Black-Hole Solutions with Scalar Hair in Einstein-Scalar-Gauss-Bonnet Theories

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arXiv:1711.07431 to appear in Phys. Rev. D.

arXiv:1711.03390 to appear in Phys. Rev. Lett.

March 30 2018

Outline

1. Black Holes in General Relativity
2. No-Scalar Hair Theorem
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Black Holes in General Relativity

- Black holes in General Relativity may be described only by three physical quantities: Mass, E/M charge and Angular Momentum.
- Black holes are very special objects: Two stars with the same mass are, in general, very different, but two black holes with the same characteristics (M, Q and J) will be identical.
- *No hair theorems:* Uniqueness theorems which state that in General Relativity only four possible solutions for black holes may exist.

No-Scalar Hair Theorem

Adding new matter/energy forms in the theory could lead to new Black holes solutions.

The simplest matter form is a Scalar field coupled to the gravitational field:

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right].$$

Assumptions:

- Asymptotically flatness,
- The scalar field has the same symmetries with the spacetime,
- Minimal coupling.

Under these assumptions black holes with scalar hair could not exist [1].

[1] J. D. Bekenstein, Phys. Rev. Lett. **28** (1972) 452
J. D. Bekenstein, Phys. Rev. D **51** (1995) no.12 R6608

The Horndenski Theory

The more general scalar-tensor theory in four dimensions with second order equations of motion:

$$\mathcal{S} = \int d^4x\sqrt{-g}\,[\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5]$$

with:

$$\begin{aligned}
\mathcal{L}_2 =& G_2(\phi, X), \\
\mathcal{L}_3 =& G_3(\phi, X)\nabla^2\phi, \\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4,X}\left[(\nabla^2\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}\left[(\nabla^2\phi)^3 - 3(\nabla^2\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}$$

$$X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi, \qquad \text{and} \qquad G_{i,X} = \frac{\partial G_i}{\partial X}.$$

The Einstein Scalar Gauss-Bonnet Theory

If we make the choices:

$$\begin{aligned} G_2 =& - X + 8 f^{(4)} X^2 (3 - \ln X), \\ G_3 =& - 4 f^{(3)} X (7 - 3 \ln X), \\ G_4 =& 1 + 4 \ddot{f} X (7 - \ln X), \\ G_5 =& - 4 f \ln X, \end{aligned}$$

we get the Einstein Scalar Gauss-Bonnet Theory:

$$\boxed{S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \nabla_\mu \phi \, \nabla^\mu \phi + f(\phi) R^2{}_{GB} \right],}$$

where:

$$R^2{}_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2.$$

The equations of motion are:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu},$$

$$\nabla^2 \phi + \frac{d\, f(\phi)}{d\phi}\, R^2_{GB} = 0,$$

where:

$$T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu}\, \nabla_\rho \phi\, \nabla^\rho \phi + \frac{1}{2} \nabla_\mu \phi\, \nabla_\nu \phi + \frac{1}{2}\, (g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu})\, \eta^{\kappa\lambda\alpha\beta}\, \tilde{R}^{\rho\gamma}{}_{\alpha\beta}\, \nabla_\gamma \nabla_\kappa f(\phi),$$

$$\tilde{R}^{\mu\nu}{}_{\kappa\lambda} = \eta^{\mu\nu\rho\sigma}\, R_{\rho\sigma\kappa\lambda},$$

$$\eta^{\mu\nu\rho\sigma} = \sqrt{-g}\,\varepsilon^{\mu\nu\rho\sigma}.$$

We assume a spherically symmetric form for the line-element:

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2\left(d\theta^2 + d\varphi^2\sin^2(\theta)\right),$$

The (rr) equation can be solved for e^B:

$$e^B = \frac{-\beta \pm \sqrt{\beta^2 - 4\gamma}}{2}, \qquad \text{and} \qquad B' = -\frac{\gamma' + \beta' e^B}{2e^{2B} + \beta e^B},$$

with

$$\beta = \frac{r^2\phi'^2}{4} - (2\dot{f}\phi' + r)A' - 1, \qquad \gamma = 6\dot{f}\phi' A'.$$

If we replace the above relations in the field equations, the (t,t) and (θ,θ) equations reduce to:

$$A'' = g(r; \phi', A', \dot{f}, \ddot{f})$$

$$\phi'' = h(r; \phi', A', \dot{f}, \ddot{f})$$

Asymptotic Solutions at the horizon

Assuming that $e^B \to \infty$ at the horizon, we find that the solutions take the form:

$$e^{A(r)} = a_0(r - r_h) + ..., \qquad e^{-B(r)} = b_0(r - r_h) + ...,$$

$$\phi\,(r) = \phi_h + \phi_h'(r - r_h) + ...,$$

the regularity condition for the scalar field demands that:

$$\phi_h' = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right)$$

- The above solutions will serve as boundary conditions for our numerical integration.
- From the condition for the derivative we get a constraint for the radius and thus the mass of the black hole:

$$r_h^4 > 96\,\dot{f}_h^2$$

Asymptotic Solutions at Infinity

In the limit $r \to \infty$ we demand that the space-time is flat and the scalar field assumes a constant value. We find:

$$e^{A(r)} = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \mathcal{O}\left(1/r^4\right), \qquad e^{B(r)} = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \mathcal{O}\left(1/r^3\right),$$

$$\phi\left(r\right) = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \mathcal{O}\left(1/r^3\right).$$

