Novel Black-Hole Solutions in Einstein-Scalar-Gauss-Bonnet Theories

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Outline

- Introduction to Generalised Theories of Gravity
- The Einstein-Scalar-Gauss-Bonnet Theories
- Novel Einstein-Scalar-Gauss-Bonnet Black Holes
    - Asymptotic solutions
    - The evasion of novel No-Hair Theorem
    - Novel Black-hole solutions with scalar 'hair'
    - The old No-Hair Theorem
- Conclusions

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in collaboration with G. Antoniou (Univ. of Minnesota)
and A. Bakopoulos (Univ. of Ioannina)

Introduction

In 1915, Einstein formulated his General Theory of Relativity (GR) that allowed him to describe the form of spacetime and the motion of a body inside that spacetime

The metric tensor $g_{\mu\nu}$ defined through the spacetime's line-element

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

is the 'building block' of Einstein's theory and its form is determined by the gravitational field equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

Selecting the physical system that interests us and the corresponding energy-momentum tensor $T_{\mu\nu}$, that describes the distribution of mass and energy, we may determine from the evolution of the whole *universe* to the form of spacetime around a small *black hole*

Introduction

General Relativity is a very good theory of Gravity: mathematically beautiful and experimentally tested

But it is not a perfect theory (if there is such a thing)....

- The Standard Cosmological Model has a number of open problems: the nature of dark matter and dark energy, the coincidence problem, the spacetime singularities, the right model for inflation...
- On the gravity side, GR predicts the existence of only three families of black-hole solutions (information loss problem) and no stable wormhole solutions
- Unification with the other forces seems unlikely within the GR (GR is based on tensors and is not renormalizable)

Perhaps, all these accumulated problems point to the need for changing the theoretical framework?

Introduction

The Einstein's field equations follow from the Einstein-Hilbert action

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} \right),$$

A generalised theory of gravity could have the form

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \Phi) + \mathcal{L}_X(\Phi) \right]$$

- as part of the string effective action at low energies
- as part of a Lovelock effective theory in four dimensions
- as part of a modified scalar-tensor (Horndeski or DHOST) theory

The large number of choices have led to a huge literature in Generalised Gravitational Theories...

Introduction

Of particular importance is the quadratic Gauss-Bonnet term

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

since it has a number of attractive points:

- It leads to field equations with up to 2nd-order derivatives, and the theory is free from Ostrogradski instabilities and ghosts
- Being quadratic, it will naturally be the next important term when the curvature becomes strong (near the black-hole horizon, in the early universe, at the wormhole throat...)

A weak point: The GB term can be shown to be a total derivative term for all gravitational backgrounds in 4 dimensions, therefore, it survives in the equations of motion only if it is coupled to a field

The Einstein-Scalar-Gauss-Bonnet Theory

A generalised theory of gravity with the GB term will have the form:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + f(\phi)\,R^2_{GB} \right],$$

with $f(\phi)$ a coupling function between a scalar field ϕ and the GB term

- when $f(\phi) \sim \ln\left[2e^\phi \eta^4(ie^\phi)\right]$ and ϕ a modulus field, the theory leads to singularity-free solutions (Antoniadis, Rizos & Tamvakis, 1994)
- when $f(\phi) \sim e^\phi$ and ϕ is the *dilaton field*, the theory evades the no-hair theorems and leads to the *Dilatonic Black Holes* (Kanti, Mavromatos, Rizos, Tamvakis & Winstanley, 1996; 1998)
- when $f(\phi) \sim e^\phi$ and ϕ is the *dilaton field*, the theory also leads to stable, wormhole solutions (Kanti, Kleihaus & Kunz, 2011)

Is superstring effective theory the only theory with so interesting solutions?

The Einstein-Scalar-Gauss-Bonnet Theory

Clearly no! Because for $f(\phi) \sim \phi^2$ and ϕ *a scalar field*, singularity-free cosmological solutions again emerge (Rizos & Tamvakis, 1994; Kanti, Rizos & Tamvakis, 1998)

Are there, then, many black-hole solutions beyond the limits of GR ...? Not really:

- The old 'No-Hair Theorem' (Bekenstein, 1972; Teitelboim, 1972) managed to restrict such solutions in the context of minimally-coupled scalar-tensor theories:

  "There are no static black-hole solutions with scalar hair"

