CFT data & spontaneously broken conformal symmetry

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Outline

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Introduction and Motivation

Intoduction and (phenomenological) motivation

The Standard Model (SM) of particle physics is THE success story

- Description of a plethora of phenomena
- Its last missing piece, the Higgs boson was observed a few years ago
- So far no convincing deviations from the SM have been observed at particle physics experiments
- Moreover, the SM could be a self-consistent effective field theory up to very high energies ($\sim M_P$)

Intoduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Intoduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Compelling indications that the answer is negative!

Intoduction and (phenomenological) motivation

Experimental point of view

The SM (plus gravity) fails to accommodate in its context well established observational facts [1]

- Neutrino physics
- Dark matter
- Baryon asymmetry of the Universe

[1] Homogeneity and isotropy at large scales *can* be explained if the Higgs inflated our Universe.

Intoduction and (phenomenological) motivation

Theoretical point of view

The SM suffers from

- Landau Pole(s) but **@** energies $\gg M_{Planck}$, so usually swept under the "quantum gravity carpet"!
- Strong-CP problem
- Hierarchy problem
- Cosmological Constant problem

The last two are not a threat to its self-consistency, rather they reflect the (dramatic) failure of dimensional analysis
$\Rightarrow$ Indication that some pieces of the puzzle are not understood.

Various attempts

Proposals for addressing the hierarchy problem

- (low-energy) Supersymmetry [Fayet ‘75, ‘77 & Witten ‘81 & Dimopoulos, Georgi ‘81 & Ibanez, Ross ‘81]
- Compositeness [Weinberg ‘76, ‘79 & Susskind ‘79]
- Large extra dimensions [Arkani-Hamed, Dimopoulos, Dvali ‘98 & Randall, Sundrum ‘99]

Distinct experimental signatures above the electroweak scale differentiate them from the SM

So far no convincing deviations from the SM have been observed at particle physics experiments! (Maybe they’re waiting for us in the corner...)

Possible relevance of scale or conformal invariance

Naturalness: certain parameters might be very small, provided that the symmetry of the theory is enhanced when these are set to zero.

If this line of reasoning is applied to the symmetries of the SM at the classical level, we notice that the theory is invariant under scale and conformal transformations when Higgs mass$\rightarrow 0$ (in the absence of gravity).

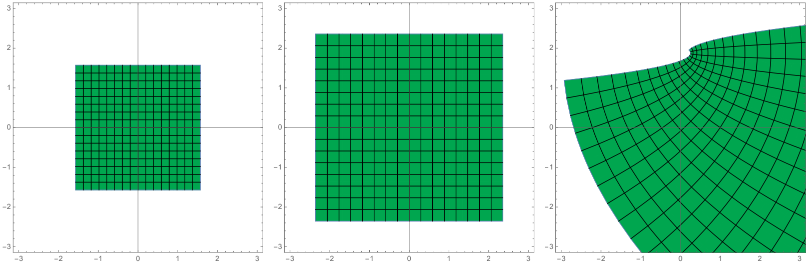
Possible relevance of scale or conformal invariance

Conformal invariance (CI) is defined as the group of coordinate transformations

$$x^\mu \to x'^\mu = F^\mu(x) \ ,$$

which leave the line element invariant up to a conformal factor

$$dx^\mu dx_\mu \to dx'^\mu dx'_\mu = \Omega(x) dx^\mu dx_\mu \ .$$



original scale trans. conformal trans.

Possible relevance of scale or conformal invariance

When this symmetry is exact it has some "peculiar" implications:

- Forbids the presence of dimensionful parameters
- No particle interpretation—the spectrum is continuous

But Nature (SM) has:

- dimensionful parameters
- particles

It certainly appears that trying to embed the SM in a conformal field theory might be a dead end...

Possible relevance of scale or conformal invariance

There is a way out! Require that the symmetry be anomaly free[2] but spontaneously broken

Consequences and *predictions*:

- Unique source for all scales
- Corrections to the Higgs mass heavily supressed if no particle thresholds between Fermi and Planck scales (technically natural)
- Presence of a gapless mode in the IR, the Goldstone boson associated with the broken symmetry—dilaton

[2] To preserve the symmetry at the loop level, one has to sacrifice renormalizability.

But...

Pretty much all considerations regarding spontaneously broken CI are for specific models—Pandora's box opens

One writes down hers/his favorite Lagrangian and studies its dynamics, so it heavily depends on taste

No attempt to study generic theories without a known explicit Lagrangian formulation, especially the implications of having the dilaton [3]

Here I will fill this gap by presenting relations on the CFT data = {operator dimensions, OPE coefficients} in the broken phase, which are universal and independent of the specifics of a system.

