Strongly-coupled Anisotropic Gauge Theories And Holography

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Based on works with:
U. Gursoy(Utrecht Univ.) and J. Pedraza(Univ. of Amsterdam) and work in progress.

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Outline

Briefly on AdS/CFT

- Gauge/Gravity duality: A way to map and answer quantum questions to gravity geometric questions.
- The initial AdS/CFT correspondence: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow$ $AdS_5 \times S^5$, is the harmonic oscillator of the gauge/gravity dualities.
- The theory is simple: Conformal, Maximally Supersymmetric, No Temperature...
- Since the discovery of the initial correspondence, there is an extensive research aiming to construct more realistic gauge/gravity dualities.

Gauge/Gravity Dualities with: Less/No Supersymmetry; Broken conformal symmetry, confinement; fundamental matter(probe and backreacting Dq branes); etc.

✓ We study Anisotropic theories in Gauge/Gravity correspondence.

Why? Attempts for Realizations in Nature

The existence of strongly coupled anisotropic systems.

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- New interesting phenomena in presence on such fiels, i.e. inverse magnetic catalysis.

  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)
- Anisotropic low dimensional materials in condensed matter.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.
  (Dumitru, Strickland, Romatschke, Baier,... 2008,...)
- Consistent top-down models. Properties of the supergravity solutions, that are dual to the anisotropic theories.
- Black hole solutions that are AdS in UV **flowing** to Lifhitz-like in IR :
  ⋆ Why there is a fixed scaling parameter z for such solutions?
  (Azeyanagi, Li, Takayanagi, 2009)
  ⋆ Other systems that have fixed scaling IR solution (e.g. in Heavy quark density). Why?
  (Kumar 2012; Faedo, Kundu, Mateos, Tarrio 2014)
  ⋆ New flows to Hyperscaling violation IR backgrounds?

Why? Even More:

- Ⓢtriking Features! Several Universality Relations predicted for the isotropic theories are violated!
  ⋆ Shear viscosity over entropy density ratio takes parametrically low values $\frac{\eta}{s} < \frac{1}{4\pi}$!
  (Rebhan, Steineder 2011; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017;...)
  ⋆ Langevin coefficients inequality for heavy quark motion in the anisotropic plasma gets inverted $\kappa_L > < \kappa_T$.
  (Gursoy, Kiritsis, Mazzanti, Nitti 2010; D.G, Soltanpanahi, 2013a, 2013b)
  ⋆ ...
  ⋆ Implications to QGP hydrodynamic simulations.

The Introduction of the Theory in One Page:

- Strongly coupled anisotropic theory.
- How the theory looks like and how to obtain it?
  - ✓ 4d $SU(N)$ gauge theory in the large N_c-limit.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O} \sim TrF^2$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.
  - ✓ On the gravity dual side we have a "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
- Eventually the gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ Solutions are RG flows:
    AdS in UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.
- Formally interesting theories. And a solid ground to study strongly coupled phenomena in presence of anisotropy.

Parts of the Theory Timeline-Related bibliography:

Non-Confining Anisotropic Theories:

(Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011; Jain, Kundu, Sen, Sinha, Trivedi, 2015;...)

Confining Anisotropic Theories:

(D.G., Gursoy, Pedraza, 2017)

Similar ideas in different context. For example:

(Gaiotto, Witten 2008; Chu, Ho, 2006; Choi, Fernadez, Sugimoto 2017;...)

How is Anisotropy introduced? A Pictorial Representation:

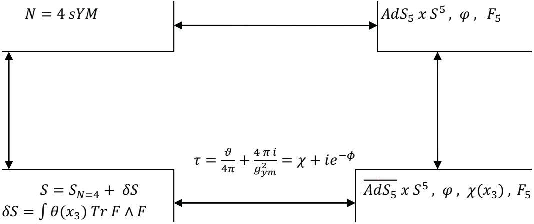
- For the Lifshitz-like IIB Supergravity solutions

$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds_{S^5}^2 .$$

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following AdS/CFT deformation.



- $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.
- Possible to compactify x_3 to get dual to the pure Chern-Simons gauge theory.

The Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

The eoms read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}Z(\phi)\partial_\mu\chi\partial_\nu\chi - \frac{1}{4}g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4}g_{\mu\nu}Z(\partial\chi)^2 + \frac{1}{2}g_{\mu\nu}V(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = \frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2 - V'(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi\right) = 0\ .$$

Where

$$V(\phi) = 12\cosh(\sigma\phi) + \left(\frac{m(\Delta)^2}{2} - 6\sigma^2\right)\phi^2, \qquad Z(\phi) = e^{2\gamma\phi},$$

(Gursoy, Kiritsis, Nitti, 2007; (Gubser, Nellore), Pufu, Rocha 2008a,b)

Remark: For $\sigma = 0, \gamma = 1, m(\Delta) = 0$ the action and the solution of eoms, are reduced of IIB supergravity.

