T-duality in (2, 1) superspace

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Athens, March 27, 2018

My picture of Ioannis

Kerkyra Sep. 2014.

T-duality 101

The bosonic nonlinear sigma model

$$\phi^i : \Sigma \to \mathcal{T} \quad : \quad x^\mu \mapsto \phi^i(x)$$

$$\int d^2x \left(g_{ij}(\phi)\partial_{+\!\!+}\phi^i\partial_{=}\phi^j \right)$$

Isometry

$$\partial g_{ij}/\partial\phi^0 = 0$$

$$\int d^2x\,\left(g_{ab}(\phi)\partial_{+\!\!\!+}\phi^a\partial_=\phi^b + 2g_{a0}(\phi)\partial_{(+\!\!\!+}\phi^a A_{=)} + g_{00}(\phi)A_{(+\!\!\!+}A_{=)}\right)$$

$$+\int d^2x\tilde{\phi}\partial_{[+\!\!\!+}A_{=]}$$

The dual action

$$\delta A_{\pm\!\!\!\!=} \Rightarrow A_{\pm\!\!\!\!=} = \frac{1}{g_{00}}(\partial_{\pm\!\!\!\!=}\tilde{\phi} - g_{a0}\partial_{\pm\!\!\!\!=}\phi^a)$$

$$\int d^2x \left(g^D_{ij}(\tilde{\phi})\partial_{+\!\!\!+}\tilde{\phi}^i\partial_{=}\tilde{\phi}^j + b^D_{ij}(\tilde{\phi})\partial_{[+\!\!\!+}\tilde{\phi}^i\partial_{=]}\tilde{\phi}^j \right)$$

$$\tilde{\phi}^0 = \tilde{\phi},\ \tilde{\phi}^a = \phi^a$$

The geometries are related by the Buscher rules.
In this case:

$$g^D_{00} = \frac{1}{g_{00}}\ , \quad g^D_{0a} = 0\ , \quad g^D_{ab} = g_{ab} - \frac{g_{0a}g_{0b}}{g_{00}}$$

$$b^D_{0a} = \frac{g_{0a}}{g_{00}}\ , \quad b^D_{ab} = 0$$

Enter SUSY

Duality in Superspace is interesting because the target space geometry of supersymmetric nonlinear sigma models is, and duality relates different such geometries.

B. Zumino 1979
L. Alvarez-Gaume and D. Freedman 1980
J. Gates, C. Hull and M. Roček 1984
C. Hull and E. Witten 1985

Susy	(1,1)	(2,2)	(2,2)	(4,4)	(4,4)
E=G+B	G, B	G	G, B	G	G, B
Geom	Riem.	Kähler	biherm.	hyperk.	bihyperc.

N.B. I. Bakas three papers from. 1995

Chiral twisted chiral duality for (Generalised) Kähler geometry

$$\{\mathbb{D}_\pm, \bar{\mathbb{D}}_\pm\} = i\partial_{\pm\!\!\!\!=}$$

$$\bar{\mathbb{D}}_\pm \phi = 0 \ , \quad \bar{\mathbb{D}}_+ \chi = \mathbb{D}_- \chi = 0$$

$$K(\phi + \bar{\phi} + V, \varphi, \bar{\varphi}) - (\chi + \bar{\chi}) V$$

The dual Lagrangian

$$\delta V : K_\phi - (\chi + \bar{\chi}) = 0$$

$$g.f. \Rightarrow V = V(\chi + \bar{\chi}, \varphi, \bar{\varphi})$$

$$K^D(\chi + \bar{\chi}, \varphi, \bar{\varphi}) = K(V(\chi + \bar{\chi}, ..), ..) - (\chi + \bar{\chi}) V(\chi + \bar{\chi}, ..)$$

No spectators

$$g_{\phi\bar{\phi}} = K,_{\phi\bar{\phi}} \ , \quad \Rightarrow \quad g^D,_{\chi\bar{\chi}} = -K^D,_{\chi\bar{\chi}} = \frac{1}{g_{\phi\bar{\phi}}}$$

Double Buscher rules? In general, the geometry becomes transparent after reduction (1, 1) superspace.

To reduce to (2, 1) we write

$$\mathbb{D}_- = D_- - iQ_- \;, \quad Q_-\phi = iD_-\phi \;, \quad Q_-\chi = -\tfrac{1}{2} iD_-\Theta$$

$$\tilde{\phi} = \bar{\phi} - V \;, \quad Q_- V_| = 2A_- - iD_- V$$

$$\nabla_-\phi = D_-\phi + iA_- \;, \quad \nabla_-\tilde{\phi} = D_-\tilde{\phi} + iA_-$$

$$i\left[K_0\nabla_-\phi - K_0\nabla_-\tilde{\phi} + K_Z D_- Z - K_{\bar{Z}} D_-\bar{Z}\right] + \bar{\Theta} iD_- V - (\Theta + \bar{\Theta})A_-$$