- This was evaded for black-hole solutions with a Yang-Mills (Volkov & Galtsov, 1989, Bizon, 1990; Greene et al, 1993; Maeda et al, 1994) or Skyrme field (Luckock & Moss, 1986; Droz et al, 1994) or for a conformally-coupled scalar field (Bekenstein, 1974)

The Einstein-Scalar-Gauss-Bonnet Theory

- The "novel No-Hair Theorem" was then formulated (Bekenstein, 1995) for non-minimally-coupled scalar fields; this was extended to general scalar-tensor theories (Sotiriou & Faraoni, 2012; Hui & Nicolis, 2013)
- But these were again evaded in the case of dilatonic black holes (1996), for an exponential coupling function $f(\phi) \sim e^{\phi}$, and the shift-symmetric theory with $f(\phi) \sim \phi$ (Babichev & Charmousis, 2014; Sotiriou & Zhou, 2014; Benkel et al, 2016)

Therefore, even in the context of generalised gravitational theories only a small number of black-hole solutions have been found

Can we add more 'members' to this 'elite' number of theories that evade the no-hair theorems and lead to novel black holes?

Novel Einstein-Scalar-GB Black-Hole Solutions

We will consider again the following theory

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + f(\phi)\,R^2_{GB} \right],$$

but we will keep the coupling function $f(\phi)$ arbitrary. Assuming the following static, spherically-symmetric line-element

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

the equations of motion read

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}\,R = T_{\mu\nu}\,, \qquad \nabla^2\phi + \dot{f}(\phi)R^2_{GB} = 0$$

where

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\nabla_\gamma\partial_\kappa f,$$

Novel Einstein-Scalar-GB Black-Hole Solutions

Rearranging the equations, we obtain the coupled system

$$A'' = \frac{P}{S}\,, \qquad \phi'' = \frac{Q}{S}$$

where P, Q and S are complicated functions of $(r, \phi', A', \dot{f}, \ddot{f})$ – the metric function B is shown to be a dependent variable

For the existence of a regular black-hole horizon we demand that

$$e^{A(r)} \to 0, \quad e^{-B(r)} \to 0, \quad \phi(r) \to \phi_h$$

Employing the above in the coupled system, and demanding that ϕ'' is also finite at the horizon r_h, we find a constraint that determines ϕ'_h

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad \dot{f}_h^2 < \frac{r_h^4}{96}$$

The above hold for an otherwise arbitrary coupling function f

Novel Einstein-Scalar-GB Black-Hole Solutions

Using the constraint on ϕ'_h in the equation for A'', we find

$$A'' = -A'^2 + ... \Rightarrow A' = (r - r_h)^{-1} + \mathcal{O}(1)$$

that upon integration leads to the complete solution

$$e^A = a_1(r - r_h) + \dots, \qquad e^{-B} = b_1(r - r_h) + \dots,$$

$$\phi = \phi_h + \phi'_h(r - r_h) + \phi''_h(r - r_h)^2 + \dots$$

The above describes a regular black-hole horizon in the presence of a scalar field provided that ϕ' and the coupling function f satisfy the aforementioned constraints

The above line of thinking was developed in the analysis for the Dilatonic black holes (Kanti, Mavromatos, Rizos, Tamvakis & Winstanley, 1996) and here it was generalised for an *arbitrary* coupling function f

Novel Einstein-Scalar-GB Black-Hole Solutions

At large distances from the horizon, we assume a power series expression in $1/r$, and by substituting in the equations of motion, we find

$$e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \frac{24MD\dot{f} + M^2D^2}{6r^4} + \ldots$$

$$e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \frac{32M^3 - 5MD^2}{4r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) + \ldots$$

$$\phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \frac{12M^3D - 24M^2\dot{f} - MD^3}{6r^4} + \ldots$$

It is in order $\mathcal{O}(1/r^4)$ that the explicit form of the coupling function $f(\phi)$ first makes its appearance

Thus, a general coupling function f does not interfere with the existence of an asymptotically-flat limit for the spacetime

Novel Einstein-Scalar-GB Black-Hole Solutions

Can we smoothly connect these two asymptotic solutions? Bekenstein's *Novel No-Hair* theorem (1995) said no, because:

- "at radial infinity: T_r^r is positive and decreasing"

Indeed, even in the presence of the GB term: $T_r^r \simeq \phi'^2/4 \sim 1/r^4 + \ldots$

- "near the BH horizon: T_r^r is negative and increasing"

If true, the smooth connection of the two demands an extremum - this is *excluded* by the positivity of energy in ordinary scalar-tensor theories

