Scalar One-point Functions in AdS/dCFT

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based on [arXiv:1802.01598] (to appear in PLB) and J.Phys. A: Math.Theor. **50** (2017) 254001
[arXiv:1612.06236] with Charlotte Kristjansen and Marius de Leeuw

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Section 1

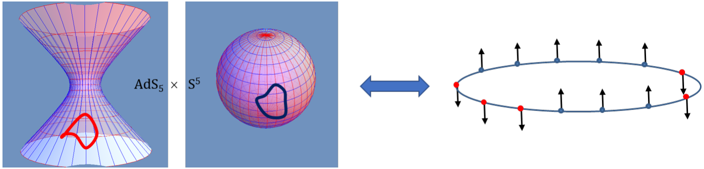
Intersecting Branes

The D3-D5 system

Let us recall the original formulation AdS/CFT correspondence:

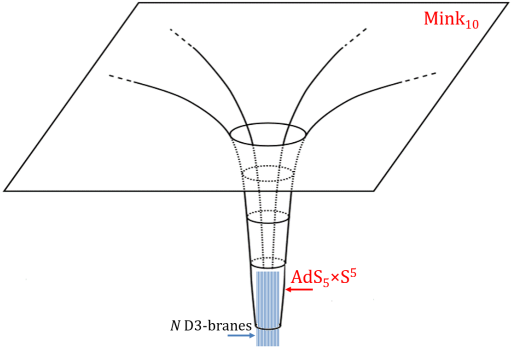
$$\left\{\text{Type IIB superstring theory on } AdS_5 \times S^5\right\} = \left\{\mathcal{N} = 4,\ SU(N) \text{ SYM theory in } 3+1 \text{ dimensions}\right\}$$

(J. Maldacena, 1998)



The D3-D5 system

IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N coincident D3-branes:



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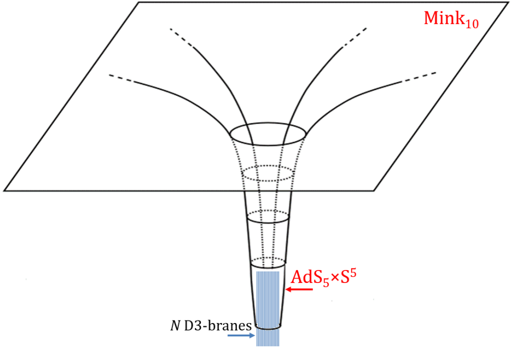

The D3-branes extend along x_1, x_2, x_3...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	●	●	●	●						

The D3-D5 system

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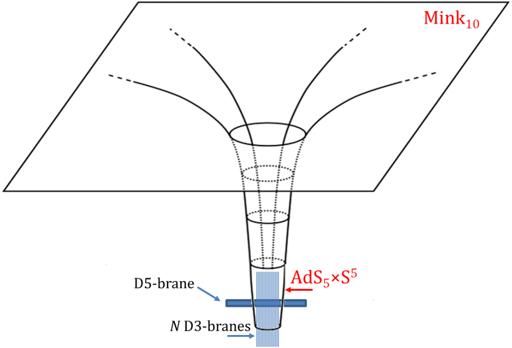
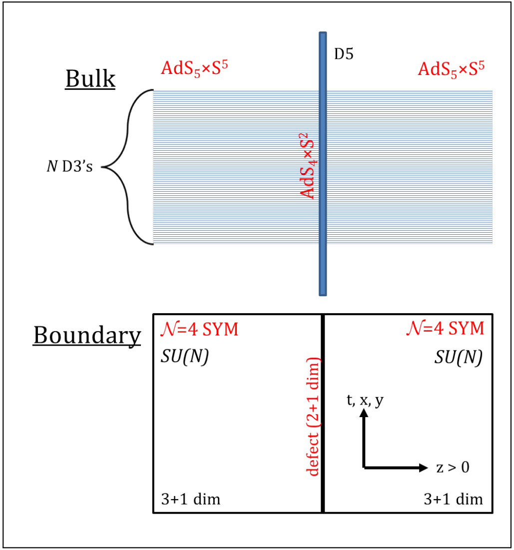


The D3-branes extend along x_1, x_2, x_3. Now insert a single (probe) D5-brane at x_3, x_7, x_8, $x_9 = 0$...

