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I. Bakas and higher-spin algebras of the W_∞ type

• One of Ioannis' most important contributions to physics is the introduction and study of higher-spin algrebras with infinite spins.

• They were introduced first in the context of two-dimensional theories

• This came a bit after Zamolodchikov introduced the W_N algebra in 2d CFTs.

THE LARGE-N LIMIT OF EXTENDED CONFORMAL SYMMETRIES

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We study the large-N limit of the operator algebra W_N generated by primary conformal fields with integer spin 1, 2, ..., N. It is shown that W_∞ provides a representation of a certain infinite dimensional (sub)algebra of the area-preserving diffeomorphisms of the 2-plane (compactified or not). We also discuss certain applications of this result to quantum field theory.

- Ioannis started from the Drinfeld-Sokolov reduction $SL(N) \to W_N$ and suggested to take the limit $N \to \infty$.

- Showed that the classical W_∞ algebra obtained is related to area-preserving diffeos.

- In a highly prescient conclusion page he suggested that:

1. W_∞ is related to quantum theories with area preserving symmetry like membranes, hydrodynamics and fermi surfaces.

2. It is related to an "enveloping" algebra of all W_N symmetries of Zamolodchikov.

3. The construction of W_N and W_∞ strings is possible and interesting.

4. Suggested that the $N \to \infty$ of Z_N parafermions, is a W_∞ theory that is related to area preserving diffeos.

5. Suggested that they are similar to the higher-spin theories of Fradkin and Vasiliev.

The Structure of the W_∞ Algebra

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Abstract. We prove rigorously that the structure constants of the leading (highest spin) linear terms in the commutation relations of the conformal chiral operator algebra W_∞ are identical to those of the $\mathrm{Diff}_0^+ \mathbb{R}^2$ algebra generated by area preserving diffeomorphisms of the plane. Moreover, all quadratic terms of the W_N algebra are found to be absent in the limit $N \to \infty$. In particular we show that W_∞ is a central extension of $\mathrm{Diff}_0^+ \mathbb{R}^2$ with non-trivial cocycles appearing only in the commutation relations of its Virasoro subalgebra. We also propose a representation of W_∞ in terms of a single scalar field in $2+1$ dimensions and discuss its significance in the context of quantum field theory.

- Fast forward to Berkeley in 1990:

Ioannis visited me, and while he came another important paper came out:

W_∞ and the Racah-Wigner algebra

C.N. Pope,* L.J. Romans† and X. Shen*

ABSTRACT

We examine the structure of a recently-constructed W_∞ algebra, an extension of the Virasoro algebra that describes an infinite number of fields with all conformal spins 2, 3, ..., with central terms for all spins. By examining its underlying $SL(2,R)$ structure, we are able to exhibit its relation to the algebras of $SL(2,R)$ tensor operators. Based upon this relationship, we generalise W_∞ to a one-parameter family of inequivalent Lie algebras $W_\infty(\mu)$, which for general μ requires the introduction of formally negative spins. Furthermore, we display a realisation of the $W_\infty(\mu)$ commutation relations in terms of an underlying associative product, which we denote with a lone star. This product structure shares many formal features with the Racah-Wigner algebra in angular momentum theory. We also discuss the relation between W_∞ and the symplectic algebra on a cone, which can be viewed as a co-adjoint orbit of $SL(2,R)$.

BOSONIC REALIZATION OF A UNIVERSAL W-ALGEBRA AND Z_∞ PARAFERMIONS*

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We construct a field theoretic representation of the universal W-algebra proposed by Pope, Romans and Shen, using a free complex boson in two dimensions. The resulting symmetry algebra is generated by conformal fields with spin $2, 3, 4, \ldots$ and has central charge $c = 2$. Highest-weight representations are also given in terms of vertex operators. Furthermore, we discuss the relation of this representation to the theory of Z_∞ parafermions.

• Our work realized the symmetry algebra as written by PRS, with a FREE complex massless boson and introduced "colored" generalizations of W_∞

• Later, Pope Romans and Shen proposed $W_{1+\infty}$

• A student of Ioannis, Depireux, realized it in terms of a FREE Dirac Fermion.

