Critical Renormalization Group Flows from Extra Dimensional Theories

Fotis Koutroulis

Departement of Physics
N.T.U. of Athens

Outline

Introduction

- Gauge-Higgs Unification (GHU) was first introduced by N. Manton [1], D. Fairlie[2] and Y. Hosotani [3].
- Our approach originates from a Non-perturbative description of GHU (NPGHU)[4]
- Our goal is to project this picture on the continuum language and to analyze the Renormalization Group (RG) flows.

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[1] *A New Six-Dimensional Approach to the Weinberg-Salam Model*, Nucl.Phys. **B158** 141.

[2] *Fields and the Determination of the Weinberg Angle*, Phys.Lett. **B82** 97

[3] *Dynamical Gauge Symmetry Breaking as the Casimir Effect*, Phys.Lett. **B129** 193

[4] *Non-perturbative definition of five-dimensional gauge theories on the $\mathbb{R}^4 \times S^1/Z_2$ orbifold*, Nucl.Phys. **B719** 121 – 139,
F. Knechtli, B. Bunk, and N. Irges, *Gauge theories on a five-dimensional orbifold, PoS*, **LAT2005** 280,
N. Irges, F. Knechtli, and M. Luz, *The Higgs mechanism as a cut-off effect*, JHEP **0708** 028

Introduction

- A $5d$ Yang-Mills model defined on a hypercubic lattice, anisotropic in the fifth dimension. The anisotropy parameter then is $\gamma = a_4/a_5$.
- Resulting geometry: a $5d$ bulk with two $4d$ boundaries, located at the end-points of the fifth dimension. On the bulk lies a $5d$ $SU(2)$, while boundary orbifold conditions breaks this group down to a $U(1)$ subgroup and a complex scalar.
- The phase diagram of this theory is determined according to M. Alberti, N. Irges, F. Knechtli and G. Moir.[5]
- Three phases separated by first order phase transitions.

[5] M. Alberti, N. Irges, F. Knechtli and G. Moir, *Five-Dimensional Gauge-Higgs Unification: A Standard Model-Like Spectrum*, JHEP **1509** 159

Continuum limit

- In Part I the lowest non-trivial order in small lattice spacing expansion is considered.
- Boundary and Bulk are decoupled.
- Need for an orbifold lattice action.
- The lattice gauge variables consist of the links

$$U(n_M, N) = e^{ia_N g_5 \mathbf{A}_N(n_M)}, \quad \mathbf{A}_N \equiv A_N^A T^A$$

- The orbifold condition on the links read

$$\begin{aligned} (1-\Gamma)U(n_M, N) &= 0, \quad \Gamma \equiv \mathcal{R}\mathcal{T}_g \\ \mathcal{R} n_M &= \bar{n}_M \equiv (n_\mu, -n_5) \\ \mathcal{R} U(n_M, \nu) &= U(\bar{n}_M, \nu) \\ \mathcal{R} U(n_M, 5) &= U(\bar{n}_M - \hat{5}, 5) \\ \mathcal{T}_g U(n_M, N) &= g U(n_M, N) g^{-1} \end{aligned}$$

Continuum limit

- The Euclidean anisotropic orbifold lattice action reads $S_{S^1/\mathbb{Z}_2} = S^{b-H}_{S^1/\mathbb{Z}_2} + S^{bulk}_{S^1/\mathbb{Z}_2}$ with

$$S^{b-H}_{S^1/\mathbb{Z}_2} = \frac{1}{2N_C}\sum_{n_\mu}\left[\beta_4 \sum_{\mu<\nu}\frac{1}{2}\mathrm{tr}\left\{1 - U^{U(1)}_{\mu\nu}(n_\mu, 0)\right\} + \beta_5 \sum_\mu \mathrm{tr}\left\{1 - U^H_{\mu 5}(n_\mu, 0)\right\}\right]$$

$$S^{bulk}_{S^1/\mathbb{Z}_2} = \frac{1}{2N_C}\sum_{n_\mu, n_5}\left[\beta_4 \sum_{\mu<\nu}\mathrm{tr}\left\{1 - U_{\mu\nu}(n_\mu, n_5)\right\} + \beta_5 \sum_\mu \mathrm{tr}\left\{1 - U_{\mu 5}(n_\mu, n_5)\right\}\right]$$