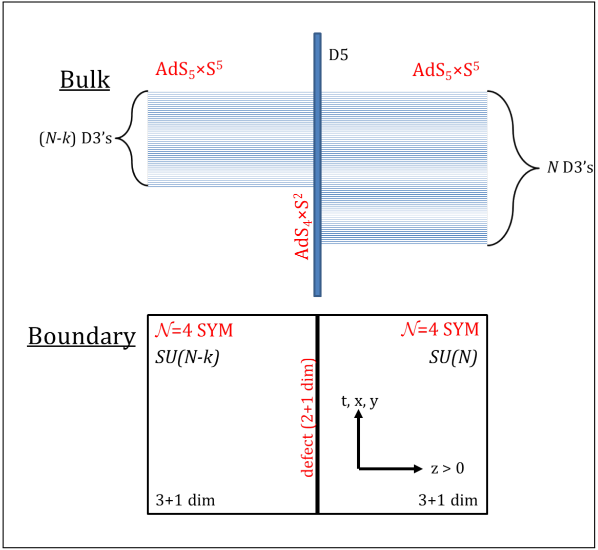
	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D5	•	•	•		•	•	•			

The D3-D5 system: description



- In the bulk, the D3-D5 system describes IIB Superstring theory on $AdS_5 \times S^5$ bisected by D5 branes with worldvolume geometry $AdS_4 \times S^2$.
- The dual field theory is still $SU(N)$, $\mathcal{N} = 4$ SYM in $3+1$ dimensions, that now interacts with a SCFT that lives on the 2+1 dimensional defect.
- Due to the presence of the defect, the total bosonic symmetry of the system is reduced from $SO(4,2) \times SO(6)$ to $SO(3,2) \times SO(3) \times SO(3)$.
- The corresponding superalgebra $\mathfrak{psu}(2,2|4)$ becomes $\mathfrak{osp}(4|4)$.

The $(D3\text{-}D5)_k$ system



- Add k units of background $U(1)$ flux on the S^2 component of the $AdS_4 \times S^2$ D5-brane.
- Then k of the N D3-branes ($N \gg k$) will end on the D5-brane.
- On the dual SCFT side, the gauge group $SU(N) \times SU(N)$ breaks to $SU(N-k) \times SU(N)$.
- Equivalently, the fields of $\mathcal{N} = 4$ SYM develop nonzero vevs...

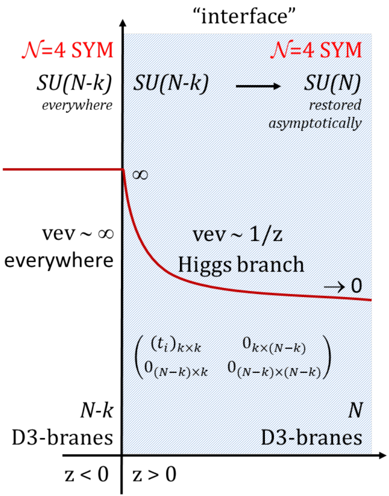
(Karch-Randall, 2001b)

Section 2

One-point Functions in the D3-D5 System

M. de Leeuw, C. Kristjansen, G. Linardopoulos, *Scalar One-point functions and matrix product states of AdS/dCFT*, [arXiv:1802.01598] (to appear in PLB)

The dCFT interface of D3-D5



- An interface is a wall between two (different/same) QFTs
- It can be described by means of classical solutions that are known as "fuzzy-funnel" solutions (Constable-Myers-Tafjord, 1999 & 2001)
- Here, we need an interface to separate the $SU(N)$ and $SU(N-k)$ regions of the $(\text{D3-D5})_k$ dCFT...
- For no vectors/fermions, we want to solve the equations of motion for the scalar fields of $\mathcal{N}=4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\Phi_i}{dz^2} = [\Phi_j, [\Phi_j, \Phi_i]], \quad i, j = 1, \ldots, 6.$$

- A manifestly $SO(3) \simeq SU(2)$ symmetric solution is given by $(z > 0)$:

$$\Phi_{2i-1}(z) = \frac{1}{z}\begin{bmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{bmatrix} \quad \& \quad \Phi_{2i} = 0,$$

Nagasaki-Yamaguchi, 2012

where the matrices t_i furnish a k-dimensional representation of $\mathfrak{su}(2)$:

$$[t_i, t_j] = i\epsilon_{ijk} t_k.$$

One-point functions

One-point functions are the most important observables in a dCFT. From them and the conformal data (Δ's, C_{ijk}'s, etc.) one can determine all the correlators of the theory (and the theory itself) by using the OPE.

