Hairy black holes and gravity wave constraints

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General Relativity

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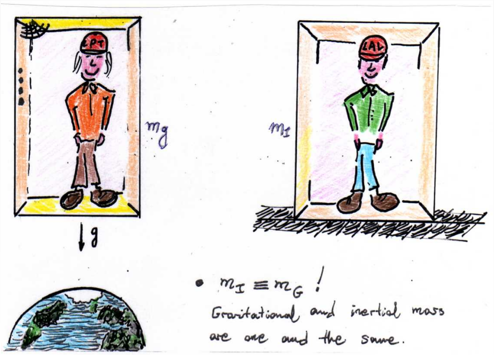

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- Mach's principle The presence of matter curves the geometry of spacetime
- Equivalence principle Locally a free-falling observer and an inertial observer are indistinguishable

- This means:
  -Gravity is a local condition of spacetime
  -Gravity sees all (including vacuum energy!)
  -In Newtonnian gravity m_I and m_G happen to be the same, in GR it is a founding principle

GR is a unique theory

- Theoretical consistency: In 4 dimensions, consider $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla\nabla g)$. Then Lovelock's theorem in $D = 4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4x \sqrt{-g^{(4)}}\,[R - 2\Lambda]$$

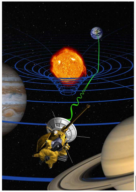
giving,

  - Equations of motion of 2^{nd}-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity!

Observational data

- Experimental consistency:
  -Excellent agreement with solar system tests and strong gravity tests on binary pulsars
  -Observational breakthrough GW170817: Non local, $40Mpc$ and strong gravity test from binary neutron stars. $c_T = 1 \pm 10^{-15}$

Time delay of light  Planetary tajectories 

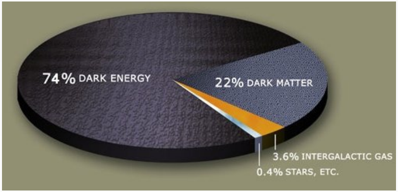
 Neutron star binary

Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the Universe today:



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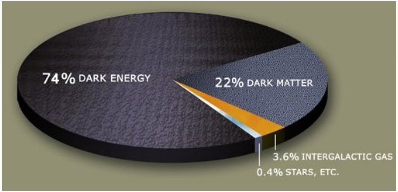

A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Universe is accelerating → Enter the cosmological constant

Simplest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10 \text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- Typical mass scale for neutrinos...
- Theoretically the cosmological constant should be huge.
- What if GR is modified at astrophysical or cosmological scales.

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Modified Gravity : General issues

- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They generically must not lead to higher derivative equations of motion. Additional degrees of freedom can lead to ghosts (Ostrogradski theorem 1850 [Woodard 2006]). Since [Gleyzes et al] we know that higher derivative EOM do not always lead to ghosts. What is essential is the number of propagating dof.
- Matter does not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit.
- Exact solutions essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm, absence of Birkhoff theorem etc.
- A modified gravity theory could tell us something about the cosmological constant problem and in particular how to screen an a priori enormous cosmological constant. Self tuning and self acceleration.

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Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973 and extended to DHOST theories [Langlois et.al] [Crisostomi et.al.]
- contain or are limits of other modified gravity theories.
- Include terms that can screen classically a big cosmological constant or give self accelerating solutions. Need a non trivial scalar field.
- Have non trivial hairy black hole solutions even around non trivial self accelerating vacua
- New: Theories are strongly constrained from gravity waves.

Jordan-Brans-Dicke theory [Sotiriou 2014]

Simplest scalar tensor theory

$$S_{\rm BD} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\varphi R - \frac{\omega_0}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - m^2(\varphi-\varphi_0)^2\right) + S_m(g_{\mu\nu},\psi)$$

- ω_0 Brans Dicke coupling parameter fixing scalar strength
- $\varphi = \varphi_0$ constant gives GR black hole solutions (with a cosmological constant) but spherically symmetric solutions are not unique (and not GR)!
- For spherical symmetry we find,

$$\gamma \equiv \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{2\omega_0 + 3 - exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}{2\omega_0 + 3 + exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}$$

- where $\gamma = 1 + (2.1 \pm 2.3)\times 10^{-5}$ from local tests.
- $\omega_0 > 40000$ in the absence of potential.
- Need a more complex version in order to screen the scalar mode locally and to obtain hairy black holes.$\rightarrow$ Higher order dervative theories.