- β_4 and β_5 are dimensionless lattice couplings given by

$$\beta_4 = \frac{2N_C a_5}{g_5^2} = \frac{\beta}{\gamma}, \quad \beta_5 = \frac{2N_C a_4^2}{a_5 g_5^2} = \beta\gamma, \quad \gamma = \frac{a_4}{a_5}$$

- Expanding the Wilson plaquettes to leading order in the lattice spacing we end up with

$$S^{b-H}_{S^1/\mathbb{Z}_2} = \sum_{n_\mu} a_4^4 a_5 \left[\frac{1}{4}\sum_{\mu,\nu} F^3_{\mu\nu}F^3_{\mu\nu} + \sum_\mu |\hat{D}_\mu \phi|^2\right]$$

$$S^{bulk}_{S^1/\mathbb{Z}_2} = \sum_{n_\mu} a_4^4 \sum_{n_5} a_5 \sum_{M,N} \frac{1}{4} F^A_{MN}F^A_{MN}$$

Leading order continuum action

- Definine: $\mathbf{A}_M \to \frac{1}{\sqrt{a_5}}\mathbf{A}_M$, $(F^3_{\mu\nu})^2 \to \frac{1}{a_5}(F^3_{\mu\nu})^2$, $|\hat{D}_\mu\phi|^2 \to \frac{1}{a_5}|\hat{D}_\mu\phi|^2$, $g = \frac{g_5}{\sqrt{a_4}}$, $\Lambda_5 = \frac{1}{a_5}$, $P(x_5) = 1 - \delta(x_5)$.
- The gauge fixed $5d$ continuum orbifold action reads

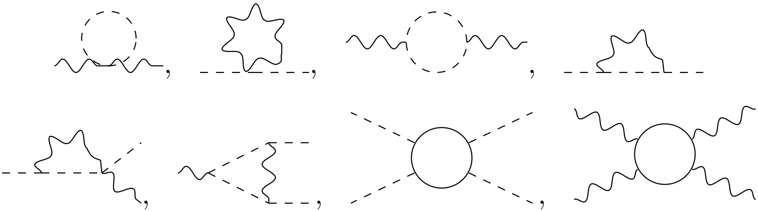
$$S_{S^1/\mathbb{Z}_2} = \int d^5x \Bigg[\Lambda_5 P(x_5) \Bigg\{ -\frac{1}{4} F^A_{MN} F^A_{MN} - \frac{1}{2\xi}(\partial_M A^A_M)^2 + \partial_M \bar{c}^C D^{CB}_M c^B \Bigg\}$$

$$+\delta(x_5) \Bigg\{ -\frac{1}{4} F^3_{\mu\nu} F^3_{\mu\nu} + |D_\mu\phi|^2 - \frac{1}{2\xi}(\partial_\mu A^3_\mu)^2 + \partial_\mu \bar{c}^3 \partial_\mu c^3 \Bigg\} \Bigg]$$

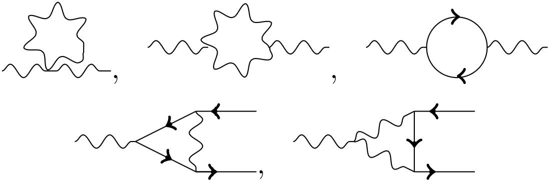
- The calculations are performed in $\xi = 1$ gauge.

1-loop Diagrams

- Quantum effects are evaluated separately for boundary and bulk theories.
- Boundary one-loop contributions



- Bulk one-loop contributions



β-functions and RGE equations

- Massive SQED has issues of renormalizability. ad hoc counter-term proposed by Salam[6]. Massless case is regularized by the free limit of the gauge invariant 1-loop Abelian-Higgs model[7].
- We use the DR version of ε-expansion, $d = 4 - \varepsilon$. $\varepsilon = 0$ for the boundary, $\varepsilon = -1$ for the bulk.
- For the boundary we use the auxiliary dimensionless coupling $\alpha_{4,0} = \frac{1}{(4\pi)^2}\mu^{d-4}g_0^2$. β-function and RGE read

$$\boldsymbol{\beta}_{\alpha_4} = \frac{2\gamma}{3}\alpha_4^2 \, (\boldsymbol{\beta}_g = \frac{g^3\gamma}{48\pi^2}), \quad \alpha_4(\mu) = \frac{3}{\gamma \ln \frac{\mu_{L4}^2}{\mu^2}}$$