• The way was open for realizations of W_∞ in many concrete systems.

• Almost 25 years later, Maldacena and Zhibhoedov proved the reverse: only (generalized) free theories have W_∞ symmetries.

• However: it provides a structure to theories that are not obviously free.

• Examples are the quantum Hall effect, D=2 non-perturbative string theory, etc.

BOSONIZATION OF NON-RELATIVISTIC FERMIONS AND W-INFINITY ALGEBRA

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We discuss the bosonization of non-relativistic fermions in one-space dimension in terms of bilocal operators which are naturally related to the generators of W-infinity algebra. The resulting system is analogous to the problem of a spin in a magnetic field for the group W-infinity. The new dynamical variables turn out to be W-infinity group elements valued in the coset W-infinity$/H$ where H is a Cartan subalgebra. A classical action with an H gauge invariance is presented. This action is three-dimensional. It turns out to be similar to the action that describes the color degrees of freedom of a Yang–Mills particle in a fixed external field. We also discuss the relation of this action with the one recently arrived at in the Euclidean continuation of the theory using different coordinates.

Fermions in the lowest Landau level. Bosonization, W_∞ algebra, droplets, chiral bosons

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We present field theoretical descriptions of massless (2+1) dimensional nonrelativistic fermions in an external magnetic field, in terms of a fermionic and bosonic second quantized language. An infinite dimensional algebra, W_∞, appears as the algebra of unitary transformations which preserve the lowest Landau level condition and the particle number. In the droplet approximation it reduces to the algebra of area-preserving diffeomorphisms, which is responsible for the existence of a universal chiral boson lagrangian independent of the electrostatic potential. We argue that the bosonic droplet approximation is the strong magnetic field limit of the fermionic theory. The relation to the $c=1$ string model is discussed.

• Boris Khesin (student of Arnold), then professor at Berkeley Math department had introduced the log of derivative cocycle to introduce central terms in algebras of pseudo-differential operators.

• We teamed up.....

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Communications in Mathematical Physics

The Logarithm of the Derivative Operator and Higher Spin Algebras of W_∞ Type*

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• An important reason that made W_∞ popular was due to the expectation that the W_∞ charges would be important in the resolution of the black hole information paradox of the two dimensional black hole.

• However, as was already suspected in Ioannis' first paper, the linear W_∞ was expected to be a linearization of a non-linear universal W_∞ algebra, which would contain all non-linear W_N algebras

• We suspected that this would be the enveloping algebra of the $SL(2)_k$ current algebra.

• We started working when we were in Kyoto in July 1991, and finished it when we overlapped at CERN in August 1991.

BEYOND THE LARGE N LIMIT: NON-LINEAR W_∞ AS SYMMETRY OF THE SL(2,R)/U(1) COSET MODEL*

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ABSTRACT

We show that the symmetry algebra of the $SL(2,R)_k/U(1)$ coset model is a non-linear deformation of W_∞, characterized by k. This is a universal W-algebra which linearizes in the large k limit and truncates to W_N for $k=-N$. Using the theory of non-compact parafermions we construct a free field realization of the non-linear W_∞ in terms of two bosons with background charge. The W-characters of all unitary $SL(2,R)/U(1)$ representations are computed. Applications to the physics of 2-d black hole backgrounds are also discussed and connections with the KP approach to c=1 string theory are outlined.

- Since then W_∞ activity quieted down

- Vasiliev went ahead to develop his higher-spin theories in AdS completely unnoticed (in the 1990's)

- Things were revived in 2010 when Gaberdiel and Gopakumar considered the holography of W_N 2d CFTs and argued that they were dual to Vasiliev theories in AdS_3.

