|^{n-1}\Theta(\phi)$$

$$2\dot{H} = -\dot{\phi}^2$$

→ $H, \dot{H}, \dot{\phi} \rightarrow \text{finite}$ → $\phi^{n-1} \rightarrow \infty, \ddot{\phi} \rightarrow \infty, \ddot{H} \rightarrow \infty$ as $t \rightarrow t_s (\phi \rightarrow 0)$

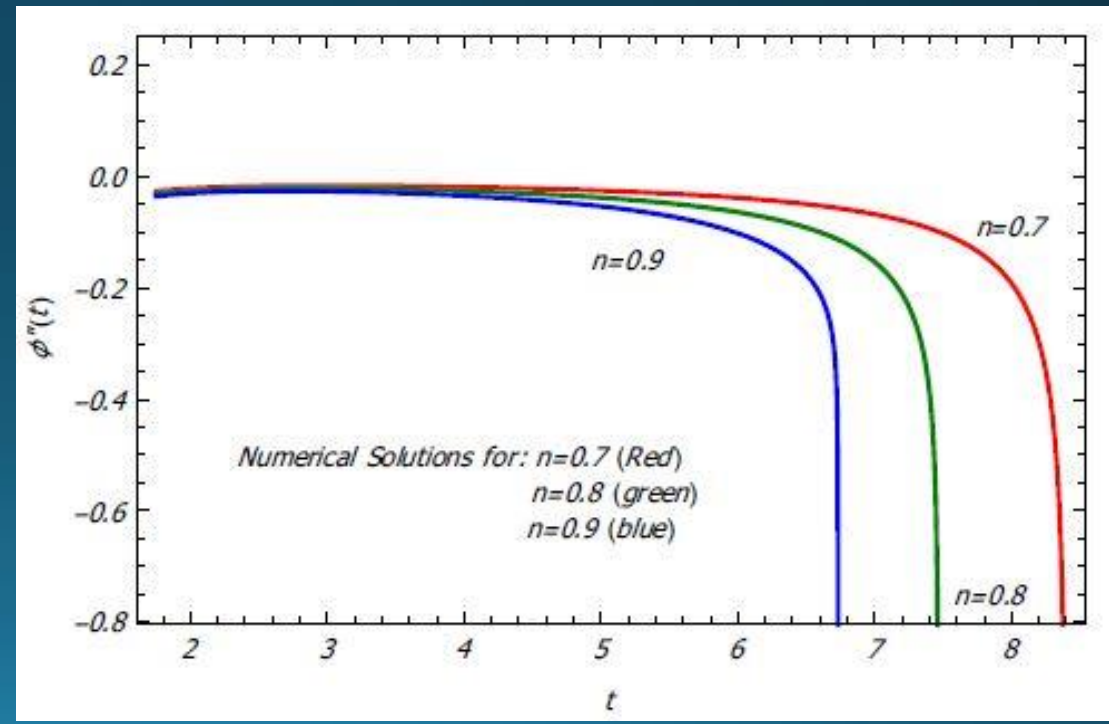
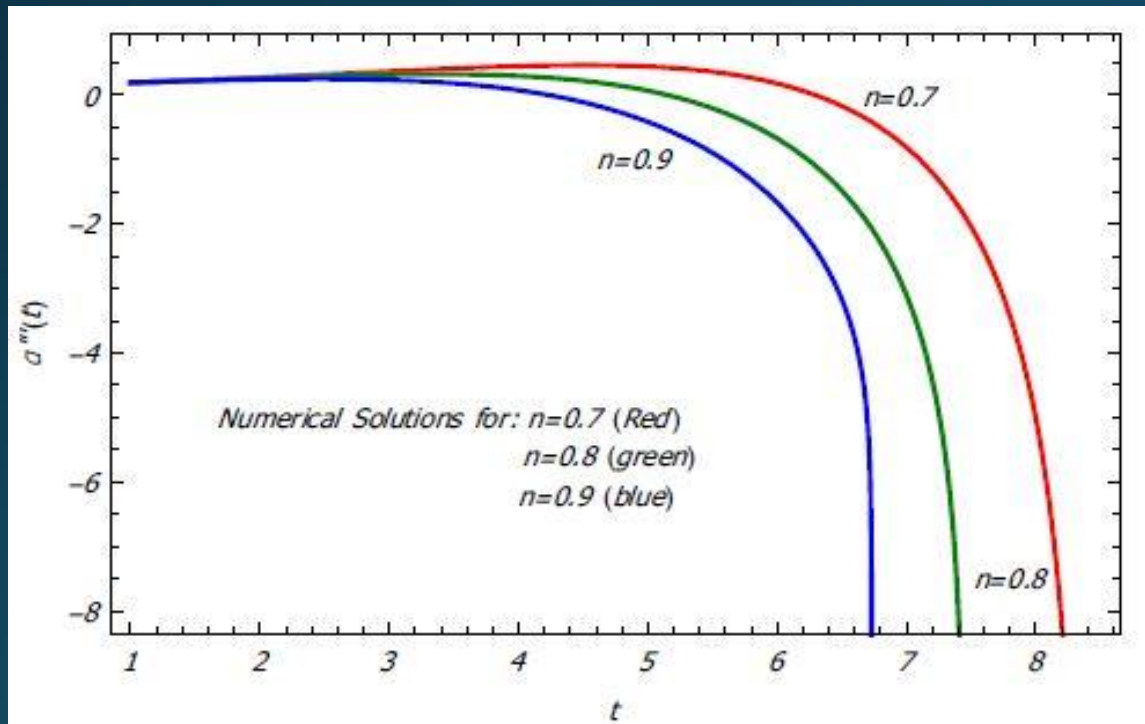
➤ $\ddot{a} \rightarrow \infty$ - a Generalized Sudden Future Singularity (GSFS)

