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Strings on Melvin Spaces and the Ω Background(?)

Carlo Angelantonj

Work in progress with Antoniadis
to appear soon (?)

See also
C.A., Dudas Mourad, NPB 637 (2002) 59-91
C.A., Antoniadis, Samsonyan, NPB 923 (2017) 32-53

The simple mechanical model of the *vibrating string*
has been a source of stunning progress in our
understanding of the fundamental interactions.

The surprise is even bigger if we realise that we can
solve the model only on simple *flat* backgrounds†

In this talk I shall study the dynamics of strings on more
complicated spaces and will derive some implications on
their (perturbative) quantum dynamics
in the low-energy limit

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†with very few exceptions

OUTLINE

The model

Motivations

OUTLINE

The model

Motivations

THE MODEL

$$ds^2 = \left|dz + i\hbar z\,dy\right|^2 + dy^2 + d\boldsymbol{x}^2$$

$z \in \mathbb{C},\ y \in S^1(R)$

One can show that this metric, together with a constant dilaton and vanishing (NS-NS and R-R) forms is an exact string background

The solution of the world-sheet theory seems hopeless due to the presence of non Gaussian terms still …

This solution can be interpreted à la Kaluza-Klein by writing the (relevant part of the) metric as

$$L=\partial_a\rho\partial^a\rho+F(\rho)\rho^2\partial_a\varphi\partial^a\varphi+e^{2\sigma}\left(\partial_a y+A_\varphi\partial_a\varphi\right)\left(\partial^a y+A_\varphi\partial^a\varphi\right)$$

In terms of four-dimensional fields

$$ds_4^2=-dt^2+d\rho^2+F(\rho)\rho^2 d\varphi^2+dx_3^2$$

$$F(\rho)=\frac{1}{1+\hbar^2\rho^2}$$

$$A_\varphi=\hbar F(\rho)\rho^2$$

$$e^{2\sigma}=F^{-1}$$

In polar coordinates

$$ds^2 = d\rho^2 + \rho^2 (d\varphi + \hbar dy)^2 + dy^2 + d\boldsymbol{x}^2$$

$z = \rho\, e^{i\varphi}$

A simple redefinition of the angular coordinate

$$ds^2 = d\rho^2 + \rho^2 d\varphi_0^2 + dy^2 + d\boldsymbol{x}^2 = |dz_0|^2 + dy^2 + d\boldsymbol{x}^2$$

$z_0 = \rho\, e^{i\varphi_0} = \rho e^{i(\varphi + \hbar y)}$

… a free (gaussian) theory subject to the identification

$$(y, \varphi_0) \sim (y + 2\pi n R, \varphi_0 + 2\pi n R \hbar)$$

These backgrounds are very well known
and are amenable to an exact CFT description