- For the bulk, in the g_5 basis, we use the auxiliary dimensionless coupling $\alpha_{5,0} = \frac{2N_C}{(4\pi)^2}\mu^{d-4}g_{5,0}^2$. β-function and RGE read

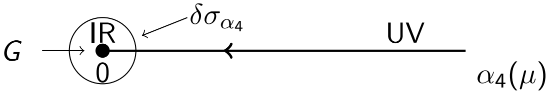
$$\boldsymbol{\beta}_{\alpha_5} = \alpha_5 - \frac{11\mathcal{C}_A\alpha_5^2}{3N_C} \, (\boldsymbol{\beta}_{g_5} = \frac{g_5\mu^{\frac{1}{2}}}{2} - \frac{11\mathcal{C}_A}{48\pi^2}g_5^3\mu^{\frac{3}{2}}), \;\; \alpha_5(\mu) = \frac{3N_C\mu}{11\mathcal{C}_A\alpha_{5,M}(\mu-M)+3N_C M}\alpha_{5,M}$$

[6] A. Salam, *Renormalized S-Matrix for Scalar Electrodynamics*, Phys. Rev. **86**, 5

[7] N. Irges and F. Koutroulis, *Renormalization of the Abelian-Higgs model in the R_ξ and Unitary gauges and the physicality of its scalar potential*, Nucl. Phys. **B924** 178

Fixed Points and anomalous dimensions

- For massless SQED the coupling and its associated operator of interest is g and $\mathcal{O}_{A\phi\bar{\phi}} = A_\mu \phi \partial_\mu \bar{\phi}$.
- The anomalous dimensions are $\gamma_{A_\mu} = -\frac{1}{16\pi^2}\frac{2g^2\gamma\mu^{-\varepsilon}}{3\varepsilon}$ and $\gamma_\phi = \frac{1}{16\pi^2}\frac{4g^2\gamma\mu^{-\varepsilon}}{\varepsilon}$. $\Delta_{\mathcal{O}_{A\phi\bar{\phi}}} = d_{\mathcal{O}_{A\phi\bar{\phi}}} = 4$ and $\gamma_{\mathcal{O}_{A\phi\bar{\phi}}} = 0$.
- Only a Gaussian fixed point (G), at $g = 0(\alpha_4 = 0)$.
- The RG flow for the marginally irrelevant boundary coupling, $\alpha_4(\mu)$ reads

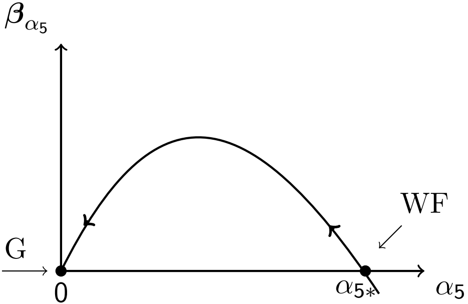


Fixed Points and anomalous dimensions

- For the 5d $SU(2)$ the coupling and its associated operator of interest is g_5 and $\mathcal{O}_{AAA} = g_5(\partial_M A_N)A_M A_N$. We specify for $N_C = 2$ and $\mathcal{C}_A = 2$.
- There is a Gaussian fixed point (G), at $g_5 = 0(\alpha_5 = 0)$ and a Wilson-Fisher (WF) fixed point at $g_{5*} = 4\pi\sqrt{\frac{3}{44}}\mu^{-\frac{1}{2}}(\alpha_{5*} = \frac{3}{11})$.
- On G: $\gamma_{A_M} = 0$, $\Delta_{\mathcal{O}_{AAA}} = d_{\mathcal{O}_{AAA}} = 5.5$ and $\gamma_{\mathcal{O}_{AAA}} = 0$.
- On WF: $\gamma_{A_M} = -0.45$, $\Delta_{\mathcal{O}_{AAA}} = 4$, $d_{\mathcal{O}_{AAA}} = 5.5$ and $\gamma_{\mathcal{O}_{AAA}} = -1.5$
- The critical exponents as the WF fixed point is approached are $\nu = 1$ and $\eta = -0.9$.
- The RG flow direction of β_{α_5} as a function of α_5, as $\mu \to \mu_* = \infty$, reads