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An AdS_3 dual for minimal model CFTs

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We propose a duality between the 2d $\mathcal{W}_N$ minimal models in the large N 't Hooft limit, and a family of higher spin theories on AdS_3. The 2d conformal field theories (CFTs) can be described as Wess-Zumino-Witten coset models, and include, for $N = 2$, the usual Virasoro unitary series. The dual bulk theory contains, in addition to the massless higher spin fields, two complex scalars (of equal mass). The mass is directly related to the 't Hooft coupling constant of the dual CFT. We give convincing evidence that the spectra of the two theories match precisely for all values of the 't Hooft coupling. We also show that the renormalization group flows in the 2d CFT agree exactly with the usual AdS/CFT prediction of the gravity theory. Our proposal is in many ways analogous to the Klebanov-Polyakov conjecture for an AdS_4 dual for the singlet sector of large N vector models.

Holographic RG-flows at finite curvature

- Holography provides a map between large-N QFTs and string theories.

- QFT RG Flow is radial evolution in asymptotically AdS spaces.

- RG-flow is a 1st order diff equation, SG equations are second order.

- Recently we found several examples of exotic flows for which the second nature of equations is important

- Minimal setup: Einstein Dilaton gravity

$$S = \int d^{d+1}x\sqrt{g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

- Solution ansatz:

$$ds^2 = du^2 + e^{2A(u)}\ \zeta_{\mu\nu}\ dx^\mu dx^\nu \quad , \quad \phi(u)$$

- $\zeta_{\mu\nu}$ is a maximally symmetric metric of unit "radius": S^d, dS_d, AdS_d (or flat).

- Such solutions are dual to the ground state of a QFT (flow), living on the metric $\zeta_{\mu\nu}$.

Quantum field theories on curved manifolds

- There are many reasons to be interested in QFTs over curved manifolds

♠ Compact manifolds like S^n are important to regularize massless/CFTs in the IR.

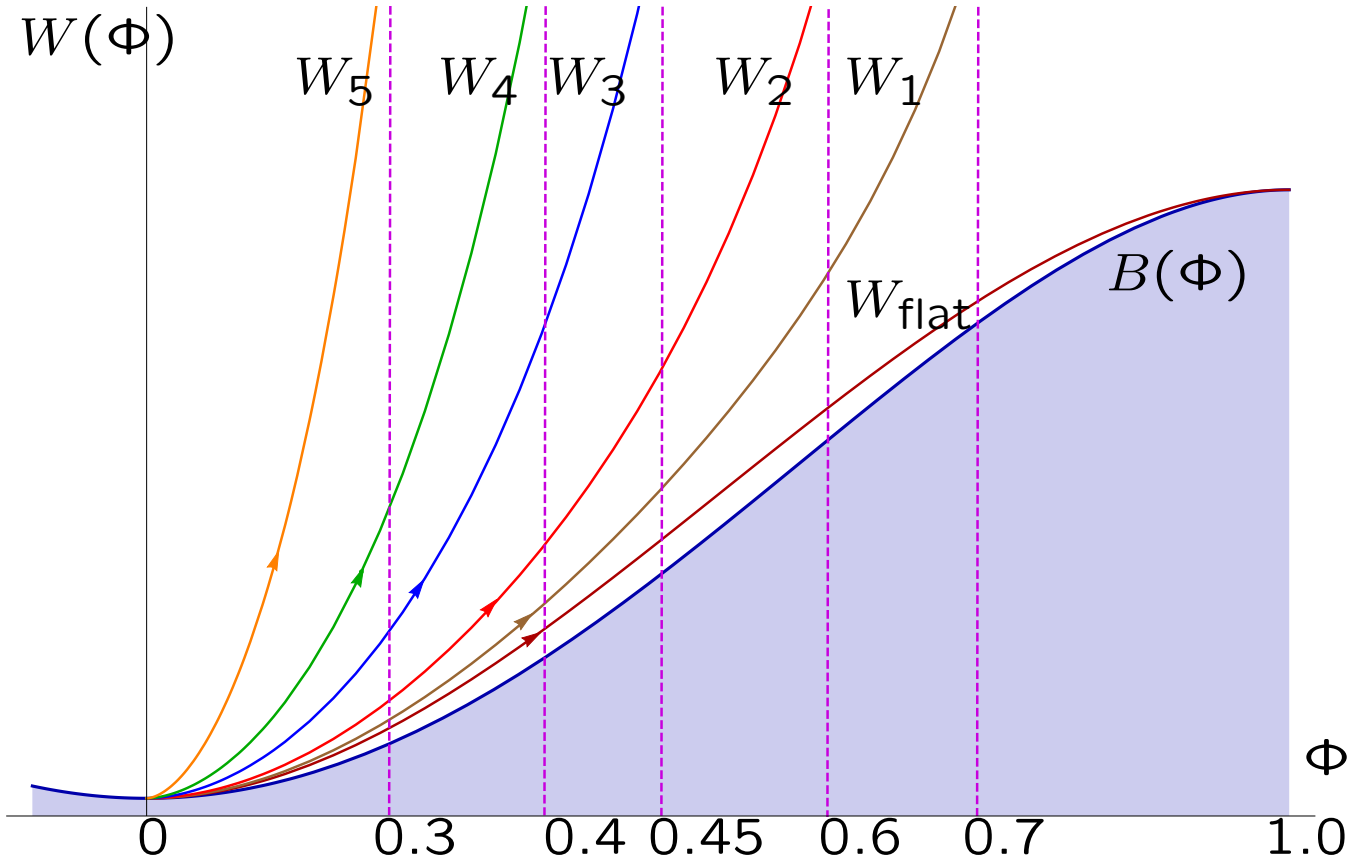
♠ QFT on deSitter manifolds is interesting due to the fact we live in a patch of de Sitter.