Freely acting orbifolds

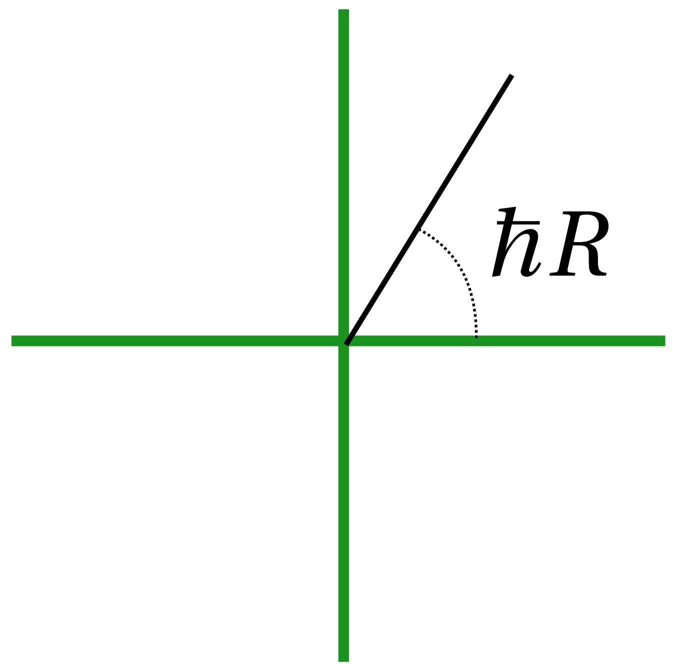
Fluxbranes

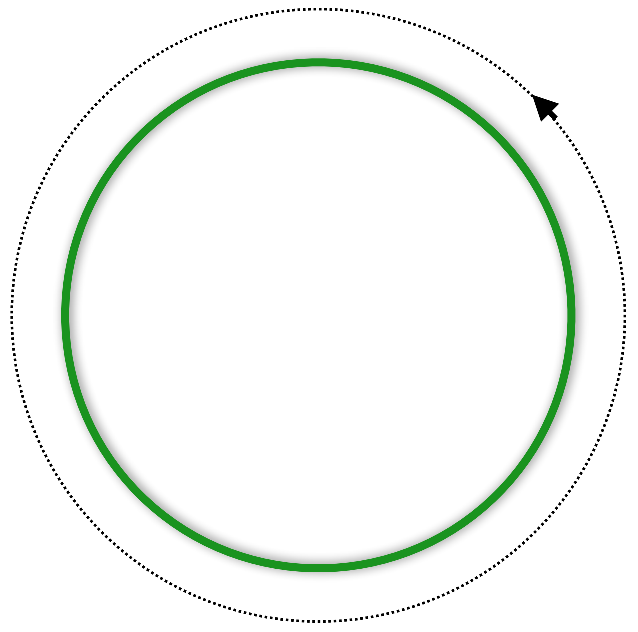
Coordinate dependent compactifications

Scherk-Schwarz reduction

Melvin backgrounds

These backgrounds are very well known
and are amenable to an exact CFT description





(for $z \in T^2$ the angle must be quantised
as in familiar orbifold compactifications)

The partition function for type II superstrings simply reads

$$\mathscr{Z} = \frac{1}{\tau_2^3}\,\frac{1}{|\eta|^{12}}\sum_{\tilde{k},n} e^{-\frac{\pi R^2}{\tau_2}|\tilde{k}+\tau n|^2}\,\mathscr{Z}(\tilde{k},n)$$

with

$$\mathscr{Z}(0,0) = \frac{1}{\tau_2}\,\frac{1}{|\eta|^4}\left|\sum_{a,b}\eta_{a,b}\,\frac{\theta\big[{a \atop b}\big]^4}{\eta^4}\right|^2$$

$$\eta_{a,b} = (-1)^{2a+2b+4ab}$$

$$\mathscr{Z}(\tilde{k},n) = \left|\sum_{a,b}\eta_{a,b}\,e^{-2\pi i b\hbar R n}\,\frac{\theta\big[{a \atop b}\big]^3}{\eta^3}\,\frac{\theta\big[{a+\hbar R n \atop b+\hbar R\tilde{k}}\big]}{\theta\big[{1/2+\hbar R n \atop 1/2+\hbar R\tilde{k}}\big]}\right|^2$$

[Russo, Tseytlin, 1995]
[Dudas, Mourad, 2002]
[C.A., Dudas, Mourad, 2002]

One can also study D-branes on this background

$$\mathscr{A} = \frac{1}{\tau_2^3}\frac{1}{\eta^6}\sum_{\tilde{k}} e^{-\pi \tilde{k}^2 R^2/\tau_2}\,\mathscr{A}(\tilde{k})$$

$$\mathscr{A}(0) = \frac{1}{\tau_2}\frac{1}{\eta^2}\sum_{a,b}\eta_{a,b}\frac{\theta[{}^a_b]^4}{\eta^4}$$

$$\mathscr{A}(\tilde{k}) = \frac{1}{\sin(\pi\hbar R\tilde{k})}\sum_{a,b}\eta_{a,b}\frac{\theta[{}^a_b]^3}{\eta^3}\frac{\theta[{}^{\;a}_{b+\hbar R\tilde{k}}]}{\theta[{}^{\;1/2}_{1/2+\hbar R\tilde{k}}]}$$

… and orientifolds (involving O-planes at angles)

