Relativistic effects in atom gravimeters

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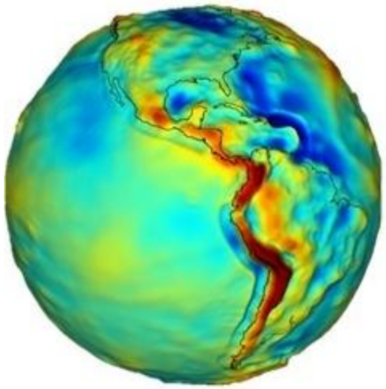
Outline

- Research motivation
- AG's Working principle
- An analytical study method
- Conclusion

1. Research motivation

● Applications of gravimeters

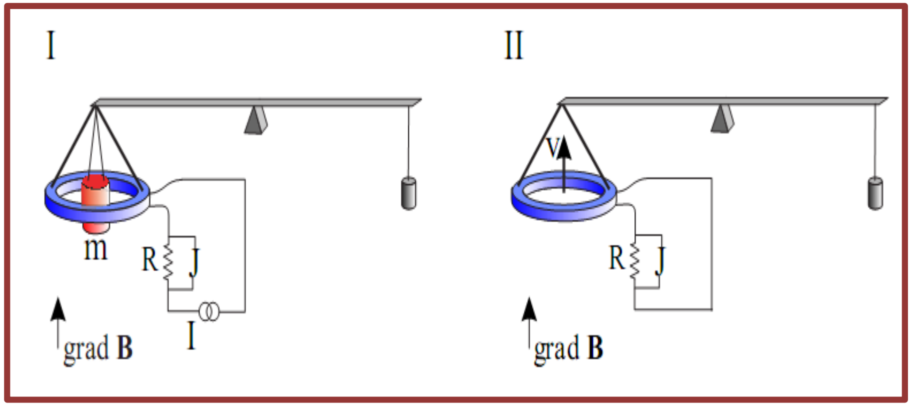
Gravimeter: measure the gravitational acceleration *g*



Geophysics

Resources exploration

Gravity navigation



Other scientific researches (such as Watt balance)

1. Research motivation

● Classification of gravimeters

Relative gravimeters

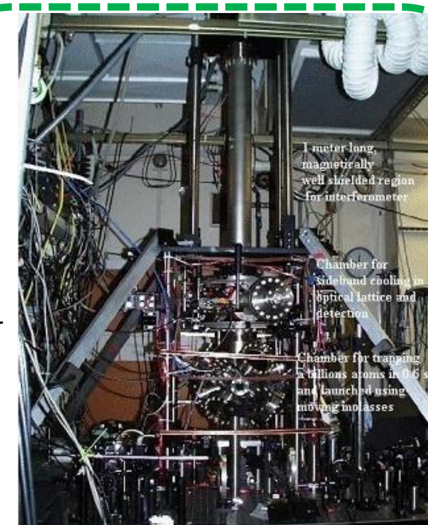
CG-5

GWR

Accuracy—μGal
$1\mu Gal = 1\times10^{-8}\ m/s^2$

Absolute gravimeters

FG-5



AG

1. Research motivation

Necessity for establishing a relativistic model of AG

High-precision gravimeter ⟹ High-precision tide model; High-precision geoid; Post-Newton gravity theory; ……

Measured gravity acceleration:

Newtonian effects

$$g_{\text{measured}} = g_0 + \boxed{\text{temperature, pressure, Earth's rotation, gravity gradient….}}$$

$$+ \; B\frac{v}{c} \; + \; \left[C\left(\frac{v}{c}\right)^2 \; + \; D\frac{\phi}{c^2} + \ldots\ldots \right]$$

Finite speed of light (FSL) effect	Second-order Doppler effect	General Relativity
10 μGal	**10^{-8} μGal**	**10^{-7} μGal ($\Delta\phi/c^2$)**

Relativistic effect

⇓ **Model**

Establish a more complete relativistic model for AG, and improve the precision of AG.

1. Research motivation

● Research status for the AG's relativistic model

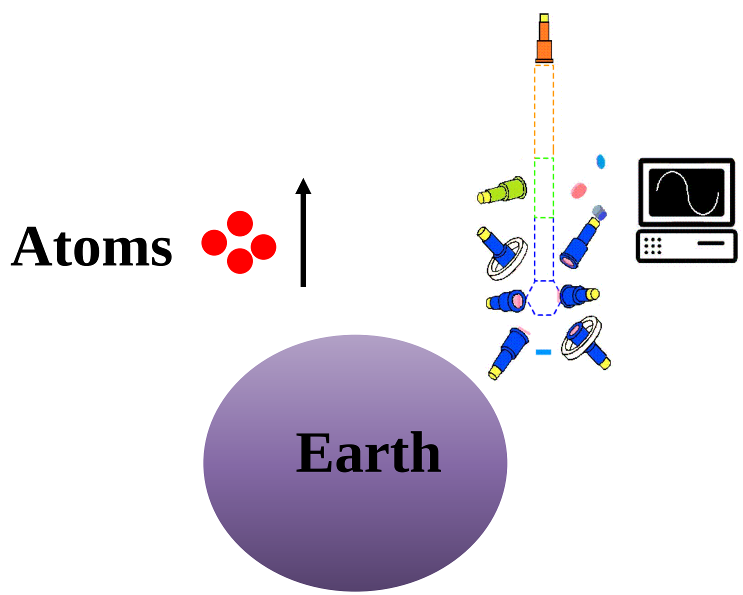
Measured gravity acceleration:

$$g_{\text{measured}} = g_{\text{N}} + \Delta g_{\text{PN}} + \Delta g_{\text{AG}}$$

Relativistic effects

Post-Newton effect of gravitational field[1]

Relativistic effects related to the instruments Δg_{AG}



[1] S.Wajima *et al.*, Physical Review D, 55 (1997)1964.

1. Research motivation

● Research status for the AG's relativistic model

$$g_{\text{measured}}^{\text{peters}} = g_0\left[1+2\frac{v(T)}{c}\frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2}\right]$$ [2-3]

[2] A. Peters *et al.*, Metrologia 38 (2001) 25-61.
[3] B. Cheng *et al.*, Phys. Rev. A 92 (2015) 063617.

FSL effect

$$g_{\text{measured}}^{\text{Dimopoulos}} = g_0\left[1+3\frac{v(T)}{c}\right]$$ [4]

Disagree !

GR effects

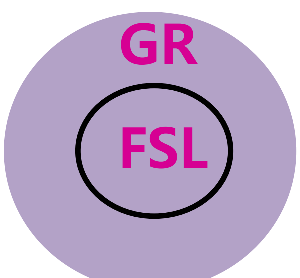
[4] S. Dimopoulos *et al.*, PRD 78 (2008) 042003.

Scalar expression!

	GR phase shift	Size (rad)	Interpretation
1.	$-k_{\text{eff}}gT^2$	3×10^8	Newtonian gravity
2.	$-k_{\text{eff}}(\partial_r g)v_LT^3$	-2×10^3	1st gradient
3.	$-\frac{7}{12}k_{\text{eff}}(\partial_r g)T^4$	9×10^2	
4.	$-3k_{\text{eff}}g^2T^3$	-4×10^1	finite speed of light and
5.	$-3k_{\text{eff}}gv_LT^2$	4×10^1	Doppler shift corrections
6.	$-\frac{k_{\text{eff}}^2}{2m}(\partial_r g)T^3$	-7×10^{-1}	1st gradient recoil
7.	$(\omega_{\text{eff}}-\omega_a)gT^2$	-4×10^{-1}	detuning
8.	$(2-2\beta-\gamma)k_{\text{eff}}g\phi T^2$	-2×10^{-1}	GR (nonlinearity)
9.	$-\frac{3k_{\text{eff}}^2}{2m}gT^2$	2×10^{-2}	
10.	$-\frac{7}{12}k_{\text{eff}}v_L^2(\partial_r^2 g)T^4$	8×10^{-3}	2nd gradient
11.	$-\frac{35}{4}k_{\text{eff}}(\partial_r g)gv_LT^4$	6×10^{-4}	
12.	$-4k_{\text{eff}}(\partial_r g)v_L^2T^3$	-3×10^{-4}	
13.	$2\omega_a g^2T^3$	2×10^{-4}	
14.	$2\omega_a gv_LT^2$	-2×10^{-4}	
15.	$-\frac{7k_{\text{eff}}^2}{12m}v_L(\partial_r^2 g)T^4$	7×10^{-6}	2nd gradient recoil
16.	$-12k_{\text{eff}}g^2v_LT^3$	-7×10^{-6}	
17.	$-7k_{\text{eff}}g^3T^4$	4×10^{-6}	
18.	$-5k_{\text{eff}}gv_L^2T^2$	3×10^{-6}	GR (velocity-dependent force)
19.	$(2-2\beta-\gamma)k_{\text{eff}}\partial_r(g\phi)v_LT^3$	2×10^{-6}	GR 1st gradient
20.	$\frac{7}{12}(4-4\beta-3\gamma)k_{\text{eff}}\phi(\partial_r g)gT^4$	-2×10^{-6}	GR