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- Our dCFT is dual to the $(\text{D3-D5})_k$ probe brane system.

One-point functions

One-point functions are the most important observables in a dCFT. From them and the conformal data (Δ's, C_{ijk}'s, etc.) one can determine all the correlators of the theory (and the theory itself) by using the OPE.

- Our dCFT is dual to the $(\text{D3-D5})_k$ probe brane system.
- Our goal is to calculate the one-point functions of $\mathfrak{so}(6)$ highest-weight eigenstates:

$$\langle \mathcal{O}(z, \mathbf{x}) \rangle = \frac{C}{z^{\Delta}}, \qquad C = \frac{1}{\sqrt{L}} \left(\frac{8\pi^2}{\lambda} \right)^{L/2} \cdot \frac{\langle \text{MPS} | \Psi \rangle}{\langle \Psi | \Psi \rangle^{\frac{1}{2}}}, \qquad k \ll N \to \infty,$$

where

$$\langle \text{MPS} | \Psi \rangle = z^M \cdot \sum_{1 \leq x_k \leq L} \psi(x_k) \cdot \text{Tr} \left[t_3^{x_1 - 1} \mathcal{W} t_3^{x_2 - x_1 - 1} \mathcal{Y} t_3^{x_3 - x_2 - 1} \overline{\mathcal{W}} t_3^{x_4 - x_3 - 1} \overline{\mathcal{Y}} \ldots \right]$$

and Ψ is an eigenstate of the $\mathfrak{so}(6)$ Hamiltonian, with

$$\langle \Psi | \Psi \rangle^{\frac{1}{2}} = \sqrt{\sum_{1 \leq x_k \leq L} \psi^2(x_k)}.$$

The $\mathfrak{su}(2)$ determinant formula

For the vacuum overlap we find:

$$\langle \text{MPS}|0\rangle = \text{Tr}\left[t_3^L\right] = \zeta\left(-L, \frac{1-k}{2}\right) - \zeta\left(-L, \frac{1+k}{2}\right), \qquad \zeta(s,a) \equiv \sum_{n=0}^{\infty} \frac{1}{(n+a)^s},$$

where $\zeta(s,a)$ is the Hurwitz zeta function. For M balanced excitations the overlap becomes:

$$C_k(\{u_j\}) \equiv \frac{\langle \text{MPS}|\{u_j\}\rangle_k}{\sqrt{\langle\{u_j\}|\{u_j\}\rangle}} = C_2(\{u_j\}) \cdot \sum_{j=(1-k)/2}^{(k-1)/2} j^L \left[\prod_{l=1}^{M/2} \frac{u_l^2\left(u_l^2 + k^2/4\right)}{\left[u_l^2 + (j-1/2)^2\right]\left[u_l^2 + (j+1/2)^2\right]}\right]$$

where

$$C_2(\{u_j\}) \equiv \frac{\langle \text{MPS}|\{u_j\}\rangle_{k=2}}{\sqrt{\langle\{u_j\}|\{u_j\}\rangle}} = \left[\prod_{j=1}^{M/2} \frac{u_j^2 + 1/4}{u_j^2} \frac{\det G^+}{\det G^-}\right]^{\frac{1}{2}},$$

and the $M/2 \times M/2$ matrices $G_{jk}^{\pm}$ and $K_{jk}^{\pm}$ are defined as:

$$G_{jk}^{\pm} = \left(\frac{L}{u_j^2 + 1/4} - \sum_n K_{jn}^{+}\right)\delta_{jk} + K_{jk}^{\pm} \qquad \& \qquad K_{jk}^{\pm} = \frac{2}{1+(u_j-u_k)^2} \pm \frac{2}{1+(u_j+u_k)^2}.$$