[Dudas, Mourad, 2002]
[C.A., Dudas, Mourad, 2002]

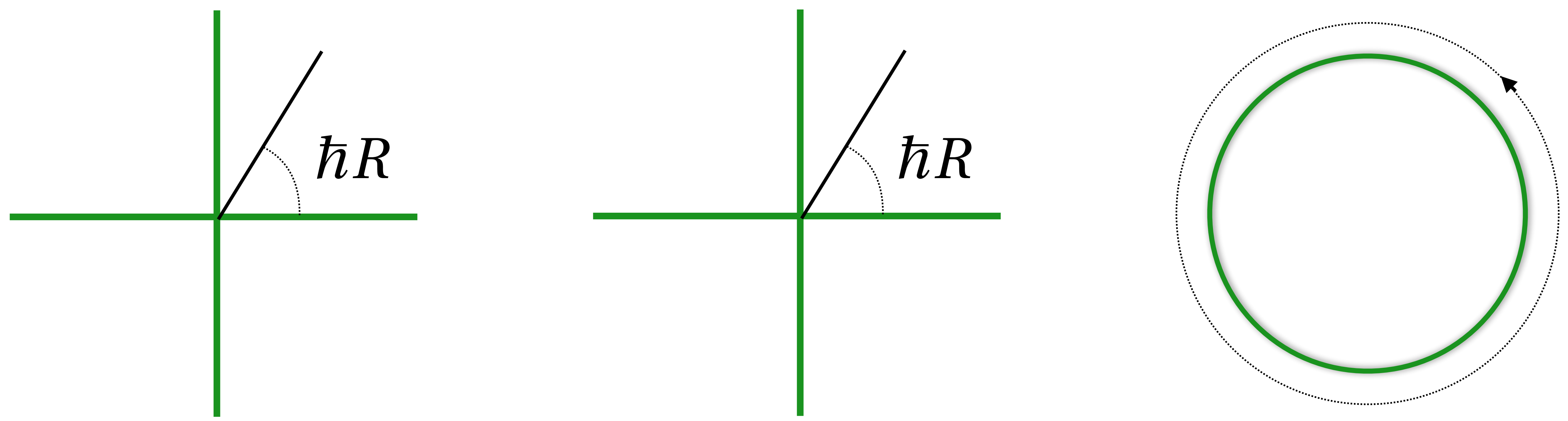
This background breaks all (space-time) supersymmetries.
In fact the Riemann (Jacobi) identity implies

$$\sum_{a,b} \eta_{a,b} \theta \left[{a \atop b} \right]^3 \theta \left[{a \atop b+\hbar R\tilde{k}} \right] \propto \vartheta \left[{1/2 \atop 1/2+\hbar R\tilde{k}/2} \right]^4$$

which does not vanish.