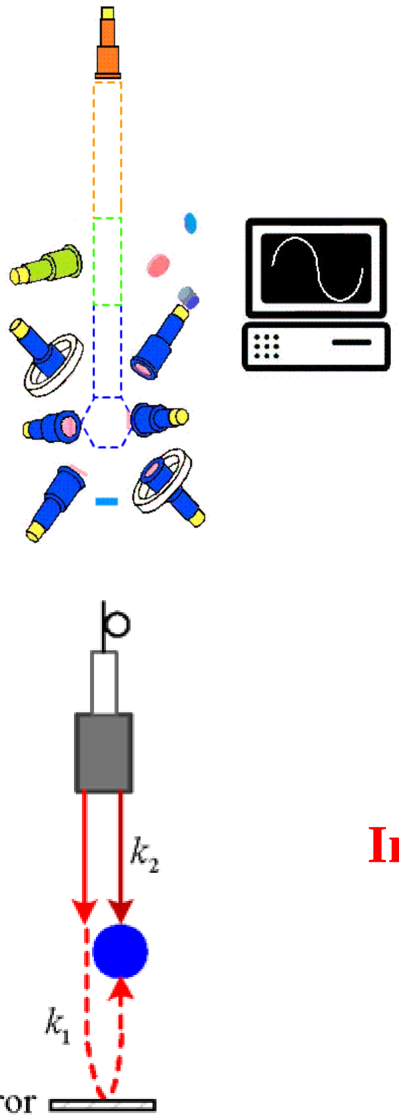
Need to explore:

- **Disagreement for FSL effect;**
- **More complete relativistic model**

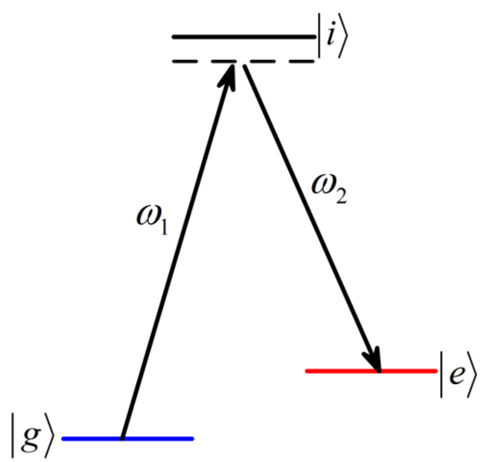


2. AG's Working principle

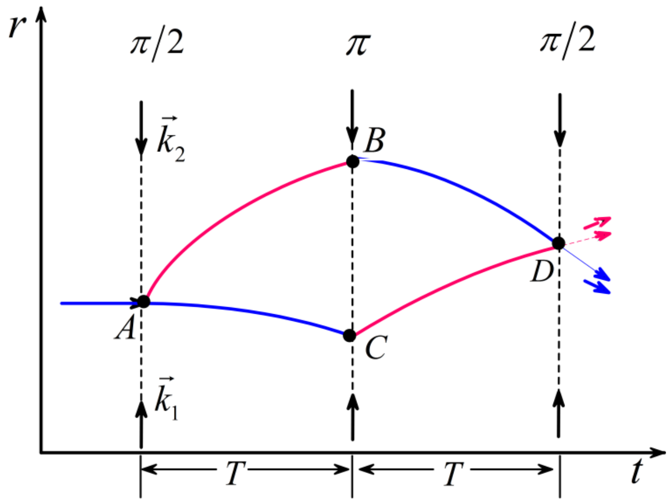
Three-Raman-laser-pulse atom gravimeter



Rabi oscillation



Split, reflect, recombine atomic wave packets



Interference phase → **gravitational acceleration**

3. An analytical study method

◆ Conventional calculating method[4]

Solve geodesic equation

$$\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=0$$

Trajectories of atoms and photons

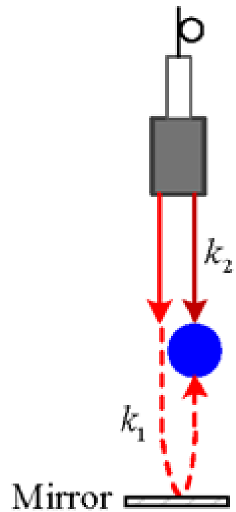
5 intersections

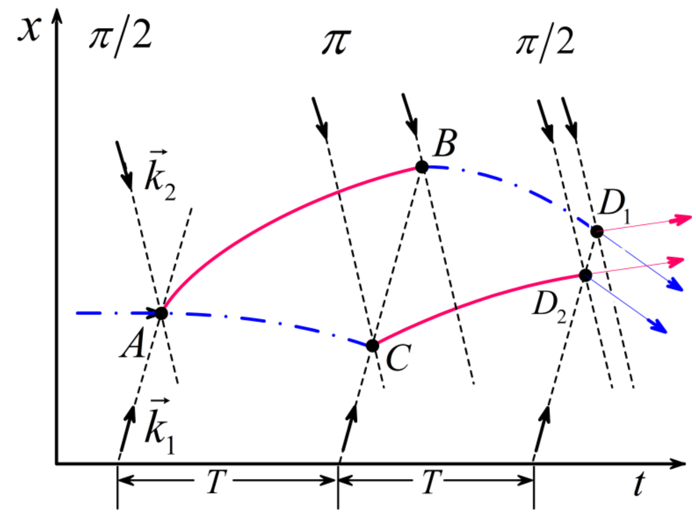
Path integral

Phase shift $\Delta\phi_{\text{tot}}=\Delta\phi_{\text{propagation}}+\Delta\phi_{\text{laser}}+\Delta\phi_{\text{separation}}$