Buhl-Mortensen, de Leeuw, Kristjansen, Zarembo, 2015

The $\mathfrak{su}(3)$ determinant formula

Moving to the $\mathfrak{su}(3)$ sector, let us define the following Baxter functions Q and R :

$$Q_1(x) = \prod_{i=1}^{M} (x - u_i)\,, \qquad Q_2(x) = \prod_{i=1}^{N_+} (x - v_i)\,, \qquad R_2(x) = \prod_{i=1}^{2\lfloor N_+/2 \rfloor} (x - v_i)\,.$$

All the one-point functions in the $\mathfrak{su}(3)$ sector are then given by

$$\boxed{C_k(\{u_j; v_j\}) = T_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\, Q_1(i/2)}{R_2(0)\, R_2(i/2)} \cdot \frac{\det G^+}{\det G^-}}}$$

de Leeuw-Kristjansen-GL, 2018

where $u_i \equiv u_{1,i}$, $v_j \equiv u_{2,j}$ and

$$T_n(x) = \sum_{a=-n/2}^{n/2} (x + ia)^L \frac{Q_1(x + i(n+1)/2)\, Q_2(x + ia)}{Q_1(x + i(a + 1/2))\, Q_1(x + i(a - 1/2))}.$$

The validity of the $\mathfrak{su}(3)$ formula has been checked numerically for a plethora of $\mathfrak{su}(3)$ states.

The $\mathfrak{su}(3)$ determinant formula

For $N_+ = 0$ the $\mathfrak{su}(3)$ formula reduces to the $\mathfrak{su}(2)$ formula that we saw before:

$$C_k(\{u_j\}) = \left[Q_1(0)\, Q_1(i/2) \cdot \frac{\det G^+}{\det G^-}\right]^{1/2} \cdot \sum_{a=(1-k)/2}^{(k-1)/2} \frac{a^L Q_1(ik/2)}{Q_1(i(a+1/2))Q_1(i(a-1/2))},$$

For $k = 2$ it reduces to a known $\mathfrak{su}(3)$ formula:

$$C_k(\{u_j; v_j\}) = 2^{1-L} \cdot \sqrt{\frac{Q_1(i/2)}{Q_1(0)} \frac{Q_2^2(i/2)}{R_2(0)\, R_2(i/2)} \cdot \frac{\det G^+}{\det G^-}},$$

de Leeuw-Kristjansen-Mori, 2016

where, for $A_\pm = A_1 \pm A_2$, $B_\pm = B_1 \pm B_2$, $C_\pm = C_1 \pm C_2$ and

$$\phi_{1,i} = -i \log\left[\left(\frac{u_{1,i} - i/2}{u_{1,i} + i/2}\right)^L \prod_{j\neq i}^{N_1} \frac{u_{1,i} - u_{1,j} + i}{u_{1,i} - u_{1,j} - i} \prod_{k=1}^{N_2} \frac{u_{1,i} - u_{2,k} - \frac{i}{2}}{u_{1,i} - u_{2,k} + \frac{i}{2}} \prod_{l=1}^{N_3} \frac{u_{1,i} - u_{3,l} - \frac{i}{2}}{u_{1,i} - u_{3,l} + \frac{i}{2}}\right]$$

$$\phi_{2,i} = -i \log\left[\prod_{l\neq i}^{N_2} \frac{u_{2,i} - u_{2,l} + i}{u_{2,i} - u_{2,l} - i} \prod_{k=1}^{N_1} \frac{u_{2,i} - u_{1,k} - \frac{i}{2}}{u_{2,i} - u_{1,k} + \frac{i}{2}}\right],$$

The $\mathfrak{su}(3)$ determinant formula

For $N_+ = 0$ the $\mathfrak{su}(3)$ formula reduces to the $\mathfrak{su}(2)$ formula that we saw before:

$$C_k(\{u_j\}) = \left[Q_1(0)\, Q_1(i/2) \cdot \frac{\det G^+}{\det G^-}\right]^{1/2} \cdot \sum_{a=(1-k)/2}^{(k-1)/2} \frac{a^L Q_1(ik/2)}{Q_1(i(a+1/2))Q_1(i(a-1/2))},$$