♠ QFT in AdS is also interesting as this is part of the bulk dynamics in holography.

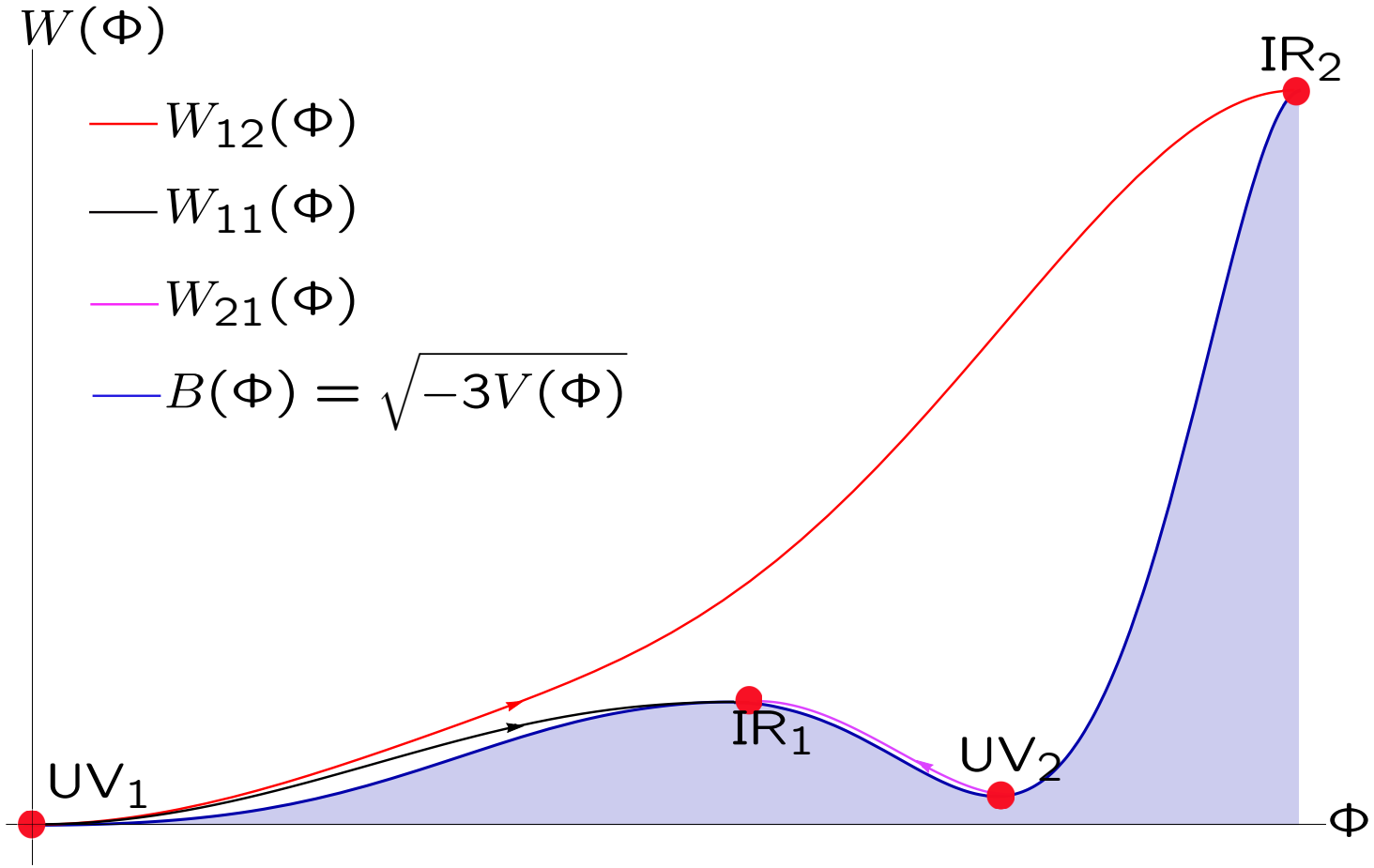
♠ The induced effective gravitational action as a function of curvature can serve as a Hartle-Hawking wave-function for three-metrics.

