Strongly-coupled Anisotropic Gauge Theories And Holography

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Based on works with:
U. Gursoy(Utrecht Univ.) and J. Pedraza(Univ. of Amsterdam) and work in progress.

Talk given for: HEP 2018, NTUA, Athens, March 29, 2018
Outline

1. Introduction
2. The Theory
3. Stable and Physical Theories
4. Phase Transitions
5. Probing the Theory
6. Conclusions
Briefly on AdS/CFT

- Gauge/Gravity duality: A way to map and answer quantum questions to gravity geometric questions.
- The initial AdS/CFT correspondence: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow AdS_5 \times S^5$, is the harmonic oscillator of the gauge/gravity dualities.
- The theory is simple: Conformal, Maximally Supersymmetric, No Temperature...
- Since the discovery of the initial correspondence, there is an extensive research aiming to construct more realistic gauge/gravity dualities.

Gauge/Gravity Dualities with: Less/No Supersymmetry; Broken conformal symmetry, confinement; fundamental matter(probe and backreacting D$q$ branes); etc.

✓ We study Anisotropic theories in Gauge/Gravity correspondence.
Why? Attempts for Realizations in Nature

The existence of strongly coupled anisotropic systems.

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- New interesting phenomena in presence on such fiels, i.e. inverse magnetic catalysis.
  
  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)
- Anisotropic low dimensional materials in condensed matter.
Why? More:

- **Weakly coupled vs strongly coupled** anisotropic theories.
  
  \[(Dumitru, Strickland, Romatschke, Baier, ... 2008, ...)]

- Consistent top-down models. **Properties of the supergravity solutions**, that are dual to the anisotropic theories.

- Black hole solutions that are **AdS in UV flowing** to Lifhitz-like in IR:
  
  ★ Why there is a **fixed scaling parameter** $z$ for such solutions?
  
  \[(Azeyanagi, Li, Takayanagi, 2009)]

  ★ Other systems that have fixed scaling IR solution (e.g. in Heavy quark density). Why?
  
  \[(Kumar 2012; Faedo, Kundu, Mateos, Tarrio 2014)]

  ★ New flows to **Hyperscaling violation** IR backgrounds?
Why? Even More:

- **Striking Features!** Several **Universality Relations** predicted for the isotropic theories are **violated**!
  - Shear viscosity over entropy density ratio takes **parametrically low values** $\frac{\eta}{s} < \frac{1}{4\pi}$!
  (Rebhan, Steineder 2011; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017;...)
  - **Langevin coefficients** inequality for heavy quark motion in the anisotropic plasma gets inverted $\kappa_L > \kappa_T$.
  (Gursoy, Kiritsis, Mazzanti, Nitti 2010; D.G, Soltanpanahi, 2013a, 2013b)
  - ...
  - **Implications to QGP hydrodynamic simulations.**
Strongly coupled anisotropic theory.

How the theory looks like and how to obtain it?

- 4d $SU(N)$ gauge theory in the large $N_c$-limit.
- Its dynamics are affected by a scalar operator $\mathcal{O} \sim TrF^2$.
- Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.
- On the gravity dual side we have a "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.

Eventually the gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.

- Solutions are RG flows:
  AdS in UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

Formally interesting theories. And a solid ground to study strongly coupled phenomena in presence of anisotropy.
Parts of the Theory Timeline-Related bibliography:

**Non-Confining Anisotropic Theories:**
(Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011; Jain, Kundu, Sen, Sinha, Trivedi, 2015;...)

**Confining Anisotropic Theories:**
(D.G., Gursoy, Pedraza, 2017)

**Similar ideas in different context. For example:**
(Gaiotto, Witten 2008; Chu, Ho, 2006; Choi, Fernandez, Sugimoto 2017;...)
How is Anisotropy introduced? A Pictorial Representation:

- For the Lifshitz-like IIB Supergravity solutions
  \[ ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds^2_{S^5}. \]

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

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- Which equivalently leads to the following AdS/CFT deformation.

\[ N = 4 \text{ sYM} \rightarrow \text{AdS}_5 \times S^5, \varphi, F_5 \]

\[ \tau = \frac{\vartheta}{4\pi} + \frac{4\pi i}{\theta_{\text{YM}}} = \chi + ie^{-\phi} \]

\[ S = S_{N=4} + \delta S \]
\[ \delta S = \int \theta(x_3) \text{Tr} F \wedge F \]

\[ \text{AdS}_5 \times S^5, \varphi, \chi(x_3), F_5 \]

- \( dC_8 \sim \ast d\chi \) with the non-zero component \( C_{x_0 x_1 x_2 S^5} \).
- Possible to compactify \( x_3 \) to get dual to the pure Chern-Simons gauge theory.