For $k = 2$ it reduces to a known $\mathfrak{su}(3)$ formula:

$$C_k(\{u_j; v_j\}) = 2^{1-L} \cdot \sqrt{\frac{Q_1(i/2)}{Q_1(0)} \frac{Q_2^2(i/2)}{R_2(0)\, R_2(i/2)} \cdot \frac{\det G^+}{\det G^-}}.$$

de Leeuw-Kristjansen-Mori, 2016

We have defined:

$$G \equiv \frac{\partial \phi_I}{\partial u_J} = \begin{bmatrix} A_1 & A_2 & B_1 & B_2 & D_1 \\ A_2 & A_1 & B_2 & B_1 & D_1 \\ B_1^t & B_2^t & C_1 & C_2 & D_2 \\ B_2^t & B_1^t & C_2 & C_1 & D_2 \\ D_1^t & D_1^t & D_2^t & D_2^t & D_3 \end{bmatrix}, \quad G^+ = \begin{pmatrix} A_+ & B_+ & D_1 \\ B_+^t & C_+ & D_2 \\ 2D_1^t & 2D_2^t & D_3 \end{pmatrix}, \quad G^- = \begin{pmatrix} A_- & B_- \\ B_-^t & C_- \end{pmatrix}.$$

The $\mathfrak{su}(3)$ determinant formula

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de Leeuw-Kristjansen-Mori, 2016

Here are some more properties of one-point functions in $\mathfrak{su}(3)$:

- One-point functions vanish if M or $L + N_+$ is odd.
- Because $Q_3 \cdot |\mathrm{MPS}\rangle = 0$ all 1-point functions vanish unless all the Bethe roots are fully balanced:

$$\left\{u_1, \ldots, u_{M/2}, -u_1, \ldots, -u_{M/2}, 0\right\}, \qquad \left\{v_1, \ldots, v_{N_+/2}, -v_1, \ldots, -v_{N_+/2}, 0\right\}.$$

The $\mathfrak{so}(6)$ determinant formula

The one-point function in the $\mathfrak{so}(6)$ sector is given by

$$C_k\left(\{u_j;v_j;w_j\}\right)=\mathbb{T}_{k-1}(0)\cdot\sqrt{\frac{Q_1(0)\,Q_1(i/2)\,Q_1(ik/2)\,Q_1(ik/2)}{R_2(0)\,R_2(i/2)\,R_3(0)\,R_3(i/2)}\cdot\frac{\det G^+}{\det G^-}}$$

where $u_i\equiv u_{1,i},\ v_j\equiv u_{2,j},\ w_k\equiv u_{3,k}$ and

$$\mathbb{T}_n(x)=\sum_{a=-n/2}^{n/2}(x+ia)^L\,\frac{Q_2(x+ia)\,Q_3(x+ia)}{Q_1(x+i(a+1/2))\,Q_1(x+i(a-1/2))}.$$

de Leeuw-Kristjansen-GL, 2018

This formula has also been verified numerically. The $M/2\times M/2$ matrices $G^\pm_{jk}$ and $K^\pm_{jk}$ are defined as:

$$G^\pm_{ab,jk}=\delta_{ab}\delta_{jk}\left[\frac{Lq_a^2}{u_{a,j}^2+q_a^2/4}-\sum_{c=1}^{3}\sum_{l=1}^{\lceil N/2\rceil}K^+_{ac,jl}\right]+K^\pm_{ab,jk},\qquad K^\pm_{ab,jk}=\mathbb{K}^-_{ab,jk}\pm\mathbb{K}^+_{ab,jk}$$

$$\mathbb{K}^\pm_{ab,jk}\equiv\frac{M_{ab}}{(u_{a,j}\pm u_{b,k})^2+\frac{1}{4}M_{ab}^2}.$$