However, in the Einstein-scalar-Gauss-Bonnet theories with general f, the second clause is not true. Instead, we find that

$$sign(T_r^r)_h = -sign(\dot{f}_h \phi'_h) = 1 \mp \sqrt{1 - 96\dot{f}^2/r_h^4)} > 0$$

The regularity of the horizon automatically guarantees the positivity of T_r^r

Novel Einstein-Scalar-GB Black-Hole Solutions

But we should also check the sign of $(T_r^r)'_h$ - for this we find that

$$(T_r^r)'_h < 0 \qquad iff \qquad \partial_r(\dot{f}\phi')|_{r_h} > 0$$

This merely demands that the negative value of $(\dot{f}\phi')$ near the horizon should start decreasing with r (in absolute value) in order to smoothly connect with the constant scalar field at infinity

It turns out that this is automatically satisfied in all the solutions found without the need of any fine-tuning of the parameters

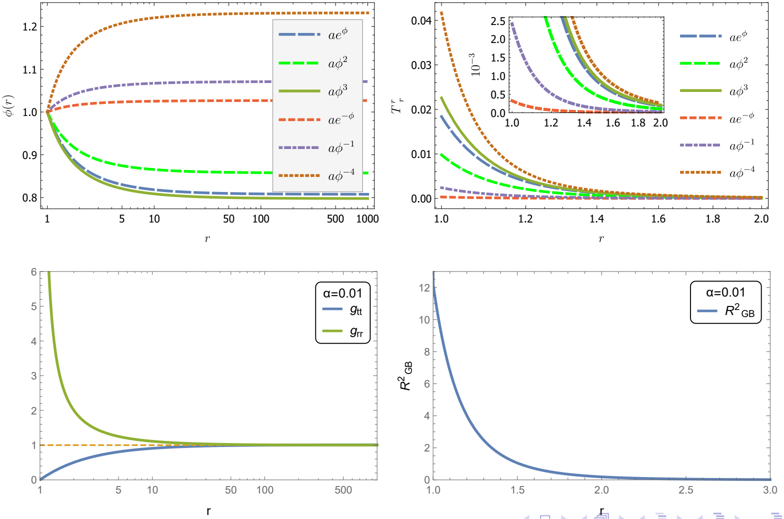
Thus, selecting the form of $f(\phi)$ and choosing any value of ϕ_h that satisfies the constraint

$$\dot{f}_h^2 < \frac{r_h^4}{96},$$

the constraint for a regular horizon uniquely determines the second input parameter ϕ'_h for the numerical integration, and the black-hole solutions may be found...

Novel Einstein-Scalar-GB Black-Hole Solutions

(Antoniou, Bakopoulos & Kanti, 1711.03390, 1711.07431)



Novel Einstein-Scalar-GB Black-Hole Solutions

- Doneva & Yazadjiev, 1711.01187 : $f(\phi) = 1 - e^{-\phi^2}$
- Silva, Sakstein, Gualtieri, Sotiriou & Berti, 1711.02080 : $f(\phi) = a\phi^2$

- **Old No-Hair Theorem:** it uses the scalar equation

$$\int d^4x \sqrt{-g}\, f(\phi) \left[\nabla^2\phi + \dot{f}(\phi)\, R^2_{GB}\right] = 0$$

Integrating by parts, we obtain

$$\int d^4x \sqrt{-g}\, \dot{f}(\phi) \left[\partial_\mu\phi\, \partial^\mu\phi - f(\phi)\, R^2_{GB}\right] = 0$$

Since, $\partial_\mu\phi\, \partial^\mu\phi > 0$, the above holds only for $f(\phi)\, R^2_{GB} > 0$. Silva et al, with a slightly different manipulation, found instead that $\ddot{f}\, R^2_{GB} > 0$

Then, can we really trust this *integral* constraint? The novel no-hair theorem is instead a *local* constraint that is easily checked

Conclusions

- The Generalised Theories of Gravity may be the way forward in gravitational physics that removes mathematical irregularities and physical puzzles
- General Einstein-scalar-Gauss-Bonnet theories have been intensively studied over the last decades
- Although the new black-hole solutions that emerge in the context of these theories were very few, our work has shown that there is nothing exceptional in them
- Under mild constraints, related to the regularity of the horizon, an asymptotically-flat solution with scalar hair always emerges
- We have found a plethora of such solutions with a variety of characteristics (radial profile of the field, size, scalar charge, entropy) - for details, please, see the talk by A. Bakopoulos on Friday afternoon