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The Anisotropic Theory

The generalized **Einstein-Axion-Dilaton action** with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

\[
S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left[ R - \frac{1}{2} (\partial\phi)^2 + V(\phi) - \frac{1}{2} Z(\phi)(\partial\chi)^2 \right].
\]

The eoms read

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2} \partial_{(\mu} \phi \partial_{\nu)} \phi + \frac{1}{2} Z(\phi) \partial_{(\mu} \chi \partial_{\nu)} \chi - \frac{1}{4} g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4} g_{\mu\nu} Z(\partial\chi)^2 + \frac{1}{2} g_{\mu\nu} V(\phi),
\]

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi) = \frac{1}{2} \partial_{\phi} Z(\phi)(\partial\chi)^2 - V'(\phi),
\]

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} \chi) = 0.
\]

Where

\[
V(\phi) = 12 \cosh(\sigma\phi) + \left(\frac{m(\Delta)^2}{2} - 6\sigma^2\right) \phi^2,
\]

\[
Z(\phi) = e^{2\gamma\phi},
\]

*(Gursoy, Kiritsis, Nitti, 2007; Gubser, Nellore, Pufu, Rocha 2008a,b)*

**Remark:** For \(\sigma = 0, \gamma = 1, m(\Delta) = 0\) the action and the solution of eoms, are reduced of IIB supergravity.
Solutions of the Generalized Einstein-Axion-Dilaton Action

- The **background solution**

\[ ds^2 = \frac{1}{u^2} \left( -\mathcal{F}(u)B(u)\, dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)} \right), \]

\[ \chi = \alpha x_3, \quad \phi = \phi(u), \]

- **Solutions**: e.g. For \( \Delta = 4 \):
  - AdS in UV flowing to Hyperscaling Lifshitz-like violation geometries in IR:

\[ ds^2 = u^{-\frac{2\Delta}{3}} \left( -u^2z (f(u)dt^2 + dx_{1,2}^2) + \tilde{\alpha}u^2dx_3^2 + \frac{du^2}{f(u)u^2} \right), \]
The values of \((\theta, z)\) dependence on \((\gamma, \sigma)\)

\[
z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)}, \quad \theta = \frac{3\sigma}{2\gamma}.
\]

**Special case:** \((\sigma = 0, \gamma = 1)\) supergravity truncated action with a single solution \((\theta = 0, z = 3/2)\).

The scaling factors \(z\) and \(\theta\) are determined by the constants in the Axion-Dilaton Coupling and the Potential. This is the reason that in the particular setup the supergravity solutions have them fixed.
Fixing \((\gamma, \sigma)\) and \(\alpha\) and \(u_h\) we get the metric flow from boundary to horizon:

\[
ds^2 = \frac{e^{-\frac{1}{2} \phi(u)}}{u^2} \left(-FB \, dt^2 + dx_1^2 + dx_2^2 + \mathcal{H} \, dx_3^2 + \frac{du^2}{\mathcal{F}}\right),\]

![Graph showing the metric flow from boundary to horizon.](image)
An exact solution

The potential and the axion-dilaton coupling

\[ V(\phi) = 6e^{\sigma \phi}, \quad Z(\phi) = e^{2\gamma \phi}. \]

A Lifshitz-like anisotropic hyperscaling violation background which may accommodate a black hole

\[ ds_s^2 = \alpha^2 C_R e^{\frac{\phi(u)}{2}} u^{-\frac{2\theta}{3z}} \left( -u^2 (f(u) dt^2 + dx_i^2) + C_Z u^2 dx_3^2 + \frac{du^2}{f(u)\alpha^2 u^2} \right), \]

where

\[ f(u) = 1 - \left( \frac{u_h}{u} \right)^{3+(1-\theta)/z}, \quad e^{\frac{\phi(u)}{2}} = u^{\frac{\sqrt{\theta^2+3z(1-\theta)-3}}{\sqrt{6z}}}, \]

\[ C_R = \frac{(3z - \theta)(1 + 3z - \theta)}{6z^2}, \quad C_Z = \frac{z^2}{2(z - 1)1 + 3z - \theta}, \]

\[ z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)}, \quad \theta = \frac{3\sigma}{2\gamma}. \]
We have obtained the theories, are they physical and stable?