The $\mathfrak{so}(6)$ determinant formula

The one-point function in the $\mathfrak{so}(6)$ sector is given by

$$C_k\left(\{u_j; v_j; w_j\}\right) = \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\, Q_1(i/2)\, Q_1(ik/2)\, Q_1(ik/2)}{R_2(0)\, R_2(i/2)\, R_3(0)\, R_3(i/2)} \cdot \frac{\det G^+}{\det G^-}}$$

where $u_i \equiv u_{1,i}$, $v_j \equiv u_{2,j}$, $w_k \equiv u_{3,k}$ and

$$\mathbb{T}_n(x) = \sum_{a=-n/2}^{n/2} (x+ia)^L \frac{Q_2(x+ia)\, Q_3(x+ia)}{Q_1(x+i(a+1/2))\, Q_1(x+i(a-1/2))}.$$

de Leeuw-Kristjansen-GL, 2018

More properties of one-point functions in $\mathfrak{so}(6)$:

- One-point functions vanish if M or $L+N_+ + N_-$ is odd.
- Because $Q_3 \cdot |\text{MPS}\rangle = 0$, all 1-point functions vanish unless all the Bethe roots are fully balanced:

$$\left\{u_1, \ldots, u_{M/2}, -u_1, \ldots, -u_{M/2}, 0\right\}$$

$$\left\{v_1, \ldots, v_{N_+/2}, -v_1, \ldots, -v_{N_+/2}, 0\right\}, \qquad \left\{w_1, \ldots, w_{N_-/2}, -w_1, \ldots, -w_{N_-/2}, 0\right\}.$$

Section 3

One-point Functions in the D3-D7 System

M. de Leeuw, C. Kristjansen, G. Linardopoulos, *One-point functions of non-protected operators in the $SO(5)$ symmetric D3-D7 dCFT*. J.Phys. A:Math.Theor., **50** (2017) 254001, [arXiv:1612.06236]

$SO(5)$ vacuum overlap

For the vacuum overlap we have found:

$$\langle \mathrm{MPS}|0\rangle = \mathrm{Tr}\left[G_5^L\right] = \sum_{j=1}^{n+1}\left[j\,(n-j+2)\,(n-2j+2)^L\right].$$

Changing variables $j \leftrightarrow (n+2-j)$, an overall factor $(-1)^L$ comes out, leading the vacuum overlap to zero for L odd. Equivalently, we may write

$$\langle \mathrm{MPS}|0\rangle = 2^L\left[\frac{(n+2)^2}{4}\left(\zeta\left(-L,-\frac{n}{2}\right)-\zeta\left(-L,\frac{n}{2}+1\right)\right)-\left(\zeta\left(-L-2,-\frac{n}{2}\right)-\zeta\left(-L-2,\frac{n}{2}+1\right)\right)\right],$$

where the Hurwitz zeta function is defined as:

$$\zeta\left(s,\mathrm{a}\right) \equiv \sum_{n=0}^{\infty}\frac{1}{(n+\mathrm{a})^s}.$$

$SO(5)$ vacuum overlap

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Changing variables $j \leftrightarrow (n+2-j)$, an overall factor $(-1)^L$ comes out, leading the vacuum overlap to zero for L odd. Equivalently, we may write

$$\langle \text{MPS}|0\rangle = \begin{cases} 0, & L \text{ odd} \\ 2^L \cdot \left[\frac{2}{L+3}\,B_{L+3}\left(-\frac{n}{2}\right) - \frac{(n+2)^2}{2(L+1)}\,B_{L+1}\left(-\frac{n}{2}\right)\right], & L \text{ even}, \end{cases}$$

by using the relationship between the Hurwitz zeta function and the Bernoulli polynomials $B_m(x)$.

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by using the relationship between the Hurwitz zeta function and the Bernoulli polynomials $B_m(x)$.

In the large-n limit we find:

$$\langle \text{MPS}|0\rangle \sim \frac{n^{L+3}}{2\,(L+1)\,(L+3)} + O\left(n^{L+2}\right), \qquad n \to \infty.$$

Overlap properties

- The overlaps $\langle \text{MPS}|\Psi\rangle$ of all the highest-weight eigenstates vanish unless:

$$\#\mathcal{W} = \#\overline{\mathcal{W}}, \qquad \#\mathcal{Y} = \#\overline{\mathcal{Y}}.$$

Therefore the only $\mathfrak{so}\,(6)$ eigenstates that have nonzero one-point functions are those with:

$$N_1 = 2N_2 = 2N_3 \equiv M \;\; (\text{even})\,.$$

Evidently, all one-point functions vanish in the $\mathfrak{su}\,(2)$ and $\mathfrak{su}\,(3)$ subsectors.