$$\int_{t_A}^{t_B}L_{\text{upper}}dt-\int_{t_A}^{t_C}L_{\text{lower}}dt$$

Gravitational acceleration





$\vec{k}_1$ Light: control light

$\vec{k}_2$ Light: background light

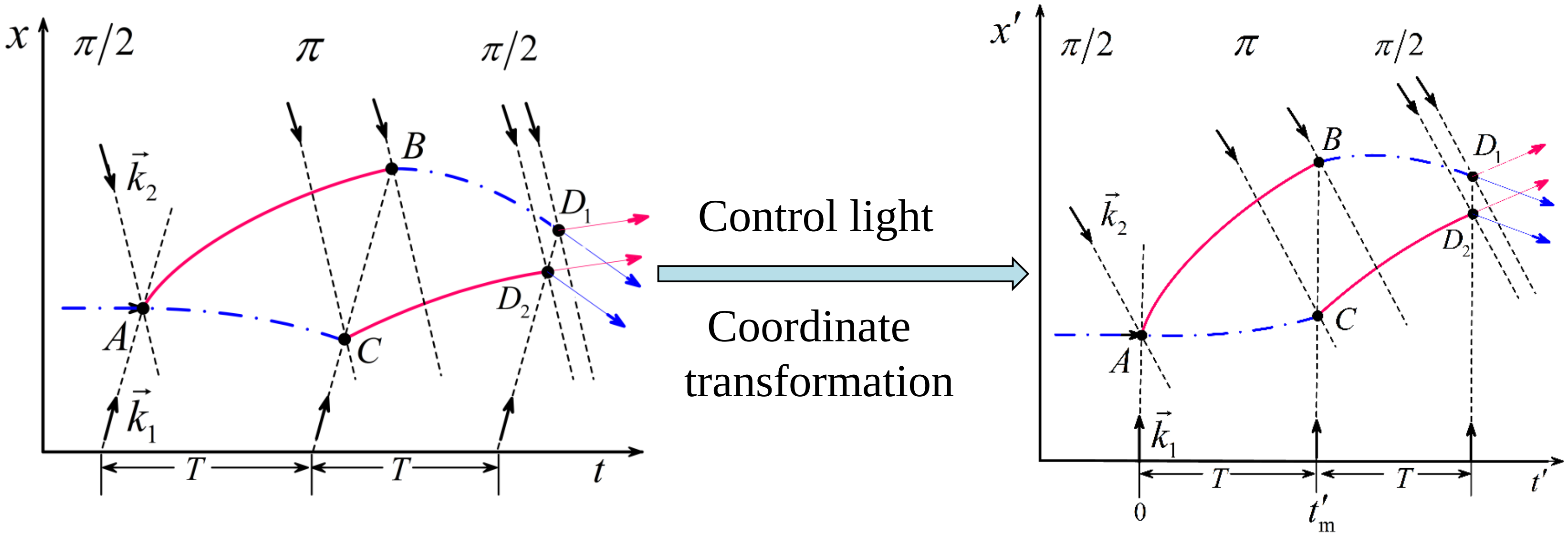
Refer the space-time diagram

[4] S. Dimopoulos *et al.*, PRD 78 (2008) 042003.

3. An analytical study method

◆ An analytical study method

Idea: transfer FSL effect into atom's Lagrangian



$$\int_{t_A}^{t_B} L_{\text{upper}}\,dt - \int_{t_A}^{t_C} L_{\text{lower}}\,dt \;\Rightarrow\; \int_0^{t'_m}\left(L'_{\text{upper}} - L'_{\text{lower}}\right)dt' \quad \text{simplify the calculation}$$

After transformation:

- Motion equation of light $\vec{k}_1$:

  $$dx'/dt' = \infty$$

- Lagrangian of atoms: $L' = L'_0 + L'_1$

  FSL disturbance (L'_1)