Jordan versus Einstein frame

Jordan frame is the physical frame. Matter couples only to metric and the weak equivalence principle is satisfied.

$$S_{Jordan} = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\varphi\tilde{R} - \frac{\omega(\varphi)}{\varphi}\tilde{\nabla}^\mu\varphi\tilde{\nabla}_\mu\varphi - V(\varphi)\right) + S_m(\tilde{g}_{\mu\nu},\psi)$$

Consider a conformal transformation: $\tilde{g}_{ab} = \Omega^2(x)\, g_{ab}$:

$$\int d^4x\sqrt{-g}\,R = \int d^4x\sqrt{-\tilde{g}}\,(\tilde{R}\,\Omega^2 + 6\tilde{\nabla}_a\Omega\tilde{\nabla}^a\Omega), \qquad \Phi = \Phi(\varphi;\Omega)$$

and the action transforms into

$$S_{Einstein} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - \nabla^\mu\Phi\nabla_\mu\Phi - U(\Phi)) + S_m(g_{\mu\nu},\Phi,\psi)$$

- The action is GR like, but,
- Matter couples to metric and scalar!
- Non physical frame or Einstein frame. Matter in free-fall does not follow $g_{\mu\nu}$ geodesics!
- Frames are equivalent mathematically and physically as long as we know how matter couples to the metric.

Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= G_2(\phi, X), \\
L_3 &= G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]. Galileons are scalars with Galilean symmetry for flat spacetime.

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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
  $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
  Action is unique modulo integration by parts.

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- Horndeski theory admits self accelerating vacua with a non trivial scalar field in de Sitter spacetime. A subset of Horndeski theory self tunes the cosmological constant.

Galileons/Horndeski [Horndeski 1973]

$$S_H = \int d^4x\sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= K(X),\\
L_3 &= -G_3(X)\Box\phi,\\
L_4 &= G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],\\
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the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- Horndeski theory includes Shift symmetric theories where G_i's depend only on X and $\phi \to \phi + c$.
  Associated with the symmetry there is a Noether current, J^μ which is conserved $\nabla_\mu J^\mu = 0$.
  Presence of this symmetry permits a very general no hair argument

Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Warning : beyond GR Birkhoff's theorem is not valid.
Spherical symmetry thus does not guarantee staticity.

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Scalar-tensor theories and black holes

- In scalar tensor theories "regular" black hole solutions are GR black holes with a constant scalar field
- Is it possible to have non-trivial and regular scalar-tensor black holes for an asymptotically flat or $\Lambda > 0$ space-time?
- We will consider:
    - Higher order scalar tensor theory
    - Translation symmetry for the scalar field
    - A scalar field that does not have the same symmetries as the spacetime metric

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General solution for a $G_4 = 1 + \beta X$ [Babichev, Charmousis '13]

Consider, $L = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda$ For static and spherically symmetric spacetime, $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$,

The general solution of theory L for static and spherically symmetric metric and $\phi = \phi(t,r)$ is given as a solution to the following third order algebraic equation with respect to $\sqrt{k(r)}$:

$$(q\beta)^2 \left(1 + \frac{r^2}{2\beta}\right)^2 - \left(2 + (1 - 2\beta\Lambda)\frac{r^2}{2\beta}\right) k(r) + C_0 k^{3/2}(r) = 0$$

All metric and scalar functions given with respect to k and $\phi = qt + \psi(r)$:

$$h(r) = -\frac{\mu}{r} + \frac{1}{r}\int \frac{k(r)}{\beta + \eta r^2} dr, \qquad f = \frac{(\beta + \eta r^2)h}{\beta (rh)'}$$

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta + \eta r^2)} \left(q^2 \beta(\beta + \eta r^2)h' - \frac{\eta + \beta\Lambda}{2}(h^2 r^2)' \right)^{1/2}.$$

Self tuning de Sitter black hole

- Solution reads $k(r) = \frac{(\beta+\eta r^2)^2}{\beta}$
  with $q^2 = (\zeta\eta + \beta\Lambda)/(\beta\eta)$ and $C_0 = (\zeta\eta - \beta\Lambda)\sqrt{\beta}/\eta$
- $$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$
  ... $q^2\beta(\beta+\eta r^2)^2 - \left(2\zeta\beta + \left(\zeta\eta - \beta\Lambda\right)r^2\right)k + C_0 k^{3/2} = 0$
- $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta}r^2$ de Sitter Schwarzschild!
- $\psi' = \pm\frac{q}{h}\sqrt{1-h}$ and $\phi(t,r) = q\,t + \psi(r)$
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Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$\begin{aligned}
L_2 &= G_2(\phi, X), \\
L_3 &= G_3(\phi, X)\Box\phi, \\
L_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
L_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

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\end{aligned}
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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
  $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
  Action is unique modulo integration by parts.

Going beyond Horndeski [Gleyzes et.al], [Zumalacarregui et.al],[Deffayet et.al], [Langlois et.al], [Crisostomi et.al]

What is the most general scalar-tensor theory with three propagating degrees of freedom?