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Evidently, all one-point functions vanish in the $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$ subsectors.

- Because the third conserved charge Q_3 annihilates the matrix product state:

$$Q_3 \cdot |\text{MPS}\rangle = 0,$$

all the one-point functions will vanish, unless all the Bethe roots are fully balanced:

$$\left\{u_1, \ldots, u_{M/2}, -u_1, \ldots, -u_{M/2}, 0\right\}$$
$$\left\{v_1, \ldots, v_{N_+/2}, -v_1, \ldots, -v_{N_+/2}, 0\right\}, \qquad \left\{w_1, \ldots, w_{N_-/2}, -w_1, \ldots, -w_{N_-/2}, 0\right\}.$$

The $L211$ states

- We can consider eigenstates with $N_1 = 2$, $N_2 = N_3 = 1$ and arbitrary L:

$$|p\rangle = \sum_{x_1 < x_2} \left(e^{ip(x_1 - x_2)} + e^{ip(x_2 - x_1 + 1)}\right) \cdot |\ldots \underset{x_1}{\mathcal{X}} \ldots \underset{x_2}{\overline{\mathcal{X}}} \ldots\rangle - 2 \sum_{x_3} \left(1 + e^{ip}\right) \cdot |\ldots \underset{x_3}{\overline{\mathcal{Z}}} \ldots\rangle,$$

where the dots stand for $\mathcal{Z}$, and $\mathcal{X}$ is any of the complex scalars $\mathcal{W}$, $\overline{\mathcal{W}}$, $\mathcal{Y}$, $\overline{\mathcal{Y}}$.

- The momentum p is found by solving the corresponding Bethe equations:

$$e^{ip(L+1)} = 1 \;\Rightarrow\; p = \frac{4m\pi}{L+1}, \qquad m = 1, \ldots, L+1$$

- Here's the one-loop energy of the $L211$ eigenstates:

$$E = L + \frac{\lambda}{\pi^2} \sin^2\left[\frac{2m\pi}{L+1}\right] + \ldots, \qquad m = 1, \ldots, L+1$$

The $L211$ determinant formula

- The corresponding one-point function for all n is given in terms of the $n=1$ one:

$$\langle \mathcal{O}_{L211} \rangle = \left[\frac{u^2}{u^2 - 1/2} \sum_{n \bmod 2}^{n} j^L \cdot \frac{(n+2)^2 - j^2}{8} \cdot \frac{[u^2 + \frac{(n+2)j+1}{4}][u^2 - \frac{(n+2)j-1}{4}]}{[u^2 + (\frac{j+1}{2})^2][u^2 + (\frac{j-1}{2})^2]} \right] \cdot \langle \mathcal{O}_{L211}^{n=1} \rangle$$

where

$$\langle \mathcal{O}_{L211}^{n=1} \rangle = 8 \sqrt{\frac{L}{L+1} \frac{u^2 - \frac{1}{2}}{u^2 + \frac{1}{4}}} \sqrt{\frac{u^2 + \frac{1}{4}}{u^2}}, \qquad u \equiv \frac{1}{2} \cot \frac{p}{2}.$$

- The results fully reproduce the numerical values (given in units of $(\pi^2/\lambda)^{L/2}/\sqrt{L}$):

L	$N_{1/2/3}$	eigenvalue γ	n=1	n=2	n=3	n=4
2	2 1 1	6	$20\sqrt{\frac{2}{3}}$	$40\sqrt{6}$	$140\sqrt{6}$	$1120\sqrt{\frac{2}{3}}$
4	2 1 1	$5+\sqrt{5}$	$20+\frac{44}{\sqrt{5}}$	$\frac{96}{5}\left(15+\sqrt{5}\right)$	$84\left(21-\sqrt{5}\right)$	$\frac{3584}{5}\left(10-\sqrt{5}\right)$
4	2 1 1	$5-\sqrt{5}$	$20-\frac{44}{\sqrt{5}}$	$288-\frac{96}{\sqrt{5}}$	$84\left(21+\sqrt{5}\right)$	$\frac{3584}{5}\left(10+\sqrt{5}\right)$
6	2 1 1	1.50604	3.57792	324.178	11338.3	98726
6	2 1 1	4.89008	9.90466	1724.55	19513.8	120347
6	2 1 1	7.60388	61.6252	1044.86	8830.95	49114.4

Section 4

Summary & Outlook

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We have studied the tree-level 1-point functions of Bethe eigenstates in the $SU(2)$ symmetric $(\text{D3-D5})_k$ dCFT and the $SO(5)$ symmetric $(\text{D3-D7})_{d_G}$ dCFT...