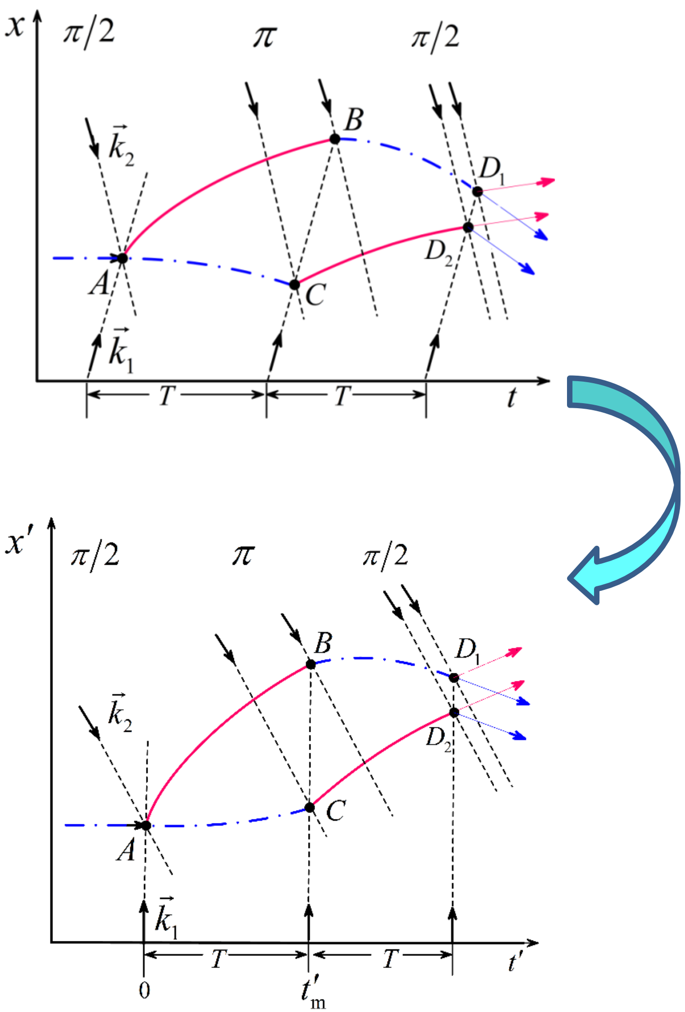
  Original form (L'_0)

3. An analytical study method

◆ Calculating the FSL effect

Calculation in Special Relativity frame

1. Coordinate transformation:



① Solve the motion equations of AG

photon: $t = \dfrac{\vec{n}\cdot\vec{x}}{c}$

atom: $L = \dfrac{1}{2} m\vec{v}^{2} + m\vec{g}\cdot\vec{r}$

② Coordinate transformation for $\vec{k}_1$

$$\begin{cases} \vec{x}' = \vec{x} \\ t' = t - \dfrac{\vec{n}\cdot\vec{x}}{c} \end{cases}$$

③ Motion equations of AG after transformation

Photon ($\vec{k}_1$) : $dx'/dt' = \infty$

atom: $L' = L_0' + L_1'$

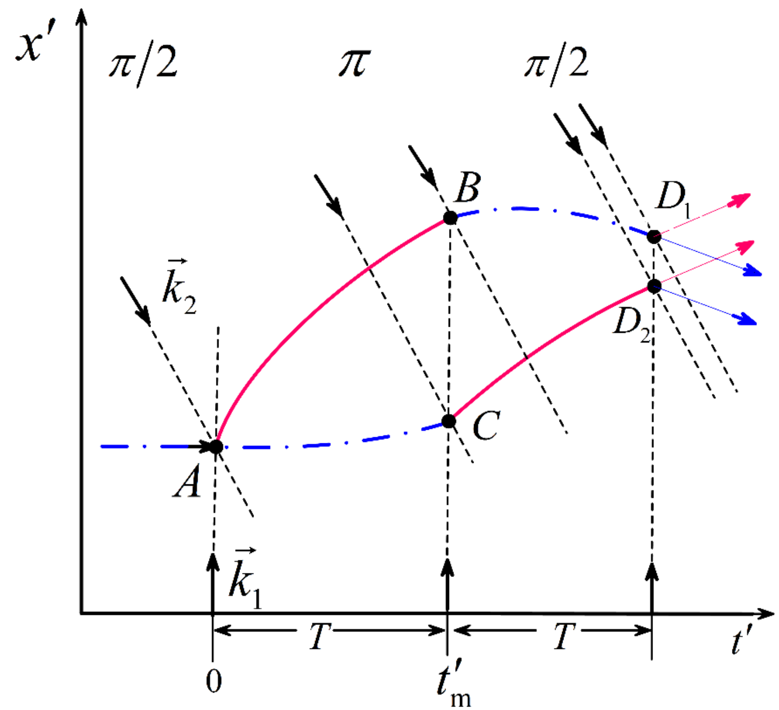
Original form

FSL disturbance

3. An analytical study method

◆ Calculating the FSL effect

2. Total phase shift：



ABCD Matrix, "Perturbation" approach, Laser phase

$$\Delta\phi_{tot} = \Delta\phi_{propagation} + \Delta\phi_{separation} + \Delta\phi_{laser} = \Delta\phi_{L'_0+separation} + \Delta\phi_{L'_1} + \Delta\phi_{laser}$$

