Newton-Cartan (Super)Gravity and Torsion

Dedicated to the memory of Ioannis Bakas

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Joint work with E. A. Bergshoeff, J. Lahnsteiner, L. Romano, J. Rosseel

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Why non-relativistic gravity?

- Unlike Einstein gravity, Newtonian gravity is not a geometric theory
  - non-relativistic gravity in an arbitrary frame?
  - Newton-Cartan: geometric formulation of Newtonian gravity; arbitrary frames
- New physical applications, mainly in condensed matter physics
  - Construction of Effective Field Theories, e.g. for the FQHE, chiral superfluids &c.
    Hoyos, Son ’12; Son ’13; Geracie, Son, Wu, Wu ’15; Hoyos, Moroz, Son ’14; Moroz, Hoyos ’15
  - Universal properties, transport phenomena

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Theory of Thermal Transport Coefficients*

J. M. LUTTINGER
Department of Physics, Columbia University, New York, New York
(Received 20 April 1964)

A simple proof of the usual correlation-function expressions for the thermal transport coefficients in a resistive medium is given. This proof only requires the assumption that the phenomenological equations in the usual form exist. It is a "mechanical" derivation in the same sense that Kubo's derivation of the expression for the electrical conductivity is. That is, a purely Hamiltonian formalism with external fields is used, and one never has to make any statements about the nature or existence of a local equilibrium distribution function, or how fluctuations regress. For completeness the analogous formulas for the viscosity coefficients and the heat conductivity of a simple fluid are given.

III. CALCULATION OF THE THERMAL TRANSPORT COEFFICIENTS

Just as the space- and time-varying external electric potential produced electric currents and density variations, so a varying gravitational field will produce, in principle,[7] energy flows and temperature fluctuations.

large one. In fact, if the gravitational field didn't exist, one could invent one for the purposes of this paper.

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    Hoyos, Son ’12; Son ’13; Geracie, Son, Wu, Wu ’15; Hoyos, Moroz, Son ’14; Moroz, Hoyos ’15
  - Universal properties, transport phenomena
- Newton-Cartan geometry is to Newton what Riemann geometry is to Einstein
  - Coupling of non-relativistic field theories to gravity
  - Distinguish geometry (background fields, symmetries) vs. gravity (EOMs)

Why torsion?

- Holography (e.g. Lifshitz) $\rightsquigarrow$ zero torsion is not allowed in CFT
  Christensen, Hartong, Obers, Rollier '14; Hartong, Kiritsis, Obers '14-'15

- To define a non-relativistic energy-momentum tensor, torsion is a key ingredient
  Luttinger '64; Gromov, Abanov '14

- Numerous applications of torsional non-relativistic gravity in condensed matter
  Gromov, Abanov '14; Geracie, Golkar, Roberts '14; Geracie, Prabhu, Roberts '16; & c.

Why supersymmetry?

- Non-relativistic supersymmetric field theories on non-trivial backgrounds?
- Localization $\rightsquigarrow$ exact partition functions for non-relativistic field theories?
- But also, what is the supersymmetrization of Newtonian gravity?

To answer such questions, first one has to construct non-relativistic supergravities

see e.g. Andringa, Bergshoeff, Rosseel, Sezgin '13; Bergshoeff, Rosseel, Zojer '15; Bergshoeff, Rosseel '16

Methods, Goals and Overview

- Construct Newton-Cartan Geometry and Gravity with arbitrary torsion
  - Gauging an algebra
  - Null-reduction of GR/conformal gravity
  - Conformal method
  - ✓ NC geometry with arbitrary torsion in any dimension
  - ✓ NC gravity with arbitrary torsion in 3D
    Bergshoeff, A.Ch., Romano, Rosseel '17

- Construct off-shell $\mathcal{N} = 2$ Newton-Cartan supergravity with arbitrary torsion
  - Null-reduction of (old-minimal) $\mathcal{N} = 1$ supergravity
  - Supersymmetric curved backgrounds? (only hints in this talk...)
  - ✓ Off-shell non-relativistic supergravity with arbitrary torsion in 3D
    Bergshoeff, A.Ch., Lahnsteiner, Romano, Rosseel; to appear