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D3-D5 dCFT

- Because $Q_3 \cdot |\text{MPS}\rangle = 0$, all 1-point functions vanish unless the Bethe roots are fully balanced:

$$\{u_{1,i}\} = \{-u_{1,i}\}, \qquad \{u_{2,i}\} = \{-u_{2,i}\}, \qquad \{u_{3,i}\} = \{-u_{3,i}\}.$$

- In $\mathfrak{su}(2)$, all 1-point functions (vacuum included) vanish if M or L is odd.
- In $\mathfrak{su}(3)$, all 1-point functions vanish if (1) M is odd or (2) $L + N_+$ is odd.
- In $\mathfrak{so}(6)$, all 1-point functions vanish if (1) M is odd or (2) $L + N_+ + N_-$ is odd.
- We have found a determinant formula for the eigenstates, valid for all values of the flux k:

$$C_k(\{u_j; v_j; w_j\}) = \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{Q_1(0)\, Q_1(i/2)\, Q_1(ik/2)\, Q_1(ik/2)}{R_2(0)\, R_2(i/2)\, R_3(0)\, R_3(i/2)} \cdot \frac{\det G^+}{\det G^-}}$$

Summary & Outlook

We have studied the tree-level 1-point functions of Bethe eigenstates in the $SU(2)$ symmetric $(\text{D3-D5})_k$ dCFT and the $SO(5)$ symmetric $(\text{D3-D7})_{d_G}$ dCFT...

D3-D7 dCFT

- Because $Q_3 \cdot |\text{MPS}\rangle = 0$, all 1-point functions vanish unless the Bethe roots are fully balanced:

$$\{u_{1,i}\} = \{-u_{1,i}\}, \qquad \{u_{2,i}\} = \{-u_{2,i}\}, \qquad \{u_{3,i}\} = \{-u_{3,i}\}.$$

- Besides the vacuum, all 1-pt functions vanish in the $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$ subsectors.
- In $\mathfrak{so}(6)$ all 1-point functions vanish unless $N_1 = 2N_2 = 2N_3 \equiv M$ (even).
- The vacuum also vanishes when $L = \text{odd}$.
- We have found a determinant formula for $L211$ eigenstates, valid for all values of the instanton number n:

$$\langle \mathcal{O}_{L211} \rangle = \left[\frac{u^2}{u^2 - 1/2} \sum_{n \bmod 2}^{n} j^L \cdot \frac{(n+2)^2 - j^2}{8} \cdot \frac{[u^2 + \frac{(n+2)j+1}{4}][u^2 - \frac{(n+2)j-1}{4}]}{[u^2 + (\frac{j+1}{2})^2][u^2 + (\frac{j-1}{2})^2]}\right] \cdot \langle \mathcal{O}_{L211}^{n=1} \rangle$$

Summary & Outlook

We have studied the tree-level 1-point functions of Bethe eigenstates in the $SU(2)$ symmetric (D3-D5)$_k$ dCFT and the $SO(5)$ symmetric (D3-D7)$_{d_G}$ dCFT...

<u>Outlook</u>

- D3-D5 integrability in terms of the boundary state and its reflection matrix.
- Identification of transfer matrix. D3-D5 integrability at higher loops & strong coupling.
- D3-D7 determinant formula for all $\mathfrak{so}(6)$ Bethe eigenstates? Integrability? (work in progress)
- Higher loop orders, Wilson loops, 2-point functions in D3-D7 à la D3-D5?
- Thermodynamic limit ($M, L \to \infty$, $M/L =$const), string theory/strong coupling side ($\lambda \to \infty$)...
- Move on to the $SU(2) \times SU(2)$ solution of D3-D7 and its β-deformation...

Ευχαριστώ!