It is beyond Horndeski but not quite DHOST yet...

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)\ ,$$

where

$$L_2 = G_2(\phi, X), \qquad L_3 = G_3(\phi, X)\Box\phi,$$

$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'},$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$+ F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

where $XG_{5,X}F_4 = 3F_5\left[G_4 - 2XG_{4,X} - (X/2)G_{5,\phi}\right]$. Beyond Horndeski acquires one extra function. BH has similar SA and ST solutions.

Conformal and disformal relations [Bellido, Zumalacarregui]

How are theories mapped under conformal and disformal transformations?

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

- Horndeski theory has G_2, G_3, G_4, G_5 free functions.
- For $C(\phi)$ and $D(\phi)$ we remain within Horndeski.
- However if we take a disformal $D(X)$ we jump to
- Beyond Horndeski (one more free function)
- Take a conformal $C(X)$ and jump to
- DHOST Type I (one more free function) [Langlois, Noui], [Crisostomi, Koyama]

In other words DHOST type I are all related to some Horndeski theory. Remaining DHOST theories are pathological [Langlois, Noui, Vernizzi]

Most general acceptable scalar tensor theories are related to Horndeski theory via a disformal and conformal transformation.

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GW170817 constraints on scalar tensor theories [Creminelli, Vernizzi], [Ezquiaga, Zumalacarregui]

- The combined observation of a gravity wave signal from a binary neutron star and its GRB counterpart constraints $c_T = 1$ to a 10^{-15} accuracy.
- For dark energy the scalar field (ST or SA) is non trivial at such distance scales (40Mpc) and generically mixes with the tensor metric perturbations modifying the light cone for gravity waves.
- For a dark energy scalar field from Horndeski the surviving theory has free $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ and $G_5 = 0$.
- For beyond Horndeski we have $G_5 = 0, F_5 = 0, 2G_{4,X} + XF_4 = 0$ and theory,

$$
\begin{aligned}
L_{c_T=1} = & \, G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi, X)\,{}^{(4)}R \\
& - \frac{4}{X} B_{4,X}(\phi, X)(\phi^\mu \phi^\nu \phi_{\mu\nu}\Box\phi - \phi^\mu \phi_{\mu\nu}\phi_\lambda \phi^{\lambda\nu}) \, ,
\end{aligned}
$$

- For DHOST we just make a conformal transformation of the above, $G_2(\phi, X) G_3(\phi, X), B_4(\phi, X), C(\phi, X)$

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L_2 &= G_2(\phi, X), \\
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Physical and disformed frames

Most general scalar tensor theory with $c_T = 1$ minimally coupled to matter parametrized by G_2, G_3, B_4, C

$$L_{c_T=1} = G_2 + G_3\Box\phi + B_4 C\, {}^{(4)}R - \frac{4B_{4,X}C}{X}\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi$$
$$+\left(\frac{4B_{4,X}C}{X} + \frac{6B_4 C_{,X}{}^2}{C} + 8C_{,X}B_{4,X}\right)\phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu}$$
$$+\frac{8C_{,X}B_{4,X}}{X}(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\ .$$

- Horndeski is related via a transformation

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$

  to the $L_{c_T=1}$ for given C and D.
- One can start with a $c_T \neq 1$ Horndeski theory and map it to a DHOST $c_T = 1$ theory for a specific function D.
- The former is what we could have called the Einstein $\rightarrow$ Horndeski frame respective to the latter, the Jordan frame...
- except that the metric is disformed in the procedure...

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The physical frame and the disformed solution [Babichev, CC, GEFarèse, Lehébel]

- The theory

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

  is excluded or it is not the physical frame.
- Solution: $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta}r^2$, $\phi = qt \pm \frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$.
- The physical frame is :

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}\,\varphi_\lambda^2}\,\varphi_\mu\varphi_\nu.$$

- Indeed the $\tilde{g}_{\mu\nu}$ frame is a beyond Horndeski theory with $c_T = 1$ for a cosmological background.
- The disformed metric is a black hole
- we have exactly $c_{grav} = 1$ for a highly curved background!
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Conclusions

- Modifying GR is a difficult task but with countable possibilities. Even more so after the GW experiments. :)))
- Scalar tensor theories are parametrized by 4 free functions which we hope will be further constrained.
- Numerous spherically symmetric solutions known. Black holes, neutron stars. One has to adjust them to acceptable theories. One has to study GW-compatible theories independently.
- Self tuning vacua and black holes can be found with $c_{grav} = 1$.
- Exact solutions are important to understand the MG theory and find novel effects. Are there solutions free of singularities? What of rotating hairy black holes and Neutron stars?

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