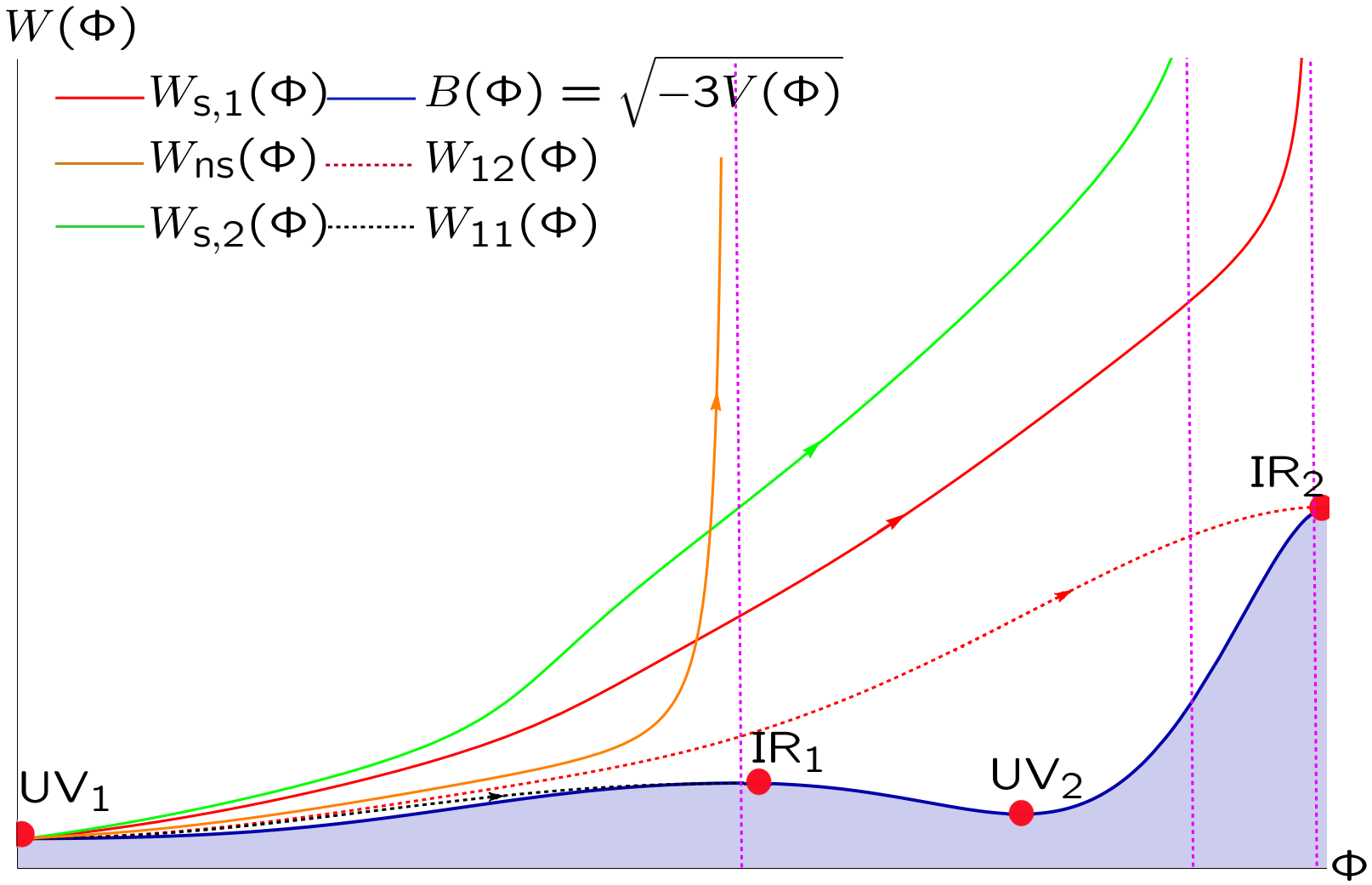
♠ Curvature, although UV-irrelevant, is IR relevant and can change importantly the IR structure of a given theory. We will see examples of quantum phase transitions driven by curvature.