Solutions of the Generalized Einstein-Axion-Dilaton Action

- The background solution

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$
$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,,$$

- Solutions: e.g. For $\Delta = 4$:
  AdS in UV flowing to Hyperscaling Lifshitz-like violation geometries in IR :

$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)\,,$$

Axion-Dilaton Coupling and Potential, rule the Scaling Coefficients

- The values of (θ, z) dependence on (γ, σ)

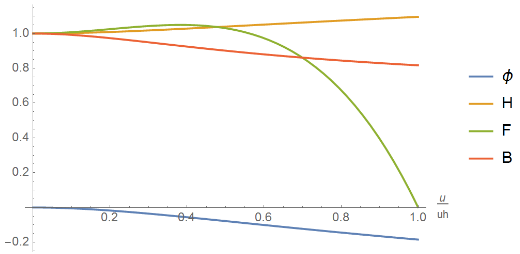
$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} , \qquad \theta = \frac{3\sigma}{2\gamma} .$$

- Special case: $(\sigma = 0,\ \gamma = 1)$ supergravity truncated action with a single solution $(\theta = 0,\ z = 3/2)$. *(Mateos, Trancanelli, 2011)*
- The scaling factors z and θ are determined by the constants in the Axion-Dilaton Coupling and the Potential. This is the reason that in the particular setup the supergravity solutions have them fixed.

Solution : The Full Flow

- Fixing (γ, σ) and α and u_h we get the metric flow from boundary to horizon:

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\, dt^2 + dx_1^2 + dx_2^2 + \mathcal{H} dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$



UV Boundary IR Horizon

An exact solution

The potential and the axion-dilaton coupling

$$V(\phi) = 6e^{\sigma\phi}, \qquad Z(\phi) = e^{2\gamma\phi}.$$

A Lifshitz-like anisotropic hyperscaling violation background which may accommodate a black hole

$$ds_s^2 = \alpha^2 C_R e^{\frac{\phi(u)}{2}} u^{-\frac{2\theta}{3z}} \left(-u^2 \left(f(u) dt^2 + dx_i^2 \right) + C_Z u^{\frac{2}{z}} dx_3^2 + \frac{du^2}{f(u)\alpha^2 u^2} \right) ,$$

where

$$f(u) = 1 - \left(\frac{u_h}{u} \right)^{3+(1-\theta)/z} , \qquad e^{\frac{\phi(u)}{2}} = u^{\frac{\sqrt{\theta^2+3z(1-\theta)-3}}{\sqrt{6}z}} ,$$

$$C_R = \frac{(3z-\theta)(1+3z-\theta)}{6z^2} , \quad C_Z = \frac{z^2}{2(z-1)1+3z-\theta} ,$$

$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} , \qquad \theta = \frac{3\sigma}{2\gamma} .$$

We have obtained the theories, are they physical and stable?

$$\Downarrow$$
$$\Downarrow$$

✓ Energy Conditions Analysis

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

✓ Local Thermodynamical Stability Analysis

$$c_\alpha = T \left(\frac{\partial S}{\partial T} \right)_\alpha \geq 0 \ , \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha} \right)_T \geq 0$$

$$\Downarrow$$
$$\Downarrow$$

YES!

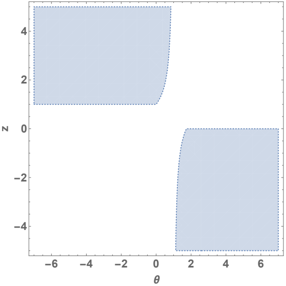
Three conditions that constrain (z, θ) and as a result (γ, σ).

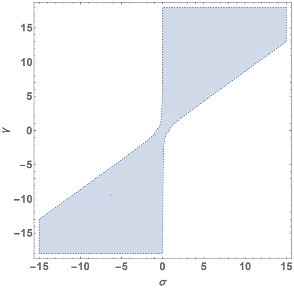
$$(z-1)(1-\theta+3z) \geq 0 \ , \qquad z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \ ,$$

$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ , \qquad \theta = \frac{3\sigma}{2\gamma} \ ,$$

$$1 - \theta + 2z \geq 0 \ .$$

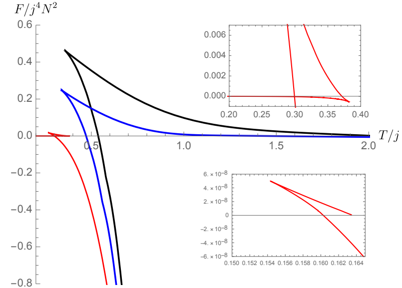
The blue region is the acceptable for the theory parameters.