⇓

⇓

✓ Energy Conditions Analysis

\[ T_{\mu\nu} N^\mu N^\nu \geq 0 \quad , \quad N^\mu N_\mu = 0 \]

✓ Local Thermodynamical Stability Analysis

\[ c_\alpha = T \left( \frac{\partial S}{\partial T} \right)_\alpha \geq 0 \quad , \quad \Phi' = \left( \frac{\partial \Phi}{\partial \alpha} \right)_T \geq 0 \]

⇓

⇓

YES!
Three conditions that constrain \((z, \theta)\) and as a result \((\gamma, \sigma)\).

\[
(z - 1)(1 - \theta + 3z) \geq 0, \quad z = \frac{2 + 4\gamma^2 - 3\sigma^2}{2\gamma(2\gamma - 3\sigma)},
\]

\[
\theta^2 - 3 + 3z(1 - \theta) \geq 0, \quad \theta = \frac{3\sigma}{2\gamma},
\]

\[
1 - \theta + 2z \geq 0.
\]

The blue region is the acceptable for the theory parameters.
Confinement/Deconfinement Phase transitions

- The free energy of the theories vs the temperature $T$ for different anisotropy ($\alpha/j=0, 1, 3$):

- Horizontal Axis: Confining Phase.
- Upper Branch: Black hole A: Deconfining Plasma Phase.
- Lower Branch: Black hole B: Deconfining Plasma Phase.
- $\alpha/j \simeq 2$: A critical value above which a richer structure in the phase diagram exist.
The Critical Temperature of the theories vs the anisotropy gives:

- The $T_c$ is reduced in presence of anisotropies of the theory.
The Proposal

- The $T_c(\alpha)$ decrease with $\alpha$, resembling the phenomenon of inverse magnetic catalysis where the confinement-deconfinement temperature decreases with the magnetic field $B$ (where an anisotropy is introduced as in our plasma).

- No charged fermionic degrees of freedom in our case; our plasma is neutral.

- Our findings suggest that the anisotropy by itself could instead be the cause of lower $T_c$ in presence of anisotropies.
\( \frac{\eta}{s} \) for our theories

- **Shear Viscosity over Entropy Density**

\[
\eta_{ij,kl} = -\lim_{\omega \to 0} \frac{1}{\omega} \text{Im} \int dt dx e^{i\omega t} \langle T_{ij}(t, x), T_{kl}(0, 0) \rangle
\]

\[
s = \frac{2\pi}{\kappa^2} A.
\]

The two-point function is obtained by calculating the response to turning on suitable metric perturbations in the bulk.

- The relevant part of the perturbed action is mapped to a Maxwell system with a mass term.

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left( -\frac{1}{4g_{\text{eff}}^2} F^2 - \frac{1}{4} m^2(u) A^2 \right),
\]

where

\[
m^2(u) = Z(\phi + \frac{1}{4} \log g_{33}) \alpha^2, \quad \frac{1}{g_{\text{eff}}^2} = g_{33}^{3/2}(u), \quad A_\mu = \frac{\delta g_{\mu 3}}{g_{33}}
\]
The shear viscosity over entropy ratio for arbitrary \((z, \theta)\).

The ratio depends on the temperature at \(\alpha / T \gg 1\) as

\[
4\pi \frac{\eta_{||}}{s} = \frac{g_{11}}{g_{33}} \sim \left( \frac{T}{\tilde{\alpha} |1 + 3z - \theta|} \right)^{2 - \frac{2}{z}}.
\]

The range of the temperature power is \([0, \infty)\).
The quark distributions for baryons in theories with magnetic fields:

- Baryon on the transverse plane and Baryon on the plane that the field lies.
- System of fundamental strings with a vertex Dp-brane, in an anisotropic gravity theory.
- The baryon dissociates at stages, depending on the proximity angle to the magnetic field direction.
Conclusions

✓ We have obtained and studied a) **Confining Anisotropic** theories. b) **Hyperscaling Lifshitz-like Anisotropic black holes with arbitrary scalings.** (1st construction in the literature)

✓ The theories are **physical and stable** for a wide range of parameters of the theory.