Fixed Points and anomalous dimensions

- The non-perturbative phase diagram of an anisotropic $5d$ $SU(2)$ Yang-Mills theory was evaluated by P. de Forcrand et al.[8] and F. Knechtli et al.[9]

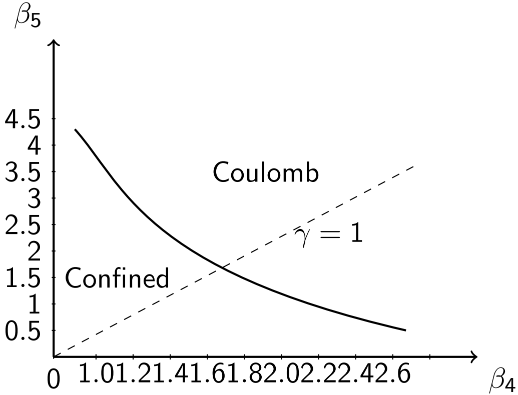


[8] P. de Forcrand, A. Kurkela and Marco Panero, *The phase diagram of Yang-Mills theory with a compact extra dimension*, JHEP **1006** 050

[9] F. Knechtli, M. Luz and A. Rago, *On the phase structure of five-dimensional SU(2) gauge theories with anisotropic couplings*, Nucl. Phys. **B856** 74-94,
F. Knechtli and E. Rinaldi, *Extra-dimensional models on the lattice* , Int. J. Mod. Phys. **A31** no.22, 1643002

Phase diagram of the bulk

- It exhibits two possible phases, a Coulomb and a Confined phase, separated by a line of first order phase transitions.



Phase diagram of the bulk

- As a quantum phase transition of first order is approached, from either side of the phase transition the system still tends to become scale invariant at one-loop order.
- Radiatively broken conformal symmetry as the mother $5d$ theory. We call our approach one of "Weak Asymptotic Safety".
- We go to the space of (β_4, β_5) couplings.
- We define a relation between $a_{4,5}$ and μ given by

$$\mu = \frac{F}{a_4} = \frac{1}{a_4}\left(f + f_q(\beta_4, \beta_5, \cdots)\right) \simeq \frac{f}{a_4}$$

Phase diagram of the bulk

- In (β_4, β_5) plain of the Coulomb phase the RGE's are

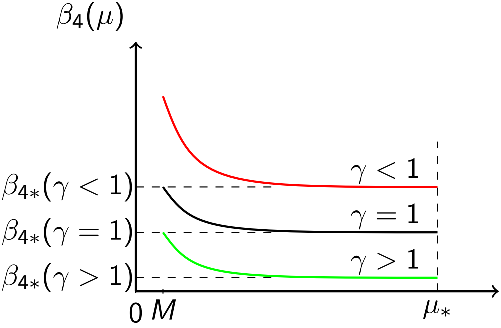
$$\beta_4(\mu) = \left(-\frac{11}{3\pi^2}\frac{f}{\gamma} + \beta_{4,M}\right)\frac{M}{\mu} + \frac{11}{3\pi^2}\frac{f}{\gamma}$$
$$\beta_5(\mu) = \left(-\frac{11}{3\pi^2}f\gamma + \beta_{5,M}\right)\frac{M}{\mu} + \frac{11}{3\pi^2}f\gamma.$$

- As $\mu \to \infty$ then $\beta_4 \to \beta_{4*} = \frac{11}{3\pi^2}\frac{f}{\gamma}$ and $\beta_5 \to \beta_{5*} = \frac{11}{3\pi^2}f\gamma$
- The knowledge of the value $\beta_{5*} = \beta_{4*} = \beta_* = 1.65$ of the $SU(2)$ coupling fixes $f = 4.44$ from Monte Carlo data.[10]

[10] M. Creutz, *Confinement and the Critical Dimensionality of Space-Time*, Phys. Rev. Lett. **43** 890,
F. Knechtli, M. Luz and A. Rago, *On the phase structure of five-dimensional SU(2) gauge theories with anisotropic couplings*, Nucl. Phys. **B856** 74-94