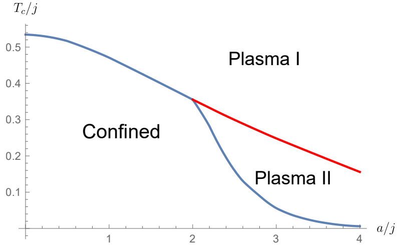
Confinement/Deconfinement Phase transitions

- The free energy of the theories vs the temperature T for different anisotropy (α/j=0,1,3):



- Horizontal Axis: Confining Phase.
- Upper Branch: Black hole A:Deconfining Plasma Phase.
- Lower Branch: Black hole B:Deconfining Plasma Phase.
- $\alpha/j \simeq 2$: A critical value above which a richer structure in the phase diagram exist.

- The Critical Temperature of the theories vs the anisotropy gives:



- The T_c is reduced in presence of anisotropies of the theory.

The Proposal

- The $Tc(\alpha)$ decrease with α, resembling the phenomenon of inverse magnetic catalysis where the confinement-deconfinement temperature decreases with the magnetic field B (where an anisotropy is introduced as in our plasma).
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Our findings suggest that the anisotropy by itself could instead be the cause of lower T_c in presence of anisotropies.

η/s for our theories

- Shear Viscosity over Entropy Density

$$\eta_{ij,kl} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dtdx e^{i\omega t}\langle T_{ij}(t,x),\, T_{kl}(0,0)\rangle$$
$$s = \frac{2\pi}{\kappa^2}A\ .$$

The two-point function is obtained by calculating the response to turning on suitable metric perturbations in the bulk.

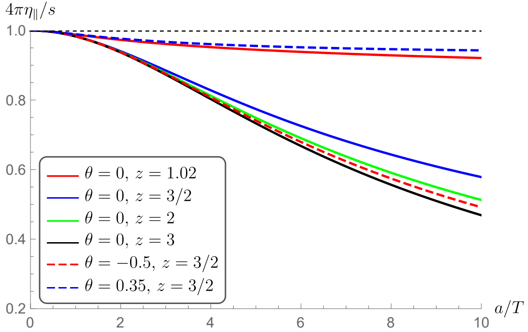
- The relevant part of the perturbed action is mapped to a Maxwell system with a mass term.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(-\frac{1}{4g_{eff}^2}F^2 - \frac{1}{4}m^2(u)A^2\right)\ ,$$

where

$$m^2(u) = Z(\phi + \frac{1}{4}\log g_{33})\alpha^2\ ,\quad \frac{1}{g_{eff}^2} = g_{33}^{3/2}(u)\ ,\quad A_\mu = \frac{\delta g_{\mu 3}}{g_{33}}$$

- The shear viscosity over entropy ratio for arbitrary (z, θ).

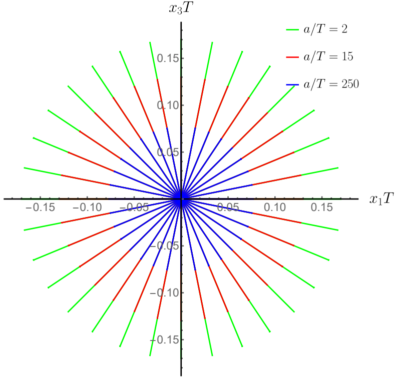


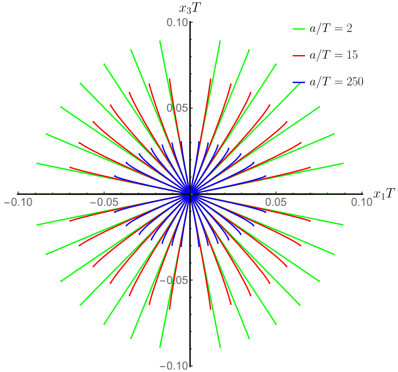
- The ratio depends on the temperature at $\alpha/T \gg 1$ as

$$4\pi\frac{\eta_{\|}}{s} = \frac{g_{11}}{g_{33}} \sim \left(\frac{T}{\tilde{\alpha}|1+3z-\theta|}\right)^{2-\frac{2}{z}} .$$

- The range of the temperature power is $[0, \infty)$.