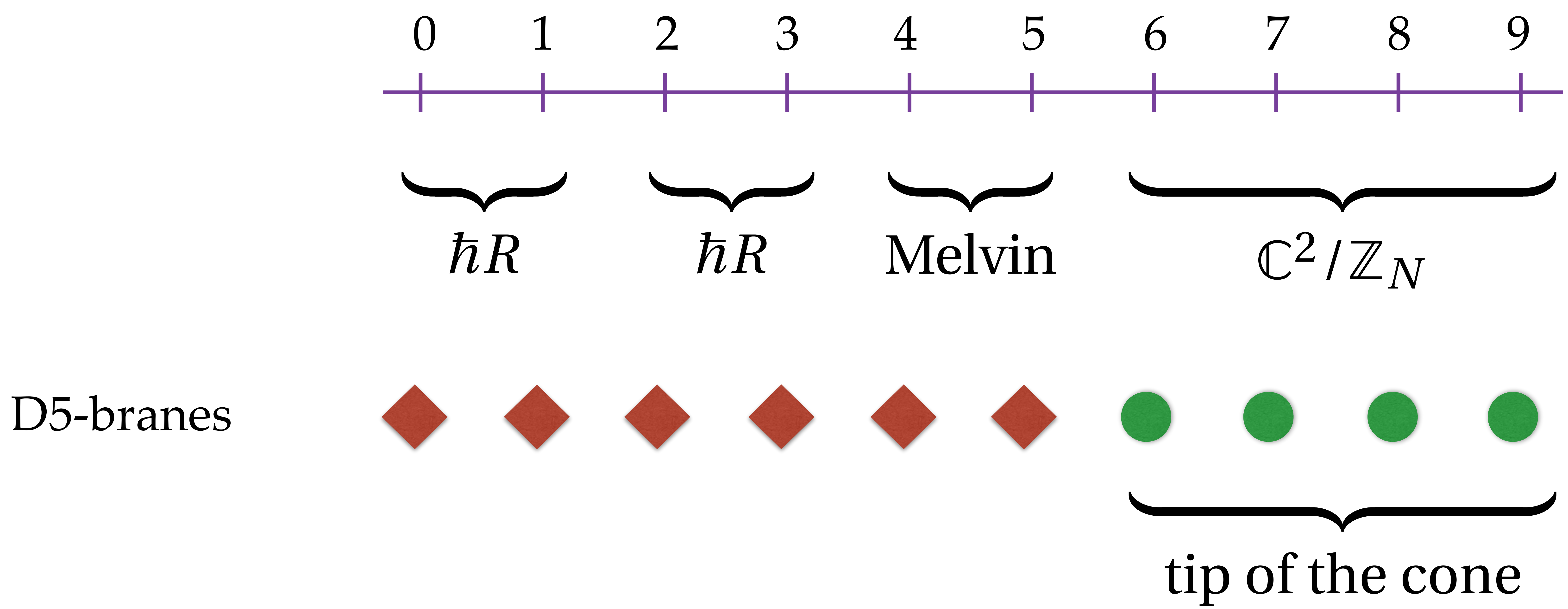
Things can be improved if one rotates more than one plane.

In the simple case of two identical (or opposite) rotations



half of the supersymmetries are preserved

Actually, for reasons that will be clear in a short-while
I am interested in the following (exact) background



The annulus partition function is simply **(!!)** the combination of branes on (freely-acting) orbifolds

$$\mathscr{A} = \sum_{\tilde{k}} e^{-\pi \tilde{k}^2 R^2/\tau_2}\, \mathscr{A}(\tilde{k})$$

$$\mathscr{A}(0) = \frac{1}{\tau_2^2}\frac{1}{N}\sum_{\ell=0}^{N-1}\sum_{a,b}\eta_{a,b}\frac{\theta[{}^{a}_{b}]^2}{\eta^6}\,\frac{\theta[{}^{a}_{b+\ell/N}]\,\theta[{}^{a}_{b-\ell/N}]}{\theta[{}^{1/2}_{1/2+\ell/N}]\,\theta[{}^{1/2}_{1/2-\ell/N}]}$$

$$\mathscr{A}(\tilde{k}) = \frac{1}{\sin^2(\pi\hbar R\tilde{k})}\frac{1}{N}\sum_{\ell=0}^{N-1}\sum_{a,b}\eta_{a,b}\frac{\theta[{}^{a}_{b+\hbar R\tilde{k}}]^2}{\theta[{}^{1/2}_{1/2+\hbar R\tilde{k}}]^2}\,\frac{\theta[{}^{a}_{b+\ell/N}]\,\theta[{}^{a}_{b-\ell/N}]}{\theta[{}^{1/2}_{1/2+\ell/N}]\,\theta[{}^{1/2}_{1/2-\ell/N}]}$$

WHY DID I DO ALL THIS?

OUTLINE

The model

Motivations

The main motivation comes from the explicit solution of Nekrasov of the (non-perturbative) dynamics of N=2 theories

$$ds^2 = A\,dz\,d\bar{z} + g_{IJ}\left(dx^I + \Omega^I{}_K x^K\,dz + \bar{\Omega}^I{}_K x^K\,d\bar{z}\right)\left(dx^J + \Omega^J{}_L x^L\,dz + \bar{\Omega}^J{}_L x^L\,d\bar{z}\right)$$

This background lifts the instanton moduli space, leaving only a finite number of isolated points as a full set of supersymmetric minima of the action. One is thus left to compute ratios of determinants near each critical point

What is (if any) the (perturbative) string description of the Ω background?