Phase diagram of the bulk

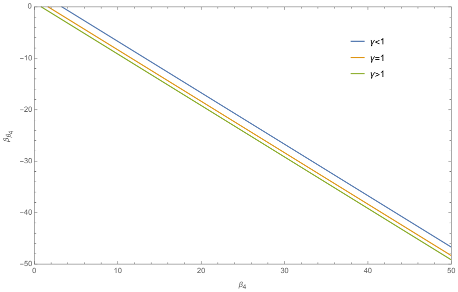
- The RG flow of $\beta_4(\mu)$ as a function of the mass scale μ for different values of anisotropy parameter γ in the continuum branch is depicted here

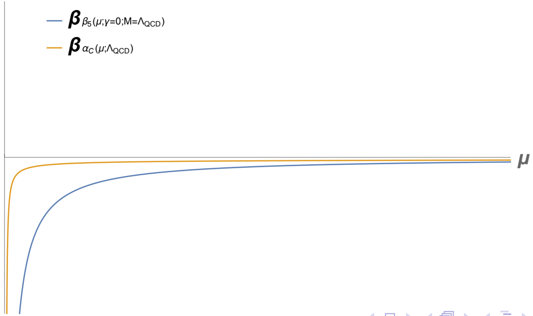


Phase diagram of the bulk

- In the Confined phase a special case is considered. For $\gamma < 1$ the 5d space breaks into approximately independently fluctuating 4d planes, called the layered phase. The perfectly layered phase is when $\beta_5 = \gamma = 0$.
- In that picture survives only an array of non-interacting 4d $SU(2)$ gauge theories.
- In (γ, β_C) plain: $\boldsymbol{\beta}_{\beta_C} = \frac{11}{3\pi^2}(\boldsymbol{\beta}_{g_C} = -\frac{44}{3}\alpha_C^2)$.
- The RGE of the coupling in the layered phase: $\beta_C(\mu) = \frac{11}{3\pi^2} \ln \frac{\mu}{\Lambda}$.

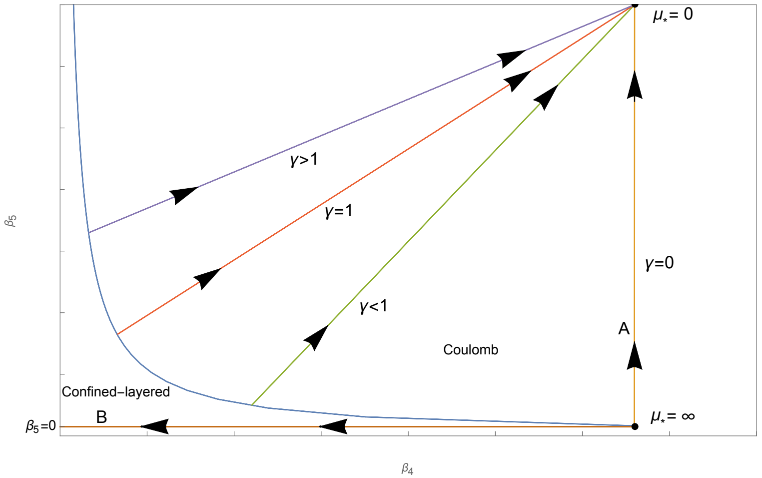
Phase diagram of the bulk





Phase diagram of the bulk

- Choosing the same boundary conditions, $M = \Lambda = \Lambda_{QCD}$, the bulk phase diagram produced from the ε-expansion procedure reads



Conclusions

- The continuum action of an anisotropic $5d$ orbifold was determined. Orbifold conditions imply a $4d$ massless scalar QED on the boundary and a $5d$ $SU(2)$ Yang-Mills model on the bulk. At the lowest non-trivial order in small lattice spacing expansion boundary and bulk are decoupled.
- The renormalization program was performed separately for the boundary and the bulk. $\boldsymbol{\beta}_g$ was obtained with the usual way. $\boldsymbol{\beta}_{g_5}$ was determined with ε-expansion procedure, expanding around $d = 4 - \varepsilon$ for $\varepsilon = -1$.
- The fixed points for both boundary and bulk couplings were determined. On the boundary there is only a Gaussian fixed point. On the bulk there are both a Gaussian and a Wilson-Fisher fixed point.
- The bulk phase diagram exhibits a Confined and a Coulomb phase. At the (β_4, β_5) plane, for $\gamma = 0$ the $5d$ space enters in layered phase. In that case, both a qualitative and a quantitative matching of the ε-expansion and of the non-perturbative phase diagrams is performed.

THANK YOU