Leaving aside my phenomenological motivation, these should be fulfilled by *all* theories with spontaneously broken CI, for example $N = 4$ SYM (completely unrealistic theory though...)

[3] Some works have studied its dynamics on cosmological settings, again model-dependent however.

CFT data and spontaneously broken conformal symmetry

Operator Product Expansion (OPE) & the spontaneous breaking of conformal symmetry

The starting point is the OPE of two scalar primary operators

$$\mathcal{O}_i(x) \times \mathcal{O}_j(0) \sim \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}} \mathcal{O}_k + \cdots ,$$

with

$$c_{ijk} = \text{OPE coefficients} ,$$
$$|x| = \sqrt{x^\mu x_\mu} ,$$
$$\Delta_{ijk} \equiv \Delta_i + \Delta_j - \Delta_k ,$$

and the ellipses stand for operators with nonzero spin, as well as descendants.

Operator Product Expansion (OPE) & the spontaneous breaking of conformal symmetry

Consider a (unitary) four-dimensional CFT in which the conformal group is spontaneously broken to its Poincaré subgroup,[4]

$$SO(4,2) \to ISO(3,1) \ .$$

This might happen, for instance, when some of the (scalar) operators of the theory acquire a nonzero vev. To put it differently, there exists a Poincaré-invariant ground state $|0\rangle$, such that

$$\langle 0|\mathcal{O}_i|0\rangle \equiv \langle \mathcal{O}_i \rangle = \xi_i \, v^{\Delta_i} \neq 0 \ ,$$

where ξ_i's are dimensionless parameters (and v carries dimension of mass).

[4] I will be discussing exclusively about four-dimensional Minkowski spacetime. Generalizations to other number of dimensions d is straightforward (apart from $d = 2$).

Operator Product Expansion (OPE) & the spontaneous breaking of conformal symmetry

Take the OPE

$$\mathcal{O}_i(x) \times \mathcal{O}_j(0) \sim \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}} \mathcal{O}_k + \cdots ,$$

and sandwich it between the symmetry-breaking vacuum $|0\rangle$. This yields the two-point correlator

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle \sim \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}} \langle \mathcal{O}_k \rangle = \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}} \xi_k v^{\Delta_k} .$$

Since the vacuum is Poincaré-invariant, it is clear that only the scalar operators contribute to the above.

Operator Product Expansion (OPE) & the spontaneous breaking of conformal symmetry

At this point, insert a complete set of states in the left-hand side of the correlator

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \sum\!\!\!\!\!\!\!\int_N \langle 0|\mathcal{O}_i(x)|N\rangle\langle N|\mathcal{O}_j(0)|0\rangle \ .$$

In the infrared limit $x \to \infty$, we will pick up only the vacuum state. As a result, the two-point function gives ($z \equiv (v|x|)^{-1}$)

$$\boxed{\xi_i\xi_j = \lim_{z\to 0}\sum_k c_{ijk}\,\xi_k\, z^{\Delta_{ijk}}}$$

Nontrivial connection between the quadratic vevs and a linear combination of vevs weighted by the OPE coefficients.

Operator Product Expansion (OPE) & the spontaneous breaking of conformal symmetry

Important remark: the OPE will also contain operators with

$$\Delta_k > \Delta_i + \Delta_j \quad \rightarrow \quad \Delta_{ijk} < 0 \ .$$

No *a priori* reason for the corresponding OPE coefficients to vanish...
Consequently, there will be terms

$$\frac{1}{z^{\#}} \ , \quad \# > 0 \ .$$

This implies that the infrared limit $(z \rightarrow 0)$ should be taken only after the series have been summed.
Unfortunately, it is not known if the series is convergent in the case of SSB.... Thus, *I will assume* that the limit exists...

Implications of the presence of a Goldstone boson

Spontaneous symmetry breaking $\rightarrow$ Goldstone theorem dictates that there are gapless modes in the IR

For the conformal group broken down to Poincaré, one scalar degree of freedom is needed to effectuate this particular symmetry breaking pattern, the *dilaton* π.