- The quark distributions for baryons in theories with magnetic fields:





- Baryon on the transverse plane and Baryon on the plane that the field lies.
- System of fundamental strings with a vertex Dp-brane, in an anisotropic gravity theory.
- The baryon dissociates at stages, depending on the proximity angle to the magnetic field direction.

Conclusions

- ✓ We have obtained and studied a)Confining Anisotropic theories. b) Hyperscaling Lifshitz-like Anisotropic black holes with arbitrary scalings. (1st construction in the literature)
- ✓ The theories are physical and stable for a wide range of parameters of the theory.
- ✓ The Confinement/Deconfinement phase transitions occur at lower critical Temperature as the anisotropy is increased!
- ✓ The anisotropy by itself could instead be the cause of the inverse magnetic catalysis.
- The shear viscosity over entropy density ratio, takes values parametrically lower than $1/4\pi$, and depends on the Temperature as $(T/\alpha)^{2-2/z}$.
- The diffusion (butterfly velocity) of chaos occurs faster than isotropic systems.
- Baryons dissociate at stages in theories with strong fields. (D.G. 2018)

Thank you!

Reminding Slide:

- The anisotropic hyperscaling violation metric

$$ds^2 = r^{-\frac{2\theta}{dz}}\left(-r^2\left(dt^2 + dy_i^2\right) + r^{\frac{2}{z}}dx_i^2 + \frac{dr^2}{r^2}\right) .$$

  which exhibits a critical exponent z and a hyperscaling violation exponent θ.

- The metric is not scale invariant

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad r \to \frac{r}{\lambda^z}, \qquad ds \to \lambda^{\frac{\theta}{d}} ds .$$

Special case: IIB Supergravity

Remark:

The ten dimensional action gives our generalized model, when the internal space is an S^5 supported by fluxes and $\sigma = 0, \gamma = 1, \Delta = 4$:

$$S = \frac{1}{2\kappa_{10^2}} \int d^{10}x \sqrt{-g} \left[R + 4\partial_M\phi\partial^M\phi - e^{2\phi}\left(\frac{1}{2}F_1^2 + \frac{1}{4\cdot 5!}F_5^2 \right) \right], \; F_1 := d\chi \,.$$

where $M = 0, \ldots, 9$ and F_1 is the axion field-strength. The equations of motion for the background are:

$$R + 4g^{MN}\left(\nabla_M\nabla_N\phi - \partial_M\phi\partial_N\phi\right) = 0\,,$$

$$R_{MN} + 2\nabla_M\nabla_N\phi + \frac{1}{4}g_{MN}e^{2\phi}\partial_P\chi\partial^P\chi - \frac{1}{2}e^{2\phi}\left(F_MF_N + \frac{1}{48}F_{MABCD}F_N{}^{ABCD} \right) = 0\,.$$

plus the Bianchi identities and self duality constraints. The axion field equation is satisfied trivially for linear axion.

Null Energy Condition

- The averaged radial acceleration between two null geodesics is

$$A_r = -4\pi T_{\mu\nu} N^\mu N^\nu \ ,$$

if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.

- This imposes the Null Energy Condition

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

leading to the following constrains:

- For the Lifshitz-like space $z \geq 1$.
- For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ .$$

Additional conditions from thermodynamics?

Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0\,, \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

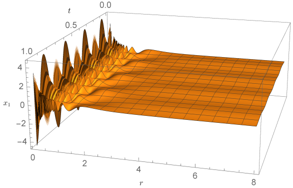
  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*

- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$

- In the linear response theory the response function $\chi(\omega)$ of the quark to the an external force, is proportional to the two point correlator of a string fluctuations divided by the applied force.



- The string fluctuations along x_1 close to the boundary are found by the monodromy patching method

$$\delta x_{1\omega}(r) = c_1\left(1 + i\omega c_0 g_{11}(r_h) + \frac{i\omega g_{11}(r_h)}{2\kappa\nu} r^{-2\kappa\nu}\right).$$

- The diffusion coefficient $D = T \lim_{\omega\to 0}\left(-i\ \omega\chi(\omega)\right)$

$$D \sim T^{2(1-\nu_i)}$$

where ν_i is determined by the asymptotics of the metric element along the fluctuation direction.

- Fluctuation-Dissipation theorem holds along each direction;The noise is white; Self energy an thermal mass of the particle depend on the properties and direction of the system... (D.G., Lee, Yeh 2018)