- Scale Factor: $a(t) = a_s + b(t_s - t) + c(t_s - t)^2 + d(t_s - t)^q$ $2 < q < 3$

As $t \rightarrow t_s \rightarrow a \rightarrow a_s$ and $\ddot{a} \rightarrow \infty$

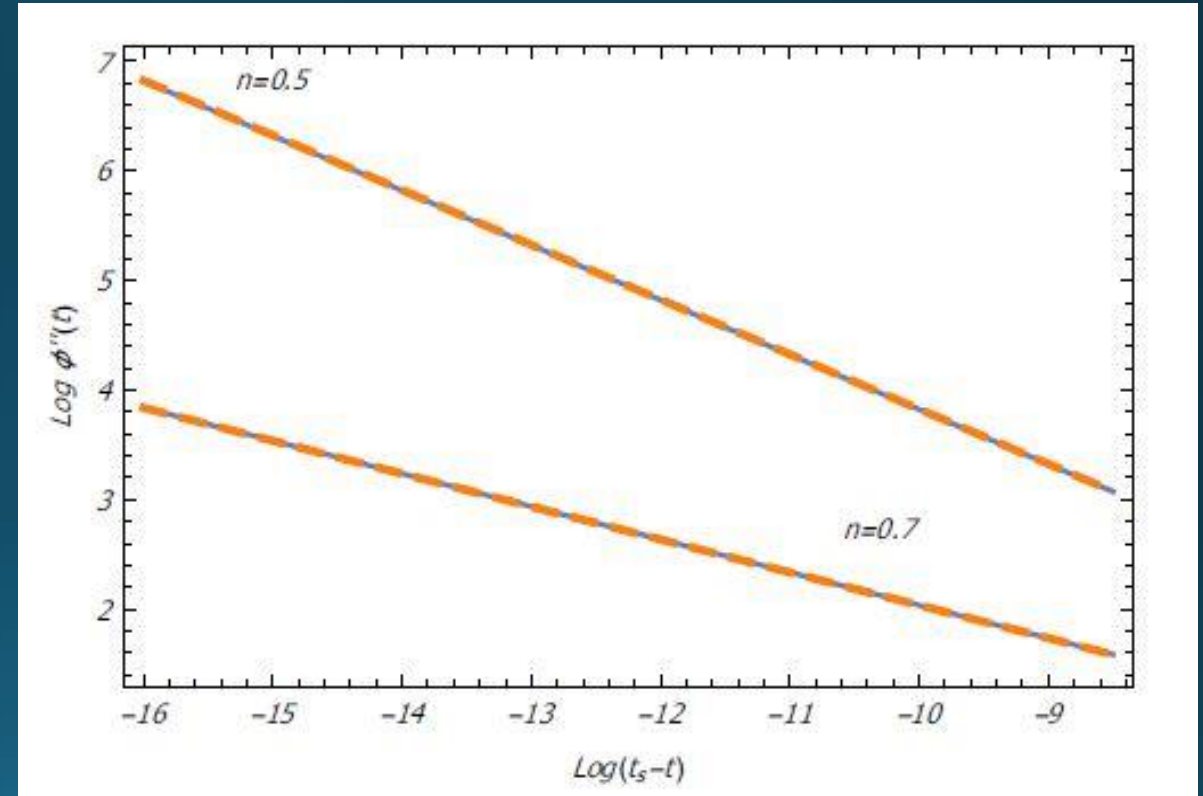
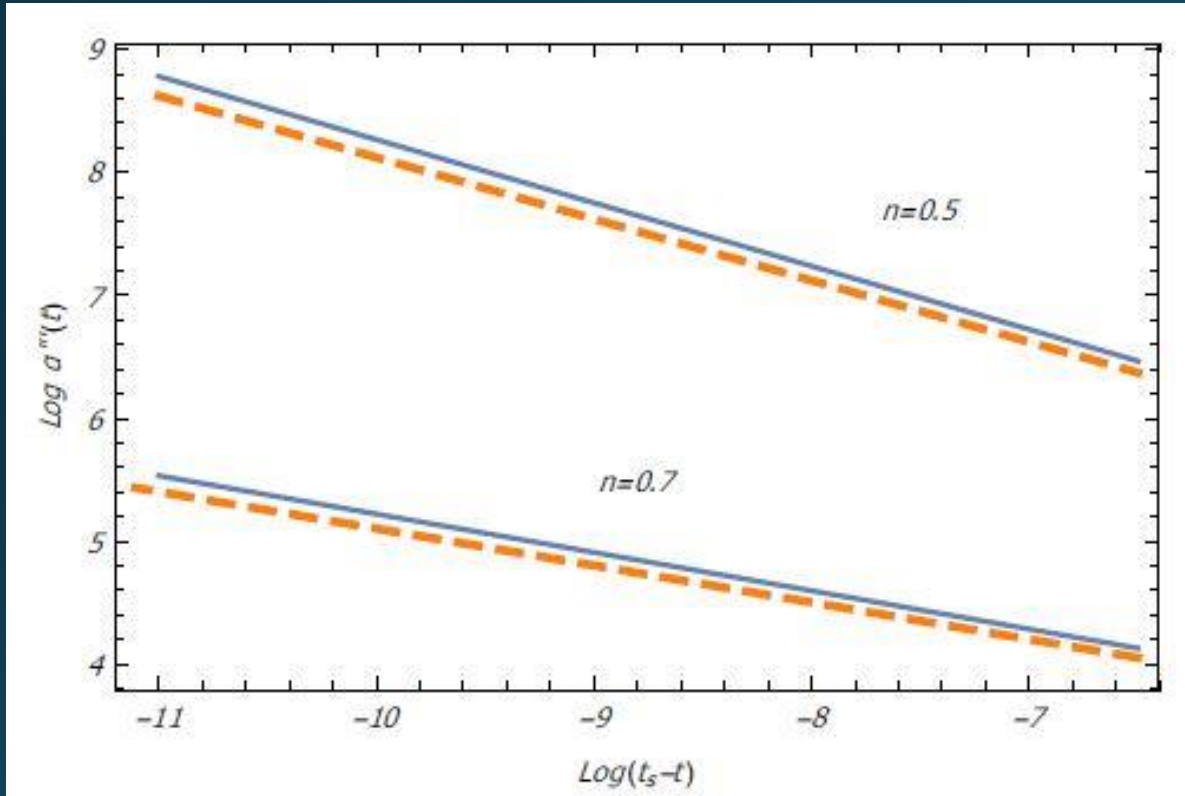
- Scalar Field: $\phi(t) = f(t_s - t) + h(t_s - t)^r$ $1 < r < 2$