◆ Calculating the FSL effect

3. FSL effect in AG：

Measured gravitational acceleration[5]:

$$g = g_0\left[1+3\frac{\vec{v}(T)\cdot\vec{e}_k}{c} - 2\frac{\vec{v}(T)\cdot\vec{e}_k}{c}\frac{\alpha_1-\alpha_2}{\vec{k}_{\text{eff}}\cdot\vec{g}_0} + \frac{2\vec{v}(T)}{c}\cdot\frac{\alpha_1\vec{n}_1-\alpha_2\vec{n}_2}{\vec{k}_{\text{eff}}\cdot\vec{g}_0}\right]$$

$$g_{\text{measured}}^{\text{Dimopoulos}} = g_0\left[1+3\frac{v(T)}{c}\right]$$

Time delay

missed

Coupling of time delay and frequency chirp

$$g_{\text{measured}}^{\text{peters}} = g_0\left[1+2\frac{v(T)}{c}\frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2}\right]$$

frequency chirp

[5] Yu-Jie Tan *et al*.. Finite-speed-of-light perturbation in atom gravimeters. **Physical Review A**, 2016, 94: 013612.

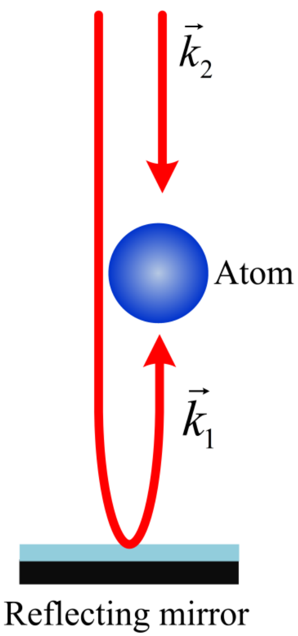
3. An analytical study method

◆ Calculating the FSL effect

$$g = g_0\left[1+\frac{\vec{v}(T)\cdot\vec{e}_k}{c}+\frac{2\vec{v}(T)}{c}\cdot\frac{\alpha_1\vec{n}_1-\alpha_2\vec{n}_2}{\vec{k}_{\text{eff}}\cdot\vec{g}_0}\right]=g_0(1+A+B)$$

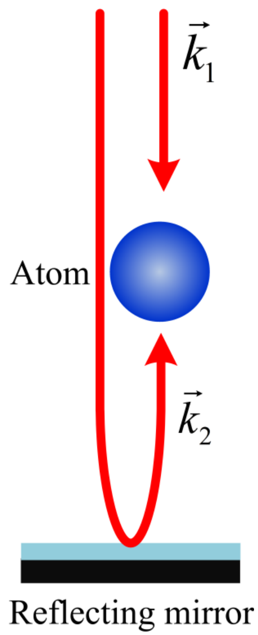
$\vec{e}_k$: direction of the control light (the light is reflected by the mirror)