The vanilla flows at finite curvature

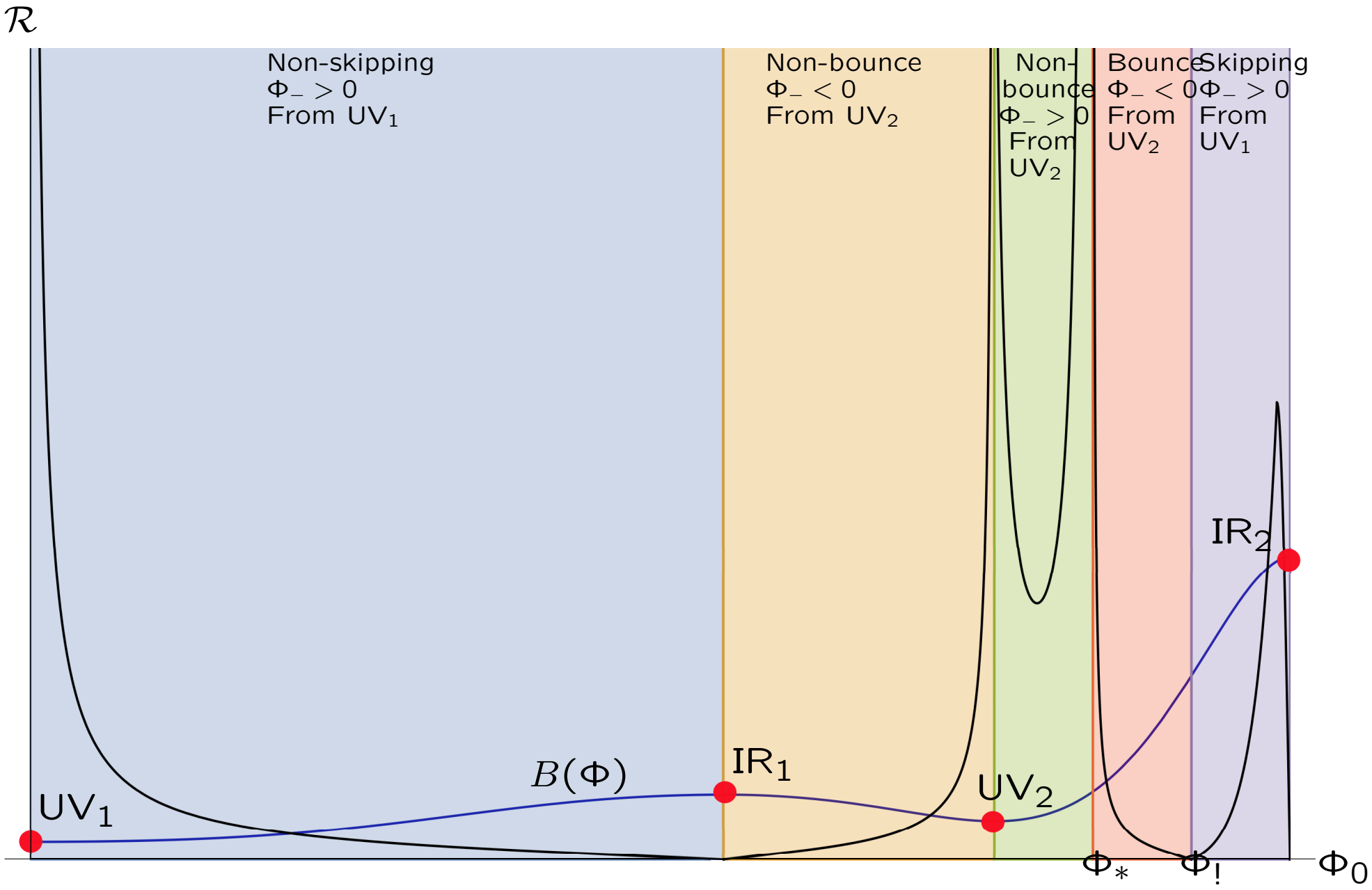


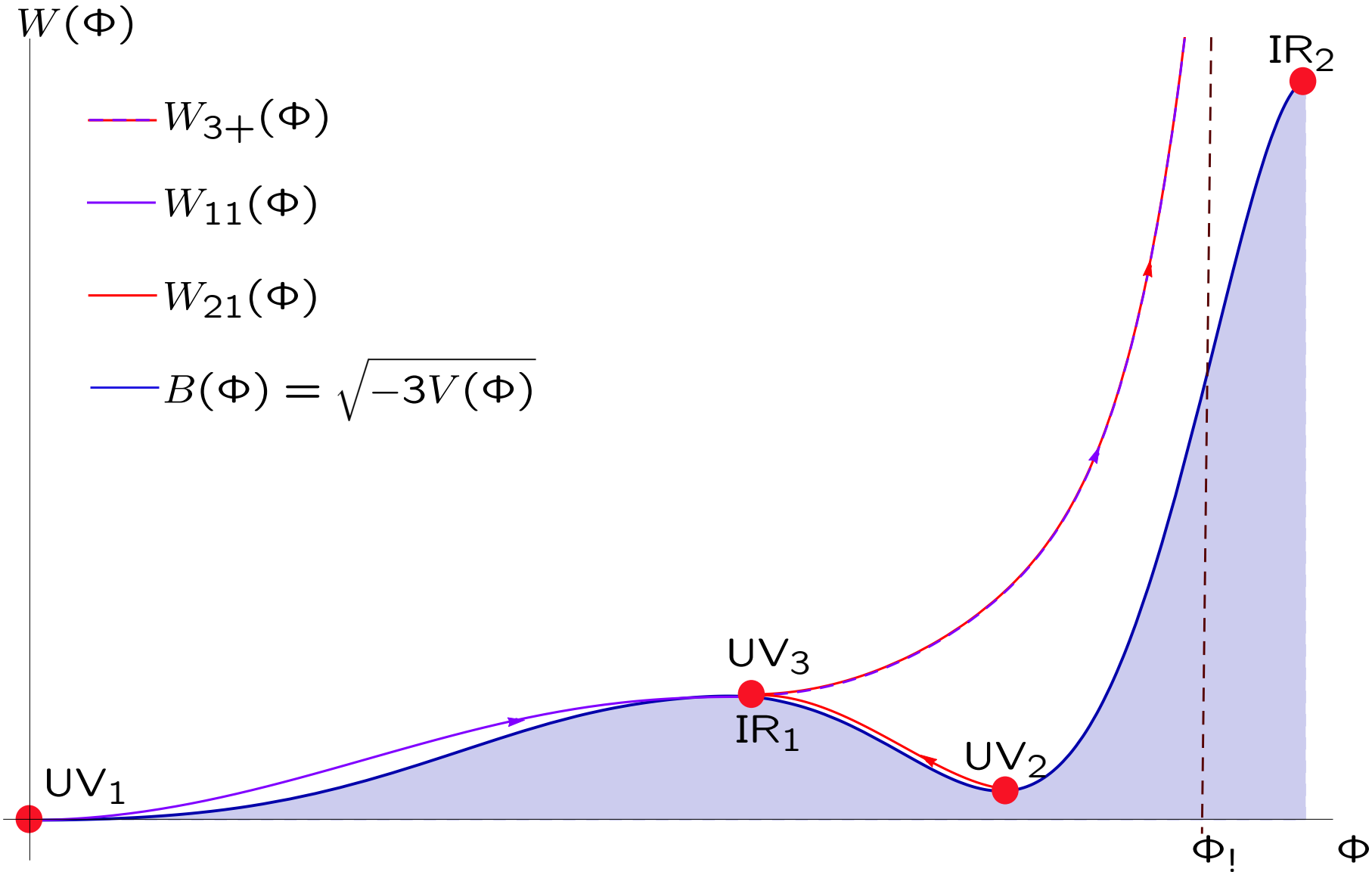
Skipping flows at finite curvature





The solid lines represent the superpotential $W(\Phi)$ corresponding to the three different solutions starting from UV_1 which exist at small positive curvature. Two of them (red and green curves) are skipping flows and the third one (orange curve) is non-skipping. For comparison, we also show the flat RG flows (dashed curves)


$\mathcal{R}$
Non-skipping
$\Phi_- > 0$
From UV_1
Non-bounce
$\Phi_- < 0$
From UV_2
Non-bounce
$\Phi_- > 0$
From UV_2
Bounce
$\Phi_- < 0$
From UV_2
Skipping
$\Phi_- > 0$
From UV_1
IR_2
$B(\Phi)$
IR_1
UV_2
UV_1
Φ_*
$\Phi_!$
Φ_0