M. Abou-Zeid, C. M. Hull, 1997, 1998

M. Abou-Zeid, C. M. Hull, U. L. ,M. Roček and R. von Unge, in prep.

The gauged Lagrangian

$$i\left[k_\alpha(\varphi,\tilde\varphi)\nabla_-\varphi^\alpha - \tilde k_{\bar\alpha}(\varphi,\tilde\varphi)\nabla_-\tilde\varphi^{\bar\alpha}\right] + \bar\Theta i D_- V - \left(\Theta+\bar\Theta\right)A_-$$

.

$$\bar{\mathbb{D}}_+\varphi^\alpha = 0\ , \quad i\frac{\partial k_\alpha}{\partial\varphi^0} - i\frac{\partial k_\alpha}{\partial\bar\varphi^0} = 0$$

$$\tilde\varphi^0 = \bar\varphi^0 + V\ , \quad \tilde\varphi^\mu = \bar\varphi^\mu$$

The dual Lagrangian

$$\delta A_- :\Rightarrow X + \Theta + \bar{\Theta} := k_0 + \bar{k}_{\bar{0}} + \Theta + \bar{\Theta} = 0$$

$$\delta V :\Rightarrow A_- = \dots$$

$$L^D = i\left(k^D_\Theta D_-\Theta - \bar{k}^D_{\bar{\Theta}} D_-\bar{\Theta} + k^D_\mu D_-\varphi^\mu - \bar{k}^D_{\bar{\mu}} D_-\bar{\varphi}^{\bar{\mu}}\right)$$

$(2,1)$

With $X := k_0 + \bar{k}_{\bar{0}}$, $Z := k_0 - \bar{k}_{\bar{0}}$, the vector potentials are related by

$$k^D_{\Theta} = -\tfrac{1}{2}\left[V + \frac{Z}{g_{0\bar{0}}}\right] , \quad k^D_{\bar{\Theta}} = -\tfrac{1}{2}\left[V - \frac{Z}{g_{0\bar{0}}}\right]$$

$$k^D_{\mu} = \left[k_{\mu} - \frac{1}{2}\frac{ZX_{,\mu}}{g_{0\bar{0}}}\right] , \quad k^D_{\bar{\mu}} = \left[\bar{k}_{\bar{\mu}} + \frac{1}{2}\frac{ZX_{,\bar{\mu}}}{g_{0\bar{0}}}\right]$$

.

The geometries are calculated from this as

$$g_{\alpha\bar{\alpha}} = \bar{k}_{\bar{\alpha},\alpha} + k_{\alpha,\bar{\alpha}} \;, \quad b_{\alpha\beta} = k_{\beta,\alpha} - k_{\alpha,\beta}$$

.

This yields the corresponding relations for the dual metric and *b*-field

$$g^D_{\Theta\bar{\Theta}} = \frac{1}{g_{0\bar{0}}}~, \quad g^D_{\mu\bar{\Theta}} = \frac{1}{g_{0\bar{0}}}[b_{\mu 0}]~, \quad g^D_{\bar{\mu}\Theta} = \frac{1}{g_{0\bar{0}}}[b_{\bar{\mu}\bar{0}}]$$

$$g^D_{\mu\bar{\mu}} = g_{\mu\bar{\mu}} - \frac{1}{g_{0\bar{0}}}(g_{\mu\bar{0}}g_{\bar{\mu}0} - b_{\mu 0}b_{\bar{\mu}\bar{0}})$$

$$b^D_{\Theta\mu} = \frac{g_{\bar{0}\mu}}{g_{0\bar{0}}}~, \quad b^D_{\bar{\Theta}\bar{\mu}} = \frac{g_{0\bar{\mu}}}{g_{0\bar{0}}}$$

$$b^D_{\mu\nu} = b_{\mu\nu} - \frac{2}{g_{0\bar{0}}}g_{\bar{0}[\mu}(b_{\nu]0})~, \quad b^D_{\bar{\mu}\bar{\nu}} = b_{\bar{\mu}\bar{\nu}} - \frac{2}{g_{0\bar{0}}}g_{0[\bar{\mu}}(b_{\bar{\nu}]\bar{0}})$$

Reduction to $(1,1)$

As for the $(2,2)$ ciral to twisted chiral duality, this looks like a doubling of the Buscher rules.

Double Buscher rules? In general, the geometry becomes transparent after reduction $(1,1)$ superspace.

The (1, 1) Lagrangian

$$\mathbb{D}_+ = D_+ - iQ_+ \ , \quad Q_+\varphi = iD_+\varphi$$

$$Q_+ V_| = 2A_+ \ , \quad Q_+ A_{-|} = -i\left(\tfrac{1}{2}d + D_- A_+\right)$$

$$2\left[K_{00}(\nabla_+\phi\nabla_-\bar{\phi} + \nabla_+\bar{\phi}\nabla_-\phi) + K_{0Z}(D_+Z\nabla_-\bar{\phi} + \nabla_+\bar{\phi}D_-Z)\right.$$

$$\left.+K_{0\bar{Z}}(D_+\bar{Z}\nabla_-\bar{\phi} + \nabla_+\bar{\phi}D_-\bar{Z}) + K_{Z\bar{Z}}(D_+ZD_-\bar{Z} + D_+\bar{Z}D_-Z)\right]$$

$$\underbrace{\frac{id}{2}\left(2K_0 + \Theta + \bar{\Theta}\right)}_{2} + \underbrace{(\Theta - \bar{\Theta})2iD_{(+}A_{-)}}_{1}$$

.