For a description in terms of RR fluxes to reconstruct the low-energy SYM and ADHM effective actions
[Billò, Frau, Fucito, Lerda, Morales, Pesando]

The Ω background of Nekrasov

$$ds^2 = A\,dz d\bar{z} + g_{IJ}\left(dx^I + \Omega^I{}_K x^K\, dz + \bar{\Omega}^I{}_K x^K\, d\bar{z}\right)\left(dx^J + \Omega^J{}_L x^L\, dz + \bar{\Omega}^J{}_L x^L\, d\bar{z}\right)$$

really looks like a Melvin space (with one/two independent parameters)

From field theory, the perturbative (one-loop) correction to the prepotential reads

$$\sum_{i,j} \int_0^\infty \frac{dt}{t} \sum_n \frac{1}{\sin^2(\hbar n t)}\, e^{-\pi t (n + a_i - a_j)^2 / R^2}$$

Moreover, Nekrasov and Okounkov observed a puzzling coincidence in the case of a single parameter background

Note that up to the terms of instanton degree zero the function $\gamma_\hbar(x|\beta;\Lambda)$ *coincides with the all-genus free energy of the type A topological string on the resolved conifold, with* βx *being the Kähler class of the* $\boldsymbol{P}^1$*, and* $\beta\hbar$ *the string coupling.*

[Nekrasov, Okounkov, 2003]

Alternatively, the perturbative contribution is captured by topological amplitudes computing (higher-derivative) F-terms of the form $F_g W^{2g}$.

Why is this true?
How to refine it to the case of two independent parameters?

$$\mathscr{A}_g = \left\langle (V^+_{\text{grav}})^2\, (V^-_{\text{grav}})^2\, V^{2g-2}_{\text{gph}} \right\rangle$$

can be "easily" computed via the generating functional

angle of rotations

$$G_{\text{Bose}}(\hbar) = \left\langle \exp \frac{\hat{\hbar}}{t} \int d^2\sigma \left[Z^1 (\bar{\partial} - \partial) Z^2 + \bar{Z}^1 (\bar{\partial} - \partial) \bar{Z}^2 \right] \right\rangle$$