As $t \rightarrow t_s \rightarrow \phi \rightarrow 0$ and $\ddot{\phi} \rightarrow \infty$



➤ Power Exponents: $r = n + 1$ $q = r + 1$ \Rightarrow $q = n + 2$

Consistent with the qualitatively expected range of r, q , for $0 < n < 1$



➤ Observationally testable prediction of this class of models

$$H(z_s) = (1 + z_s)^3$$

Scalar-Tensor Quintessence Models

• Action:

$$\mathcal{S} = \int \left[\frac{1}{2} F(\phi) R + \frac{1}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu} - V(\phi) \right] \sqrt{-g} d^4x$$

$$F = 1 - \lambda\phi$$

• Potential:

$$V(\phi) = A|\phi|^n, \quad A > 0$$

$$0 < n < 1$$

• Dynamical equations:

$$3FH^2 = \frac{\dot{\phi}^2}{2} + V - 3H\dot{F}$$

$$\ddot{\phi} + 3H\dot{\phi} - 3F_\phi \left(\frac{\ddot{a}}{a} + H^2 \right) + An|\phi|^{(n-1)}\Theta(\phi) = 0$$

$$-2F \left(\frac{\ddot{a}}{a} - H^2 \right) = \dot{\phi}^2 + \ddot{F} - H\dot{F}$$

$$\Rightarrow H, \dot{\phi}, F, \dot{F} \rightarrow \text{finite}$$

$$\Rightarrow \phi^{n-1} \rightarrow \infty, \ddot{\phi} \rightarrow \infty, \ddot{F} \rightarrow \infty$$

$$\text{as } t \rightarrow t_s (\phi \rightarrow 0)$$

➤ $\ddot{a} \rightarrow \infty$ - a Sudden Future Singularity (SFS)

• Scale Factor:

$$a(t) = a_s + b(t_s - t) + c(t_s - t)^2 + d(t_s - t)^q$$

$$1 < q < 2$$

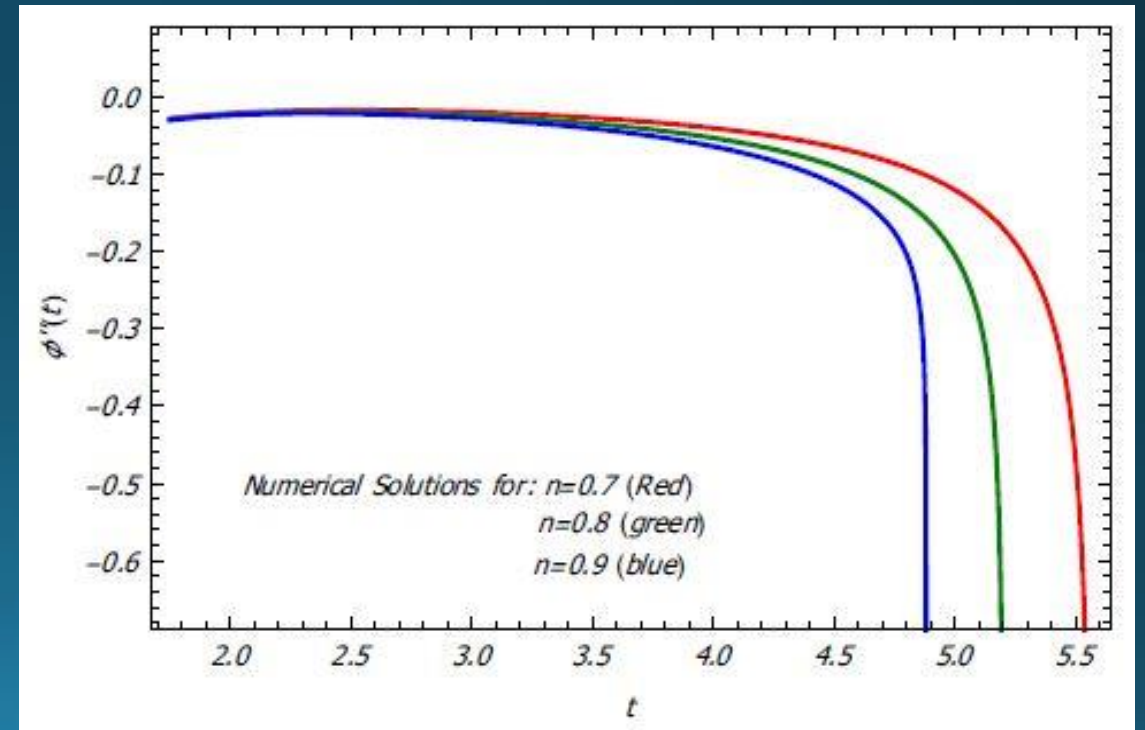
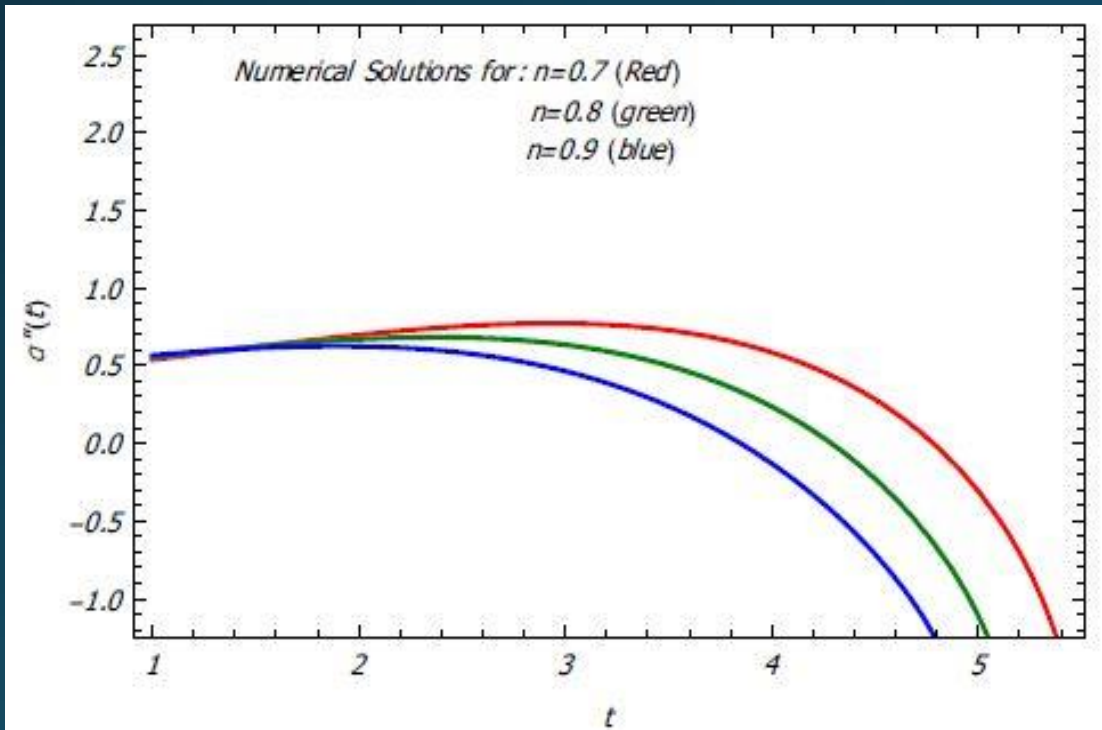
As $t \rightarrow t_s$ \rightarrow $a \rightarrow a_s$ and $\ddot{a} \rightarrow \infty$