Implications of the presence of a Goldstone boson

Presence of gapless mode in the IR implies that

$$1 \underset{x\to\infty}{\sim} |0\rangle\langle 0| + \int \frac{d^3\vec{p}}{2p_0(2\pi)^3} |\pi(p)\rangle\langle\pi(p)| \ .$$

Play the same game as before and insert this into the left-hand side of the two-point function. Fourier-transforming back to coordinate space gives

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle \underset{x\to\infty}{\sim} v^{\Delta_i+\Delta_j} \left(\xi_i\xi_j - \frac{f_i f_j}{(v|x|)^2}\right) \ ,$$

with f_i dimensionless couplings appearing in the following matrix elements

$$\langle 0|\mathcal{O}_i|\pi\rangle = f_i\, v^{\Delta_i - 1} \ .$$

Implications of the presence of a Goldstone boson

Using this relation we are lead to the second consistency condition

$$\boxed{f_i f_j = \lim_{z\to 0}\left[\frac{1}{z^2}\left(\xi_i\xi_j - \sum_k c_{ijk}\,\xi_k\,z^{\Delta_{ijk}}\right)\right]}$$

It certainly appears that I've complicated my life a lot by inserting yet another set of parameters f_i...

But these can be expressed in terms of ξ's and Δ's!

Implications of the presence of a Goldstone boson

To show that, I will work with the energy-momentum tensor $T_{\mu\nu}$.

Lorentz invariance and $\partial_\mu T^{\mu\nu} = 0$, dictate that the matrix element of $T_{\mu\nu}$ between the vacuum and the dilaton π, be of the following form,[5]

$$\langle 0|T_{\mu\nu}(0)|\pi(p)\rangle = \frac{1}{3} f_\pi v \, p_\mu p_\nu \ ,$$

with f_π the dimensionless dilaton decay constant and the factor of 1/3 was added for later convenience.

[5] Note that a term proportional to $\eta_{\mu\nu}p^2$ is also admissible in this matrix element. However, this contribution vanishes on shell.

Implications of the presence of a Goldstone boson

It is straightforward to show that the expectation value of the commutator between the energy-momentum tensor and an operator reads

$$\langle [T_{\mu\nu}, \mathcal{O}_i] \rangle = \frac{i}{3} f_\pi f_i v^{\Delta_i} \, \partial_\mu \partial_\nu G(x) \ ,$$

where $G(x)$ is the (massless) Green's function

$$G(x) = -i \int \frac{d^3\vec{p}}{2p_0(2\pi)^3} (e^{-ipx} + e^{ipx}) \ .$$

Implications of the presence of a Goldstone boson

The Noether current associated with dilatations is [6]

$$J_\mu \equiv x^\nu T_{\mu\nu} \ ,$$

so the corresponding charge is a particular moment of the energy-momentum tensor

$$D = \int d^3\vec{x}\, J_0 = i f_\pi f_i v^{\Delta_i} \ ,$$

while, by definition,

$$[D, \mathcal{O}_i] = i\Delta_i \mathcal{O}_i \quad \rightarrow \quad \langle [D, \mathcal{O}_i] \rangle = i\xi_i \Delta_i v^{\Delta_i} \ .$$

Consequently,

$$\boxed{f_i = \frac{\xi_i\, \Delta_i}{f_\pi}}$$

[6] I could equivalently consider the charge associated with special conformal transformations, but its form is more complicated. The results are obviously the same.

Consistency conditions on the CFT data

If a nontrivial solution for the constraint equations exists, this can serve as an indication that the CFT data describes a system that exhibits the symmetry breaking pattern $SO(4,2) \to ISO(3,1)$.

Consistency conditions on the CFT data

One can go ahead and find more constraints. Look for example at the energy-momentum tensor. The relevant terms in the two-point correlator of $T_{\mu\nu}$ with itself are

$$\langle T_{\mu\nu}(x) T_{\lambda\sigma}(0)\rangle = \sum_k \mathcal{T}_{\mu\nu\lambda\sigma} \frac{\xi_k v^{\Delta_k}}{|x|^{8-\Delta_k}} \ ,$$

with $\mathcal{T}_{\circ\circ\circ\circ} = \mathcal{T}_{\circ\circ\circ\circ}\left(c_{ijk}^{E-M}, x_\mu, \eta_{\mu\nu}\right)$ the most general Lorentz covariant structure consistent with the appropriate symmetries. Due to the interaction with the dilaton, one finds

$$\boxed{f_\pi = f_\pi\left(c_{ijk}^{E-M}, \Delta_i, \xi_i\right)}$$

or to put in words, the dilaton decay constant is related to the E-M tensor OPE coefficients, the Δ's and the ξ's.

Concluding remarks and open questions

- Nontrivial consistency conditions for theories with exact but spontaneously broken conformal symmetry
- Lagrangian-independent constraints that rely solely on the symmetry breaking pattern
- Leaving aside our phenomenological motivation, these relations should be true for all systems with nonlinearly realized conformal symmetry

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- How to prove convergence of the OPE?
- Ideal testing ground → Coulomb branch of $N = 4$ SYM
- Can unitarity & analyticity yield more useful relations? (in the spirit of bootstrap...)