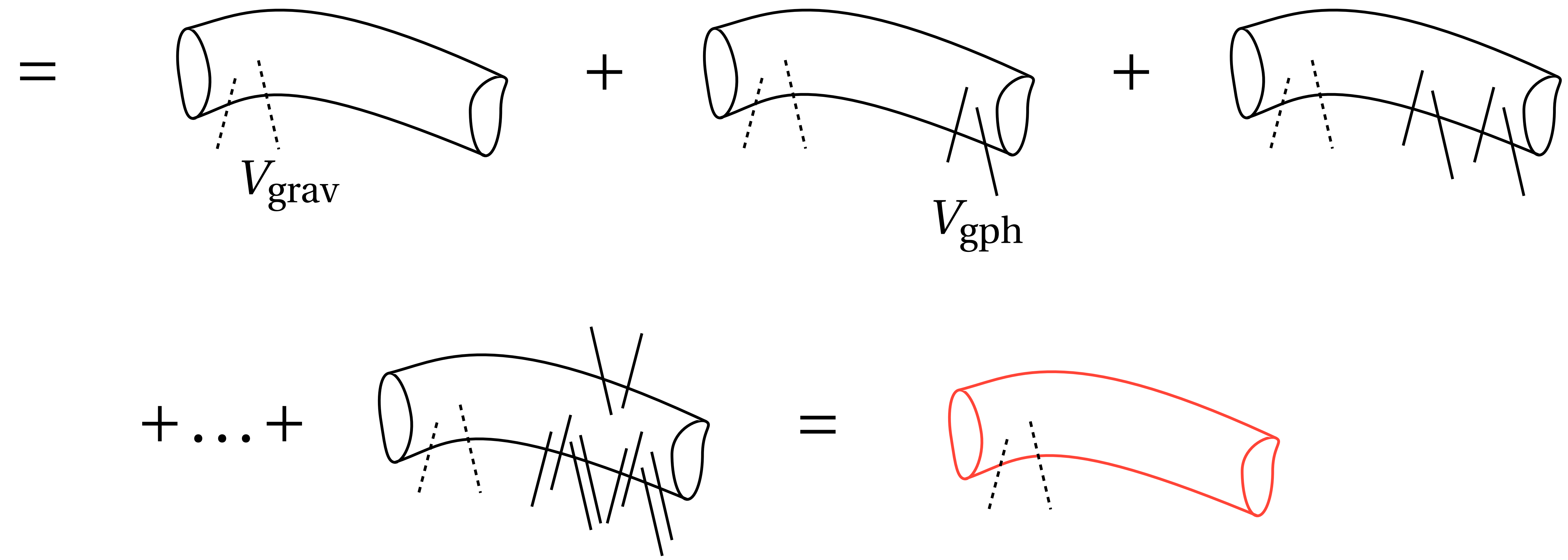
generators of rotations

similarly for fermions

[Antoniadis, Gava, Narain, Taylor, 1993]

What does it mean?

$$\mathcal{F}(\hbar) = \sum_{g=0}^{\infty} \left\langle (V^{+}_{\text{grav}})^2 \, (V^{-}_{\text{grav}})^2 \, V^{2g-2}_{\text{gph}}(\hbar) \right\rangle$$



The insertion of the graviphotons is effectively generating the (Melvin/Ω) background

OUTLOOK

a. Can we give a (perturbative) description of the Ω background?

b. String corrections to the prepotential on Ω?

c. Can we gain insight on the refinement of the topologica string?

The Galileo Galilei Institute
For Theoretical Physics
Centro Nazionale di Studi Avanzati dell'Istituto Nazionale di Fisica Nucleare
Arcetri, Firenze

www.ggi.infn.it

STRING THEORY FROM A WORLD-SHEET PERSPECTIVE

March 29, 2019 — May 10, 2019

ORGANISERS:

C. Angelantonj (Università di Torino)
Antoniadis (Bern University)
N. Berkovits (Sao Paolo University)
M.B. Green (DAMPT – Cambridge)
C. Maccaferri (Università di Torino)
Y. Okawa (Tokyo University)
R. Russo (Queen Mary College – London)
M. Schnabl (Czech Academy of Science)
B. Zwiebach (MIT)

THANK YOU

RADIATIVE CORRECTIONS

RADIATIVE CORRECTIONS

We are interested in computing one-loop corrections to the gauge coupling on the D5-branes

This has been a very active and fruitful topic in the late nineties that has contribute to our understanding of string-string dualities

RADIATIVE CORRECTIONS

An elegant way to compute these quantum corrections in open strings is to employ the background-field method

$$S = S_0 + \int_{\partial\Sigma} d\tau\, q\, F_{\mu\nu} X^{\mu} \partial_{\tau} X^{\nu}$$

This extra term only alters the boundary conditions and the string model is still exactly solvable

The modifications of the annulus amplitude are (essentially)
in the shifted masses of (charged) open strings

$$M^2 \to M^2 + (2n+1)|\epsilon| + 2\epsilon\Sigma$$

$$\pi\epsilon = \tan^{-1}(\pi q_{\mathrm{L}} B) + \tan^{-1}(\pi q_{\mathrm{R}} B)$$

Expanding the annulus amplitude for small B

$$\mathscr{A} \to \mathscr{A}(B) \simeq \mathscr{A}_0 + B^2\,\mathscr{A}_2 + B^4\,\mathscr{A}_4 + \ldots$$

Let us see what happens in the case of orbifold compactifications on flat space-time (with N=2 susy)

$$\mathscr{A} \sim B \sum_{\ell=0}^{N-1} \sum_{a,b} \eta_{a,b} \frac{\theta[{}^{a+B}_{\ b}]}{\theta[{}^{1/2+B}_{\ 1/2}]} \frac{\theta[{}^a_b]}{\eta^2} \frac{\theta[{}_{b+\ell/N}^{\ a}]\,\theta[{}_{b-\ell/N}^{\ a}]}{\theta[{}_{1/2+\ell/N}^{\ 1/2}]\,\theta[{}_{1/2-\ell/N}^{\ 1/2}]}$$