$\vec{k}_{\text{eff}}$: $\vec{k}_1-\vec{k}_2$ effective wave vector



①

$\vec{e}_k \uparrow \quad \vec{k}_{\text{eff}} \uparrow$

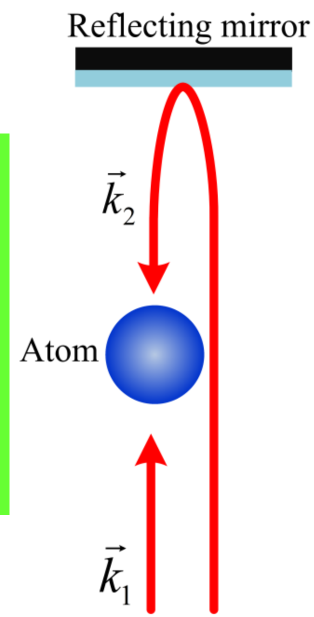


②

$\vec{e}_k \uparrow \quad \vec{k}_{\text{eff}} \downarrow$

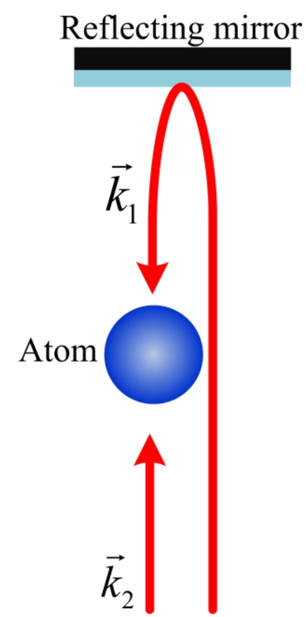
①② or ③④	B
①③ or ②④	A
①④ or ②③	A+B

Experimental design



③

$\vec{e}_k \downarrow \quad \vec{k}_{\text{eff}} \uparrow$



④

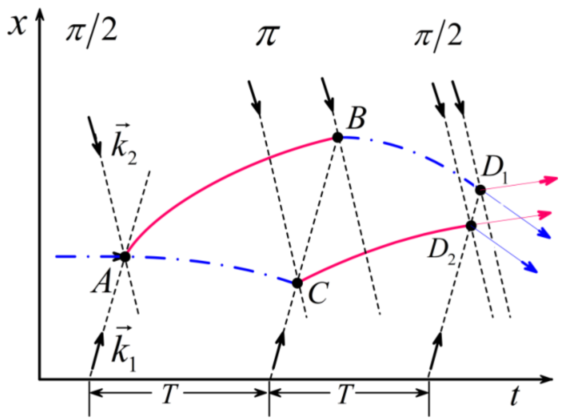
$\vec{e}_k \downarrow \quad \vec{k}_{\text{eff}} \downarrow$

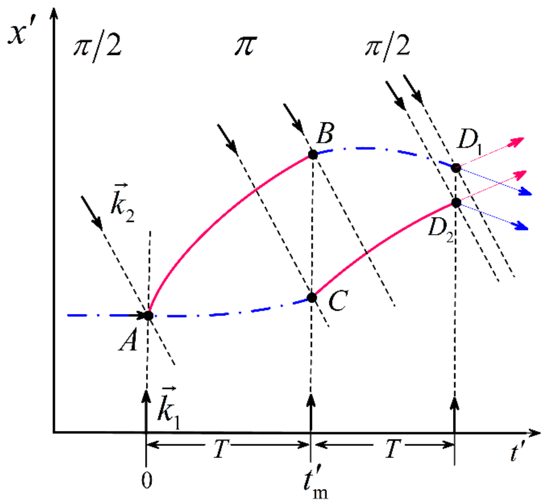
3. An analytical study method

◆ calculating the relativistic effects

Calculation in General Relativity frame

1. Coordinate transformation:





① Solve the motion equations of AG

photon: $t = f(\vec{x})$

atom: L

$$ds^2 = -\left[1+\frac{2\phi}{c^2}+\frac{2\beta\phi^2}{c^4}-\left(1-\frac{2\gamma\phi}{c^2}\right)\frac{\Omega^2 r^2\sin^2\theta}{c^2}\right](cdt)^2 +\left(1-\frac{2\gamma\phi}{c^2}\right)2\Omega r^2\sin^2\theta dt d\varphi_r +\left(1-\frac{2\gamma\phi}{c^2}\right)(dr^2+r^2 d\theta^2+r^2\sin^2\theta d\varphi_r^2)$$

② Coordinate transformation for $\vec{k}_1$

$$\begin{cases} t' = t - f(\vec{x}) \\ \vec{x}' = \vec{x} \end{cases}$$

③ Motion equations of AG after Coordinate transformation

Photon ($\vec{k}_1$) : $dx'/dt' = \infty$

atom: $L' = L'_0 + L'_1$

3. An analytical study method

◆ calculating the relativistic effects

2. Relativistic model for AG：

Relativistic phase shift[6] ：

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{Newtonian}} + \Delta\phi_{\text{relativistic}}^{\Omega-\text{independent}} + \Delta\phi_{\text{relativistic}}^{\Omega-\text{dependent}}$$

↓

Keep the sensitivity of gravitational acceleration within 10^-15 g

[6] Yu-Jie Tan *et al*.. Relativistic effects in atom gravimeters. **Physical Review D**, 2017, 95: 024002.

3. An analytical study method

TABLE I: A list of the non-relativistic phase shifts of the three-pulse interferometer, $\Delta\phi_{\rm classical}$. Column 2, vector expression of each term; column 3, a rough order estimate for the terms in column 2, and the relative sizes of each term to $k_{\rm eff}a_0T^2$ are given, with the mentioned design of sensitive to acceleration $\sim 10^{-15}a_0$, and the 2nd gradient $\partial_r\gamma_0 = 1.4\times 10^{-12}({\rm ms}^2)^{-1}$; column 4, the interpretation for the terms.