✓ The **Confinement/Deconfinement** phase transitions occur at **lower critical Temperature** as the anisotropy is increased!

✓ The **anisotropy by itself** could instead be the cause of the **inverse magnetic catalysis**.

- The **shear viscosity over entropy density ratio**, takes values parametrically **lower** than $1/4\pi$, and depends on the **Temperature** as $(T/\alpha)^{2-2/z}$.

- The **diffusion (butterfly velocity) of chaos** occurs **faster** than isotropic systems.

- **Baryons** dissociate **at stages** in theories with strong fields. (D.G. 2018)
Thank you!
The anisotropic hyperscaling violation metric

\[ ds^2 = r^{-\frac{2\theta}{dz}} \left( -r^2 (dt^2 + dy_i^2) + r^2 \frac{2}{dz} dx_i^2 + \frac{dr^2}{r^2} \right). \]

which exhibits a critical exponent \( z \) and a hyperscaling violation exponent \( \theta \).

The metric is not scale invariant

\[ t \to \lambda^z t, \quad y \to \lambda^z y, \quad x \to \lambda x, \quad r \to \frac{r}{\lambda^z}, \quad ds \to \lambda^{\frac{\theta}{d}} ds. \]
Remark:
The ten dimensional action gives our generalized model, when the internal space is an $S^5$ supported by fluxes and $\sigma = 0, \gamma = 1, \Delta = 4$:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R + 4\partial_\phi \partial^p \phi - e^{2\phi} \left( \frac{1}{2} F_1^2 + \frac{1}{4 \cdot 5!} F_5^2 \right) \right], \ F_1 := d\chi .$$

where $M = 0, \ldots, 9$ and $F_1$ is the axion field-strength. The equations of motion for the background are:

$$R + 4g^{\mu\nu} \left( \nabla_\mu \nabla_\nu \phi - \partial_\mu \phi \partial_\nu \phi \right) = 0 ,$$

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi + \frac{1}{4} g_{\mu\nu} e^{2\phi} \partial_p \chi \partial^p \chi - \frac{1}{2} e^{2\phi} \left( F_M F_N + \frac{1}{48} F_{MABCD} F_N^{ABCD} \right) = 0 .$$

plus the Bianchi identities and self duality constraints. The axion field equation is satisfied trivially for linear axion.
Null Energy Condition

- The averaged radial acceleration between two null geodesics is
  \[ A_r = -4\pi T_{\mu\nu} N^\mu N^\nu , \]
  if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.
- This imposes the Null Energy Condition
  \[ T_{\mu\nu} N^\mu N^\nu \geq 0 , \quad N^\mu N_\mu = 0 , \]
  leading to the following constrains:
  - For the Lifshitz-like space \( z \geq 1 \).
  - For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads
    \[
    (z - 1)(1 - \theta + 3z) \geq 0 , \\
    \theta^2 - 3 + 3z(1 - \theta) \geq 0 .
    \]

Additional conditions from thermodynamics?
Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

\[ c_\alpha = T \left( \frac{\partial S}{\partial T} \right)_\alpha \geq 0, \quad \Phi' = \left( \frac{\partial \Phi}{\partial \alpha} \right)_T \geq 0 \]

- \( c_\alpha \) is the specific heat: increase of the temperature leads to increase of the entropy.
- \( \Phi' \) is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. \((\text{Gubser, Mitra 2001})\)

- In the IR the positivity of the specific heat imposes

\[ c_\alpha = 1 - \theta + 2z \geq 0 \]
In the linear response theory the response function $\chi(\omega)$ of the quark to the an external force, is proportional to the two point correlator of a string fluctuations divided by the applied force.

The string fluctuations along $x_1$ close to the boundary are found by the monodromy patching method

$$\delta x_{1\omega}(r) = c_1 \left( 1 + i \omega c_0 g_{11}(r_h) + \frac{i \omega g_{11}(r_h)}{2 \kappa \nu} r^{-2\kappa \nu} \right).$$

The diffusion coefficient $D = T \lim_{\omega \to 0} (-i \omega \chi(\omega))$

$$D \sim T^{2(1-\nu_i)}$$

where $\nu_i$ is determined by the asymptotics of the metric element along the fluctuation direction.

Fluctuation-Dissipation theorem holds along each direction; The noise is white; Self energy an thermal mass of the particle depend on the properties and direction of the system...