From the Riemann identity

$$\sum_{a,b} \eta_{a,b} \prod_{i=1}^{4} \theta\left[{}^a_b\right](z_i) = -2 \prod_{i=1}^{4} \theta\left[{}^{1/2}_{1/2}\right](u_i)$$

$$u_1 = \tfrac{1}{2}(z_1 + z_2 + z_3 - z_4)\,,$$
$$u_2 = \tfrac{1}{2}(z_1 + z_2 - z_3 + z_4)\,,$$
$$u_3 = \tfrac{1}{2}(z_1 - z_2 + z_3 + z_4)\,,$$
$$u_4 = \tfrac{1}{2}(-z_1 + z_2 + z_3 + z_4)\,,$$

Let us see what happens in the case of orbifold
compactifications on flat space-time (with N=2 susy)

$$\mathscr{A} \sim B \sum_{\ell=0}^{N-1} \sum_{a,b} \eta_{a,b} \frac{\theta[{}^{a+B}_{\ b}]}{\theta[{}^{1/2+B}_{\ 1/2}]} \frac{\theta[{}^{a}_{b}]}{\eta^2} \frac{\theta[{}^{\ a}_{b+\ell/N}]\theta[{}^{\ a}_{b-\ell/N}]}{\theta[{}^{\ 1/2}_{1/2+\ell/N}]\theta[{}^{\ 1/2}_{1/2-\ell/N}]}$$

drastic simplifications of the final result
(string states decouple — BPS saturated amplitude)

$$\mathscr{A}_2 \simeq \int_0^\infty \frac{dt}{t} \sum_m e^{-\pi t (m/R)^2}$$

[Antoniadis, Bachas, Dudas, 1999]

For the Melvin case the situation is a bit more complicated.
The annulus amplitude is deformed in a similar way

$$\mathscr{A} \sim \sum_{\ell=0}^{N-1} \sum_{\tilde{k}} \frac{1}{\sin^2(\pi\hbar R\tilde{k})} \sum_{a,b} \eta_{a,b} \frac{\theta[{a+B \atop b+\hbar R\tilde{k}}]}{\theta[{1/2+B \atop 1/2+\hbar R\tilde{k}}]} \frac{\theta[{a \atop b+\hbar R\tilde{k}}]}{\theta[{1/2 \atop 1/2+\hbar R\tilde{k}}]} \frac{\theta[{a \atop b+\ell/N}]\theta[{a \atop b-\ell/N}]}{\theta[{1/2 \atop 1/2+\ell/N}]\theta[{1/2 \atop 1/2-\ell/N}]} e^{-\pi R^2\tilde{k}^2/t}$$

The Riemann identity works in a similar way,
but the contribution of the world-sheet fermions does not
cancel anymore that of the world-sheet bosons.

Taking the field theory limit (*i.e.* decoupling string states)

$$\mathscr{A}_2 \sim \sum_{i,j} \sum_{\tilde{k}\neq 0} \int_0^\infty dt\, \sqrt{t}\, e^{2i\pi(a_i - a_j)\tilde{k}}\, \frac{\cos(2\hbar R\tilde{k})}{\sin^4(\hbar R\tilde{k})}\, e^{-\pi\tilde{k}^2 R^2/t}$$

1. Taylor Expanding the trigonometric function
2. Poisson summing each single term
3. Compute the t-integral

$$\mathscr{A}_2 \sim \sum_{i,j} \sum_{\tilde{k}\neq 0} \int_0^\infty dt\,\sqrt{t}\,e^{2i\pi(a_i-a_j)\tilde{k}}\,\frac{\cos(2\hbar R\tilde{k})}{\sin^4(\hbar R\tilde{k})}\,e^{-\pi\tilde{k}^2R^2/t}$$

The one-loop correction to the gauge coupling constant of
the "N=2" theory is then expressed in terms of
an infinite series in powers of $\hbar$,
with coefficients related to the Bernoulli numbers

WHY DID I DO ALL THIS?