- The exact form of the coupling function does not enter in the expansions earlier than the order $\mathcal{O}(1/r^4)$.

The No-Scalar Hair theorem may be evaded and the two asymptotic solutions may be smoothly connected assuming that near the horizon[2]:

$$\dot{f}_h \phi'_h < 0, \qquad \text{and} \qquad \partial_r(\dot{f}\phi')|_{r_h} > 0.$$

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[2] P. Kanti talk.

G. Antoniou, A. B. and P. Kanti, arXiv:1711.03390 [hep-th], to appear in Phys. Rev. Lett.

Numerical Solutions

The system of field equations can not be solved analytically.

In order to get exact solutions, valid over the entire radial domain, we use numerical integration methods.

- Our integration starts at a distance very close to the horizon of the black hole $r \approx r_h + \mathcal{O}(10^{-5})$
- For simplicity we set $r_h = 1$.
- We use as boundary conditions the asymptotic solutions near the horizon together with the condition for the ϕ'_h
- The integration proceeds towards large values of the radial coordinate until the form of the derived solution matches the asymptotic solutions at infinity.

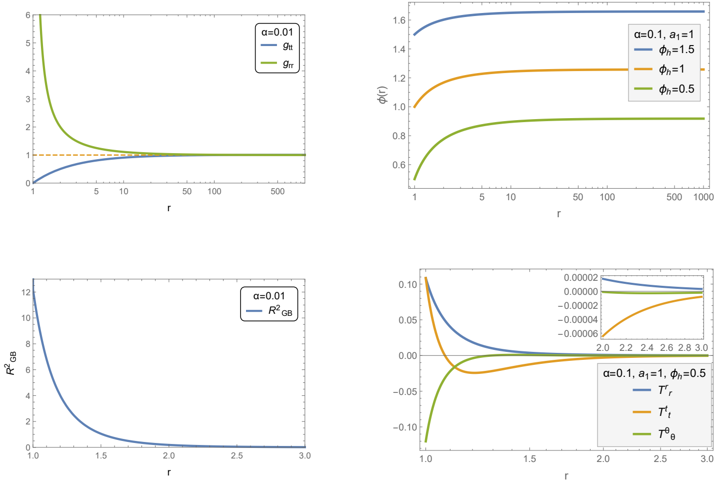
Known Numerical Solutions:

P. Kanti et al., Phys. Rev. D **54** (1996) 5049 (for exponential coupling)

T.P. Sotiriou and S.Y. Zhou, Phys. Rev. D **90**, (2014) 124063 (for linear coupling)

Solutions for $f(\phi) = \alpha e^{-\phi}$

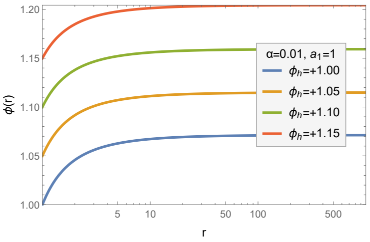
We reproduce the old results by P.Kanti et al. (1996)

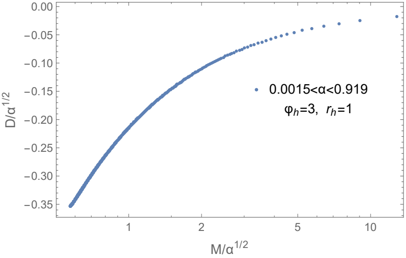


The behavior of the solutions for the scalar field, near the horizon, is dictated by the constraint:

$$\dot{f}_h \phi'_h < 0.$$

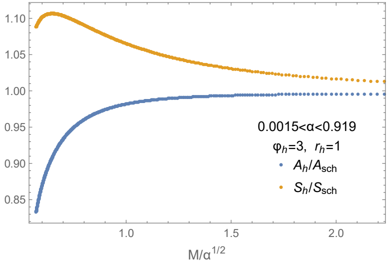
Solutions and Physical Characteristics for $f(\phi) = \alpha/\phi$