Newton-Cartan from gauging

Einstein gravity $\sim$ Gauging of the Poincaré algebra

Newton-Cartan gravity $\sim$ Gauging of the Bargmann algebra

Andringa, Bergshoeff, Panda, de Roo '10

Commutation relations

$$[J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]}\,, \qquad [J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,,$$

$$[J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,, \qquad [G_a, H] = -P_a\,, \qquad [G_a, P_b] = -\delta_{ab}M\,.$$

Transformation	Generator	Gauge Field	Gauge Parameter
Time translations	H	τ_μ	ζ
Space translations	P_a	${e_\mu}^a$	ζ^a
Spatial rotations	J_{ab}	${\omega_\mu}^{ab}$	λ^{ab}
Galilean boosts	G_a	${\omega_\mu}^a$	λ^a
Central charge	M	m_μ	σ

Transformation rules and torsion

The independent fields are $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ and they transform as 1-forms and as

$$\begin{aligned} \delta\tau_\mu &= 0 \,, \\ \delta e_\mu{}^a &= \lambda^a{}_b e_\mu{}^b + \lambda^a \tau_\mu \,, \\ \delta m_\mu &= \partial_\mu \sigma + \lambda^a e_{\mu a} \,. \end{aligned}$$

Inverse vielbeins may be defined using a set of projective invertibility relations

$$e^\mu{}_a e_\nu{}^a = \delta^\mu_\nu - \tau^\mu \tau_\nu \,, \quad e^\mu{}_a e_\mu{}^b = \delta^a_b \,, \quad \tau^\mu \tau_\mu = 1 \,, \quad e^\mu{}_a \tau_\mu = 0 \,, \quad \tau^\mu e_\mu{}^a = 0 \,.$$

For each set of generators, there is an associated curvature. In particular

$$R_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} \quad \rightsquigarrow \quad \text{the torsion}$$

$$\tau_{ab} = e^\mu{}_a e^\nu{}_b \partial_{[\mu}\tau_{\nu]}$$

$$\tau_{0a} = \tau^\mu e^\nu{}_a \partial_{[\mu}\tau_{\nu]}$$

geometric constraint	Newton-Cartan
$\tau_{0a} \neq 0 \,, \tau_{ab} \neq 0$	arbitrary torsion
$\tau_{0a} \neq 0 \,, \tau_{ab} = 0$	twistless-torsional
$\tau_{0a} = 0 \,, \tau_{ab} = 0$	zero torsion

Newton-Cartan equations of motion

- $R_{\mu\nu}(P^a)$ and $R_{\mu\nu}(M)$ are used to find expressions for $\omega_\mu{}^{ab}(\tau, e, m)$ & $\omega_\mu{}^a(\tau, e, m)$
- The remaining two curvatures are related to the equations of motion for NC
  - For zero torsion

$$\tau^\mu e^\nu{}_a R_{\mu\nu}(G^a) := R_{0a}(G^a) = 0 \quad \rightsquigarrow \quad \nabla^2\Phi = 0$$

$$R_{c0}{}^c{}_b(J) = 0\,, \quad R_{ca}{}^c{}_b(J) = 0$$

  - For arbitrary torsion, it is much harder to establish such equations... see later

Null-reduction of general relativity

Duval, Burdet, Kunzle, Perrin '85; Julia, Nicolai '95

Start with GR in $d+1$ dimensions, in the second-order formalism $\{E_M{}^A, \Omega_M{}^{AB}(E)\}$.

The spin connection and the curvature are given by the standard expressions

$$\begin{aligned} \Omega_M{}^{BA}(E) &= 2E^{N[A}\partial_{[M}E_{N]}{}^{B]} - E^{N[A}E^{B]P}E_{MC}\partial_N E_P{}^C \,, \\ \hat{R}_{MN}{}^{AB}(\Omega(E)) &= 2\partial_{[M}\Omega_{N]}{}^{AB} - 2\Omega_{[M}{}^{AC}\Omega_{N]C}{}^B \,. \end{aligned}$$

In addition, the vielbein transforms under g.c.t.s and local Lorentz transformations as:

$$\delta E_M{}^A = \zeta^N\partial_N E_M{}^A + \partial_M\zeta^N E_N{}^A + \lambda^A{}_B E_M{}^B \,.$$