• Scalar Field:

$$\phi(t) = f(t_s - t) + h(t_s - t)^r$$

$$1 < r < 2$$

As $t \rightarrow t_s$ \rightarrow $\phi \rightarrow 0$ and $\ddot{\phi} \rightarrow \infty$



➤ Power Exponents:

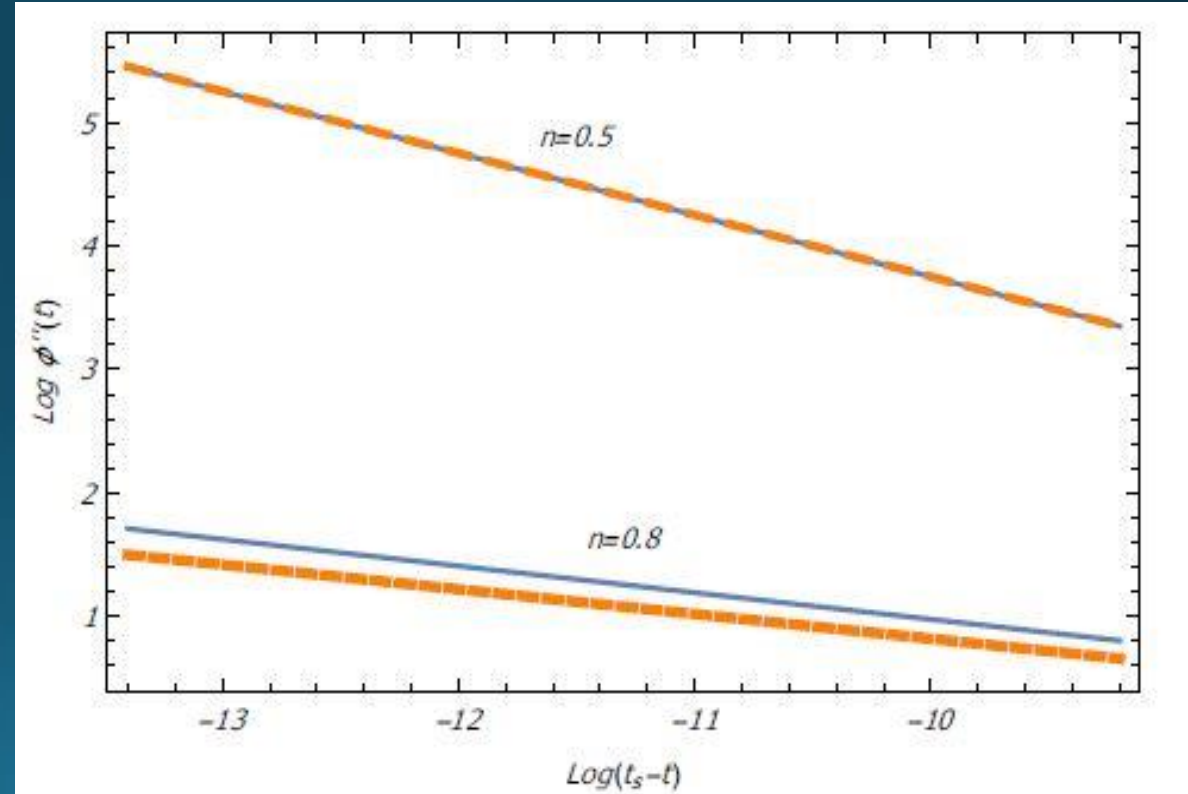
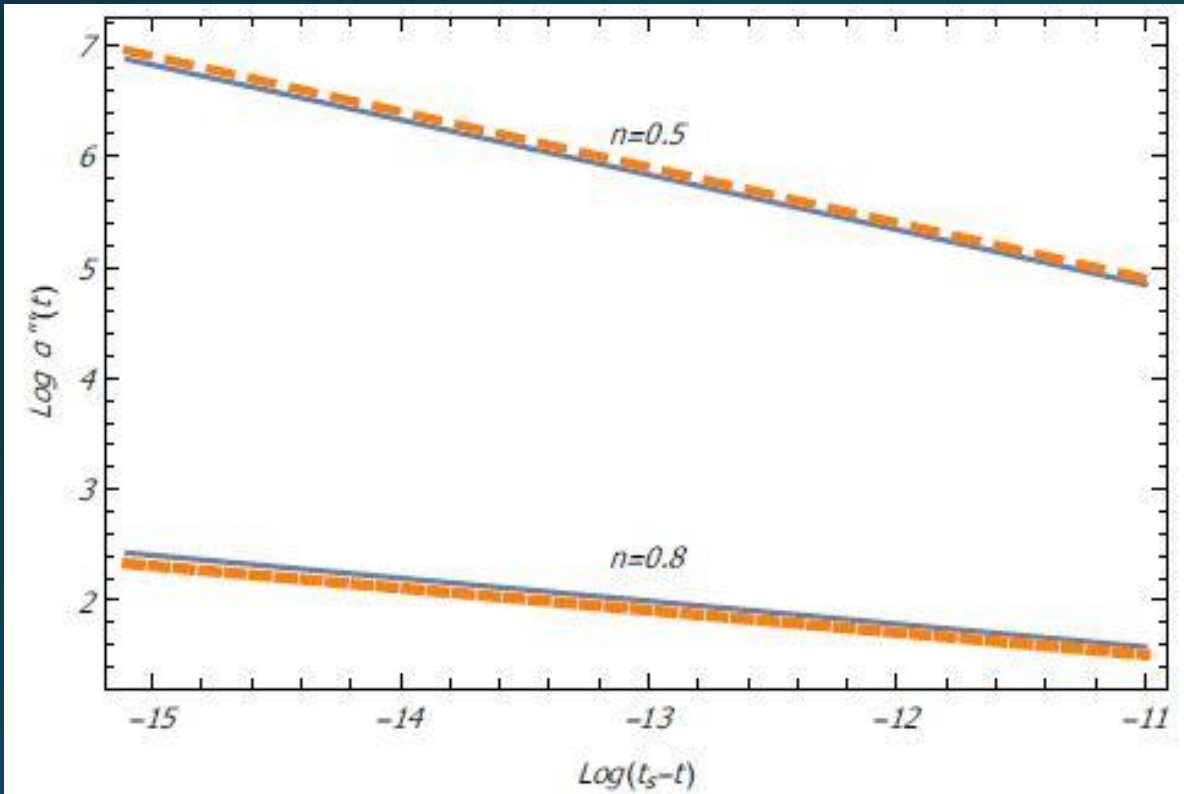
$$q = r$$

$$r = n + 1$$



$$q = n + 1$$

Consistent with the qualitatively expected range of r, q , for $0 < n < 1$



Conclusions

- For quintessence models: $\ddot{a} \rightarrow \infty$ \longrightarrow GSFS singularity.
- Scalar-tensor quintessence models: $\ddot{a} \rightarrow \infty$ \longrightarrow a stronger singularity occurs - an SFS singularity (due to divergence of Ricci scalar).
- The additional linear and quadratic terms of $t_s - t$ in the form of the scale factor play an important role as $t \rightarrow t_s$. In the scalar-tensor case the quadratic term becomes subdominant close to the singularity.
- The relations of the Hubble parameter $H(z)$, may be used as observational signatures of such singularities in this class of models.

THANK YOU