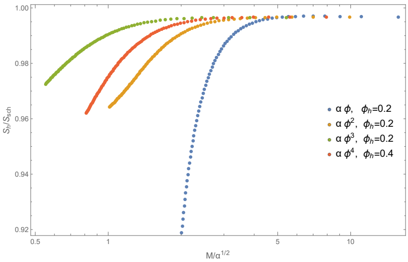


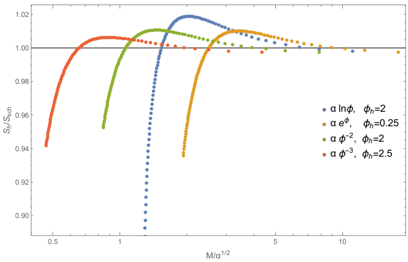
$$A_h = 4\pi r_h^2,$$

$$S_h = \frac{A_h}{4} + 4\pi\, f(\phi_h)$$



Entropy ratios for more coupling functions





For large values of the ϕ_h there is always a mass range in which the entropy ratio is above unity.

We expect the solutions with higher entropy than the Schwarzschild solution to be thermodynamically stable.

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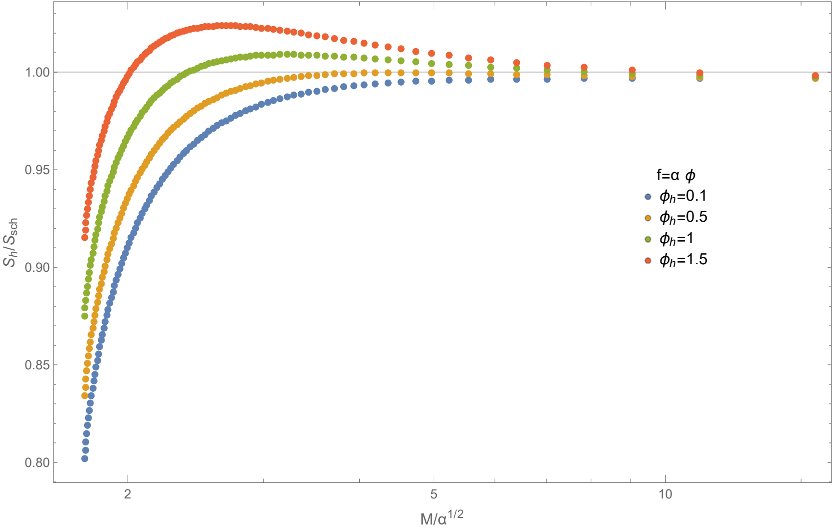
for more informations about the quadratic and the exponential coupling function:
H.O.Silva et al. arXiv:1711.02080 [gr-qc].
D. D. Doneva and S. S. Yazadjiev, arXiv:1711.01187 [gr-qc].

Conclusions

- Regular, asymptotically flat black hole solutions were found for a large number of choices for the coupling function.
- The profile for the scalar field, in all of the cases considered, was found to be regular over the entire radial domain.
- The scalar charge D was determined in each case, and its dependence on M was studied
- In all cases the scalar charge is an M-dependent quantity, and therefore our solutions have a non-trivial scalar field but with a "secondary" hair.
- In the large-mass limit, the horizon area and the entropy of all black-hole solutions approached the Schwarzschild value.
- The entropy ratio S_h/S_{Sch} may provide hints for the stability of our solutions.

Thank You!



f=α ϕ
ϕ_h=0.1
ϕ_h=0.5
ϕ_h=1
ϕ_h=1.5
S_h/S_{sch}
1.00
0.95
0.90
0.85
0.80
2
5
10
$M/\alpha^{1/2}$

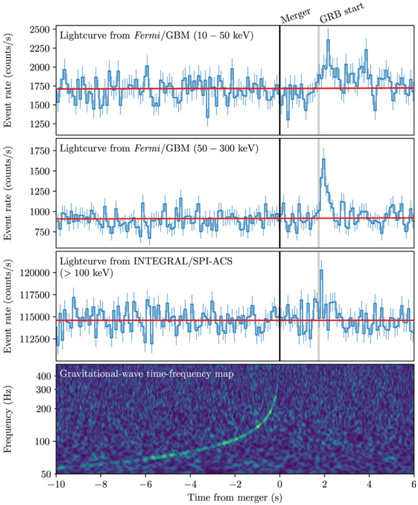
In the cosmological Friedmann-Robertson-Walker (FRW) background the propagation speed of the gravitational waves in the Horndeski theory is:

$$c_{gw}^2 = \frac{G_4 - X\left(\ddot{\phi}\, G_{5,X} + G_5'\right)}{G_4 - 2XG_{4,X} - X\left(H\dot{\phi}G_{5,X} - G_5'\right)}$$

Using the above relation for the Scalar Gauss bonnet theory we find:

$$c_{gw} \neq c,$$

This result is valid for cosmological solutions with time-dependent scalar field.



The GW170817 and GRB 170817A signals