Assume that we have a null Killing vector $\xi = \xi^M\partial_M$ for the metric $g_{MN} \equiv E_M{}^A E_N{}^B \eta_{AB}$:

$$\mathcal{L}_\xi g_{MN} = 0 \quad \text{and} \quad \xi^2 = 0 \,.$$

We choose adapted coordinates and split indices as $M = \{\mu, \upsilon\}$ and $A = \{a, +, -\}$

Notably, this implies that the metric is degenerate (the Killing vector is now $\xi = \xi^\upsilon\partial_\upsilon$):

$$g_{\upsilon\upsilon} = 0$$

Reduction Ansatz

The suitable Ansatz for the Vielbein and its inverse is cf. Julia, Nicolai '95

$$E_M{}^A = \begin{array}{c} \\ \mu \\ \upsilon \end{array} \begin{array}{c} \begin{array}{ccc} a & - & + \end{array} \\ \begin{pmatrix} e_\mu{}^a & \tau_\mu & -m_\mu \\ 0 & 0 & 1 \end{pmatrix} \end{array}, \qquad E^M{}_A = \begin{array}{c} \\ a \\ - \\ + \end{array} \begin{array}{c} \begin{array}{cc} \mu & \upsilon \end{array} \\ \begin{pmatrix} e^\mu{}_a & e^\mu{}_a m_\mu \\ \tau^\mu & \tau^\mu m_\mu \\ 0 & 1 \end{pmatrix} \end{array}.$$

Off-shell reduction yields the NC transformations for $\{e_\mu{}^a, \tau_\mu, m_\mu\}$ along with

$$\Omega_\mu{}^{ab}(E) \equiv \omega_\mu{}^{ab}(\tau, e, m) = \mathring{\omega}_\mu{}^{ab}(e, \tau, m) - m_\mu \tau^{ab}\,,$$

$$\Omega_\mu{}^{a+}(E) \equiv \omega_\mu{}^{a}(\tau, e, m) = \mathring{\omega}_\mu{}^{a}(e, \tau, m) + m_\mu \tau_0{}^{a}\,.$$

On-shell, however, one obtains the NC EOMs and $\tau_{0a} = \tau_{ab} = 0$.

Null-reduction of GR: NC geometry with arbitrary torsion, but NC gravity without torsion

Null-reduction of conformal gravity

Bergshoeff, A.Ch., Romano, Rosseel '17

Motivated by the conformal construction of Poincaré gravity

see the book of Freedman and van Proeyen for all details

Poincaré = Conformal + Scalar + gauge-fixing

The full set of gauge fields is $\{E_M{}^A, \Omega_M{}^{AB}, b_M, f_M{}^A\}$; the independent ones transform as

$$\begin{aligned} \delta E_M{}^A &= \lambda^A{}_B E_M{}^B + \lambda_D E_M{}^A \,, \\ \delta b_M &= \partial_M \lambda_D + \lambda_K^A E_{MA} \,. \end{aligned}$$

Null-reduction $\rightsquigarrow z = 2$ Schrödinger gravity with arbitrary torsion; anisotropic scaling:

$$\begin{aligned} \delta_D \tau_\mu &= 2\lambda_D \tau_\mu \,, \\ \delta_D e_\mu{}^a &= \lambda_D e_\mu{}^a \,, \end{aligned}$$

Equivalent to gauging the Schrödinger algebra (conformal extension of Bargmann)

Torsion revisited

Originally (d+1) λ_K^As. Gauge-fixing $b_\upsilon = 0$ fixes λ_K^-. Fixing (d-1) more, requires

$$\mathcal{R}_{0a}(H) := \tau^\mu e^\nu{}_a \left(2\tau_{\mu\nu} - 4b_{[\mu}\tau_{\nu]}\right) \stackrel{!}{=} 0 \;\Rightarrow\; b_a = -\tau_{0a} \,.$$

This is a conventional constraint. Zero torsion is not compatible with dilatations...

geometric constraint	Schrödinger
$\tau_{0a} \neq 0\,, \tau_{ab} \neq 0$	arbitrary torsion
$\tau_{0a} \neq 0\,, \tau_{ab} = 0$	twistless-torsional
$\tau_{0a} = 0\,, \tau_{ab} = 0$	N/A

Newton-Cartan Gravity with Arbitrary Torsion?