$W(\Phi)$
$W_{3+}(\Phi)$
$W_{11}(\Phi)$
$W_{21}(\Phi)$
$B(\Phi) = \sqrt{-3V(\Phi)}$
IR$_2$
UV$_3$
IR$_1$
UV$_2$
UV$_1$
$\Phi_!$
Φ

RG flows with IR endpoint $\Phi_0 \to \Phi_!$. When the endpoint Φ_0 approaches $\Phi_!$ flows from both UV_1 and UV_2 pass by closely to IR_1, passing through IR_1 exactly for $\Phi_0 = \Phi_!$. This is shown by the purple and red curves. Beyond IR_1 both these solutions coincide, which is denoted by the colored dashed curve. These have the following interpretation. The flows from UV_1 and UV_2 should not be continued beyond IR_1, which becomes the IR endpoint for the zero curvature flows W_{11} and W_{21}. The remaining branch (the colored dashed curve) is now an independent flow denoted by W_{3+}. This is a flow from a UV fixed point at a minimum of the potential (denoted by UV_3 above) to $\Phi_!$ and corresponds to a W_+ solution with fixed value $\mathcal{R} = R^{uv}|\Phi_+|^{-2/\Delta_+} \neq 0$. While flows from UV_1 and UV_2 can end arbitrarily close to $\Phi_!$, the endpoint $\Phi_0 = \Phi_!$ cannot be reached from UV_1 or UV_2.

Epilogue

• Ioannis Bakas played an important role in the Physics Department in Patras, especially for the students.

• Beyond physics, he was a remarkable personality: he was warm, honest, with high standards, both on moral issues and on quality.

• He will live-on in our minds, as long as we live.

• But his scientific legacy will outlive all of us.

Some photos