1. is the usual Lagrange multiplier term imposing duality between the $(1,1)$ fields $\phi - \bar{\phi}$ and $\Theta - \bar{\Theta}$. In addition to this, there is the gauged Lagrangian and the the *d*-term 2. More explicitly, the latter imposes

$$X + \Theta + \bar{\Theta} := K_0(\phi + \bar{\phi}, \varphi^\mu, \bar{\varphi}^{\bar{\mu}}) + \Theta + \bar{\Theta} = 0$$

We can either solve this for $\Theta + \bar{\Theta} = (\Theta + \bar{\Theta})(\phi + \bar{\phi}, \varphi^\mu, \bar{\varphi}^{\bar{\mu}})$ or change coordinates to $(\Theta + \bar{\Theta}, \varphi^\mu, \bar{\varphi}^{\bar{\mu}})$. The $(2,2)$ and $(2,1)$ calculations correspond to the latter choice. In conclusion, the procedure leads to a dualisation of the imaginary part of ϕ to the imaginary part of Θ along with a coordinate transformation of their real parts.

Kähler gauge transformations

$$S = \int d^2x \mathbb{D}_+ \bar{\mathbb{D}}_+ \mathbb{D}_- \bar{\mathbb{D}}_- K(\phi, \bar\phi)$$

$$K(\phi, \bar\phi) \to K + f(\phi) + \bar f(\bar\phi)$$

$(2, 2)$

$$S = \int d^2x \mathbb{D}_+ \bar{\mathbb{D}}_+ D_- i \left[k_\alpha(\varphi, \varphi) D_- \varphi^\alpha - \bar k_{\bar\alpha}(\varphi, \varphi) D_- \varphi^{\bar\alpha} \right]$$

$$k_\alpha(\varphi, \bar\varphi) \to k_\alpha + i\partial_\alpha \chi(\varphi, \bar\varphi) + \vartheta_\alpha(\varphi)$$

$(2, 1)$

The full (2, 1) T-duality

C. M. Hull, A. Karlhede, U. L. , M. Roček 1986.
M. Abou-Zeid, C. M. Hull, 1997, 1998

The gauged Lagrangian

$$\mathcal{L}_\xi k_\alpha = i\partial_\alpha \chi + \vartheta_\alpha$$

$$\left[i\left(k_\alpha D_- \varphi^\alpha - \bar{k}_{\bar\alpha} D_- \tilde{\varphi}^{\bar\alpha}\right) - A_- X\right](\varphi, \tilde{\varphi}) + \int_0^1 e^{tL} dt V \bar{\vartheta}_{\bar\mu}(\bar\varphi) D_- \bar\varphi^{\bar\mu}$$
$$+ \bar\Theta i D_- V - (\Theta + \bar\Theta) A_-$$

where

$$L_{V\cdot\bar\xi} = iV\bar\xi^{\bar\alpha} \frac{\partial}{\partial \bar\varphi^{\bar\alpha}}$$

Geometry

The dual vector potentials are

$$k^D_\Theta = -\frac{1}{2}\left[V + \frac{\hat{Z}}{g_{0\bar{0}}}\right] \;, \quad \bar{k}^D_\Theta = -\frac{1}{2}\left[V - \frac{\hat{Z}}{g_{0\bar{0}}}\right]$$

$$\bar{k}^D_\mu = \left[k_\mu - \frac{1}{2}\frac{\hat{Z}X_{,\mu}}{g_{0\bar{0}}}\right] \;, \quad k^D_{\bar{\mu}} = \left[\bar{k}_{\bar{\mu}} + i\int_0^1 dt e^{tL} V \bar{\vartheta}_{\bar{\mu}}(\bar{\varphi}) + \frac{1}{2}\frac{\hat{Z}X_{,\bar{\mu}}}{g_{0\bar{0}}}\right]$$

where

$$\widehat{Z} = Z + \chi$$

More geometry

The dual metric and B-field are related by the same "double" Buscher rules as before, with the same geometric understanding resulting from going to $(1,1)$. The only difference is that the B-field entering on the right is shifted compared to t the original one, e.g.,

$$g^D_{\mu\bar{\mu}} = g_{\mu\bar{\mu}} - \frac{1}{g_{0\bar{0}}}(g_{\mu\bar{0}}g_{\bar{\mu}0} - (b_{\mu 0} + i\vartheta_\mu)(b_{\bar{\mu}\bar{0}} - i\bar{\vartheta}_{\bar{\mu}}))$$

The shifted field has the same field strength and is thus physically equivalent to the original one.