Bergshoeff, A.Ch., Romano, Rosseel '17; cf. Afshar et al. '16 for twistless-torsion

Use a non-relativistic version of the conformal construction of relativistic gravity

- Scalar CFT with $\mathcal{L} = \frac{1}{2}\phi\partial^\mu\partial_\mu\phi$
- Coupling to conformal gravity $\partial \to D_{\text{conf}}$
- Gauge-fix $\phi = 1 \quad \rightsquigarrow \quad \mathcal{L} = R$

In our (arbitrary torsion) case

- SFT for two real scalars (one for dilatations, one for central charge trafos):

$$\partial_0\partial_0\varphi - \frac{2}{\mathrm{M}}(\partial_0\partial_a\varphi)\partial_a\chi + \frac{1}{\mathrm{M}^2}(\partial_a\partial_b\varphi)\partial_a\chi\partial_b\chi = 0\,.$$

- Coupling to Schrödinger gravity $\partial \to D_{\text{Schr}}$
- Gauge-fixing $\varphi = 1$ and $\chi = 0$, and restricting to $d = 3$, leads to a set of EOMs:

$$\mathcal{R}'_{0a}(G^a) + \text{terms}(\tau_{0a}, \tau_{ab}) = 0\,,$$
$$\mathcal{R}_{0b}{}^{ba}(\Omega) = 0\,, \quad \mathcal{R}_{ac}{}^{cb}(\Omega) = 0\,, \qquad \Omega = \mathring{\omega} + \text{terms}(\tau_{ab})\,.$$

Non-relativistic Supergravity

Apply the technique of null-reduction to off-shell (old minimal) $\mathcal{N} = 1, D = 4$ sugra

Bergshoeff, A.Ch., Lahnsteiner, Romano, Rosseel; to appear

The off-shell multiplet comprises the fields $\{E_M{}^A, \Psi_M, A_M, F\}$

For the bosons, we use the same null-reduction Ansatz as before. For the fermions:

$$\begin{aligned} \Psi_\mu &= (\psi_{\mu-} - m_\mu \psi_{v-}) \otimes \chi_+ + (\psi_{\mu+} - m_\mu \psi_{v+}) \otimes \chi_- \,, \\ \Psi_v &= \psi_{v-} \otimes \chi_+ + \psi_{v+} \otimes \chi_- \,. \end{aligned}$$

Nullity of the Killing vector ξ leads to a closed set of constraints:

$$\xi^2 = 0 \quad \Rightarrow \quad E_v{}^- = 0 \quad \Rightarrow \quad \psi_{v+} = 0 \quad \Rightarrow \quad \epsilon^{ab}\tau_{ab} = 2\sqrt{2}\,\bar{\psi}_{v-}\psi_{v-} + 12A_v$$

The reduced, non-relativistic off-shell supergravity multiplet contains the fields

$$\{\tau_\mu, e_\mu{}^a, m_\mu, A_\mu, A_v, F, \psi_{\mu\pm}, \psi_{v-}\}$$

Taking into account the constraint, the supersymmetry algebra closes, as it should

Non-relativistic supersymmetric curved backgrounds?

Relativistic: starting from off-shell sugra, rigid susy theories on curved backgrounds

Pestun '07; Festuccia, Seiberg '11; & c.

- Classified by studying Killing spinor equations in curved space

Non-relativistic (to date): studied only for torsionless off-shell Newton-Cartan sugra

Knodel, Lisbao, Liu '16

- "Small" off-shell multiplet $\{\tau_\mu, e_\mu{}^a, m_\mu, \psi_{\mu\pm}, S\}$
- Killing spinor equations $\rightsquigarrow$ solutions exist, but on flat space

In order to ameliorate this, the full off-shell multiplet (allowing torsion) is necessary

Bergshoeff, A.Ch., Lahnsteiner, Romano, Rosseel; work in progress

Epilogue

- New developments in non-relativistic (super)gravity, in many different directions
- ✓ We constructed Newton-Cartan (super)gravity with arbitrary torsion in 3D
- Systematic methods, null-reduction and conformal construction
- Curved supersymmetric backgrounds might exist, from off-shell torsional NC sugra
- A host of potential applications
  - Non-relativistic holography
  - Localization
  - Condensed matter systems, EFTs
  - $D = 4$?