	Non-relativistic phase shift	Relative size		Interpretation
1	$-T^2\vec{k}_{\rm eff}\cdot\vec{a}_0$	1	1	Acceleration due to gravity
2	$-2T^2\vec{k}_{\rm eff}\cdot[(\vec{v}_0+\vec{a}_0T)\times\vec{\Omega}]$	$v_0\Omega/a_0$	$\sim 1\times 10^{-4}$	Coriolis acceleration
3	$-T^3\vec{k}_{\rm eff}\overleftrightarrow{\gamma}_0(\vec{v}_0+\frac{7T^4}{12}\vec{a}_0T)$	γ_0T^2	$\sim 5\times 10^{-6}$	1st gradient
4	$-3T^3\{(\vec{k}_{\rm eff}\cdot\vec{\Omega})[\vec{\Omega}\cdot(\vec{v}_0+\frac{7T^4}{12}\vec{a}_0T)]\}$	$(\Omega T)^2$	$\sim 9\times 10^{-9}$	
	$+3T^3\{\Omega^2[\vec{k}_{\rm eff}\cdot(\vec{v}_0+\frac{7T^4}{12}\vec{a}_0T)]\}$			
5	$-\frac{1}{6}T^4[(\vec{\Omega}\times\vec{k}_{\rm eff})\overleftrightarrow{\gamma}_0(7\vec{v}_0+3\vec{a}_0T)]$	$\gamma_0T^2\Omega T$	$\sim 5\times 10^{-10}$	Couple (1st gradient and earth's rotation)
	$-\frac{1}{6}T^4\{\vec{k}_{\rm eff}\overleftrightarrow{\gamma}_0[(7\vec{v}_0+3\vec{a}_0T)\times\vec{\Omega}]\}$			
6	$\frac{7}{12}T^4\nabla_i\nabla_j\nabla_k\phi_0(\vec{k}_{\rm eff})_i(\vec{v}_0)_j(\vec{v}_0)_k$	$(\partial_r\gamma_0)v_0T^3$	$\sim 4\times 10^{-11}$	2nd gradient
	$+\frac{3}{4}T^5\nabla_i\nabla_j\nabla_k\phi_0(\vec{a}_0)_i(\vec{v}_0)_j(\vec{k}_{\rm eff})_k$			
	$+\frac{31}{120}T^6\nabla_i\nabla_j\nabla_k\phi_0(\vec{a}_0)_i(\vec{a}_0)_j(\vec{k}_{\rm eff})_k$			
7	$-\frac{1}{360}T^5\vec{k}_{\rm eff}\overleftrightarrow{\gamma}_0^2(90\vec{v}_0+31\vec{a}_0T)$	$(\gamma_0T^2)^2$	$\sim 3\times 10^{-11}$	Nonlinearity of the 1st gradient
8	$\frac{1}{3}T^4\Omega^2\vec{k}_{\rm eff}\cdot[(7\vec{v}_0+3\vec{a}_0T)\times\vec{\Omega}]$	$(\Omega T)^3$	$\sim 9\times 10^{-13}$	
9	$-\frac{1}{120}T^5[(\vec{k}_{\rm eff}\cdot\vec{\Omega})\vec{\Omega}\overleftrightarrow{\gamma}_0(90\vec{v}_0+31\vec{a}_0T)]$	$\gamma_0T^2(\Omega T)^2$	$\sim 5\times 10^{-14}$	
	$+\frac{1}{60}T^5[\Omega^2\vec{k}_{\rm eff}\overleftrightarrow{\gamma}_0(90\vec{v}_0+31\vec{a}_0T)]$			
	$-\frac{1}{90}T^5\{(\vec{\Omega}\times\vec{k}_{\rm eff})\overleftrightarrow{\gamma}_0[(90\vec{v}_0+31\vec{a}_0T)\times\vec{\Omega}]\}$			
	$-\frac{1}{120}T^5\{(\vec{k}_{\rm eff}\overleftrightarrow{\gamma}_0\vec{\Omega})[\vec{\Omega}\cdot(90\vec{v}_0+31\vec{a}_0T)]\}$			

Have been given by others!

3. An analytical study method

TABLE II: A list of the earth rotation-independence relativistic phase shifts $\Delta\phi^{\Omega-\text{independent}}_{\text{relativistic}}$ of the three-pulse interferometer (with $\omega_1-\omega_2 = 6.8$ GHz).

	Relativistic phase shift	Relative size		Interpretation
1	$-[3\vec{k}_{\text{eff}}\cdot\vec{a}_0 + 4\alpha_2](v_n + a_n T)T^2/c$	v_0/c	$\sim 4\times10^{-8}$	Finite speed of light
2	$2\,(1-\beta)\,\vec{k}_{\text{eff}}\cdot\vec{a}_0\phi_0 T^2/c^2$	ϕ_0/c^2	$\sim 7\times10^{-10}$	Vanish for GR ($\beta=1$)
3	$(-2\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\vec{v}_0 v_n + k_{\text{eff}}\vec{v}_0\overleftrightarrow{\gamma}_0\vec{v}_0)T^3/c$	$\frac{\gamma_0 v_0 T}{a_0}\frac{v_0}{c}$	$\sim 2\times10^{-13}$	SR 1st gradient
4	$-\frac{7}{12}[(\vec{k}_{\text{eff}}\times\vec{n}_0\times\vec{a}_\perp)\overleftrightarrow{\gamma}_0\vec{v}_0 + (\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\times\vec{n}_0\times\vec{a}_\perp)\vec{v}_0]T^4/c$	$\gamma_0 T^2\frac{v_0}{c}$	$\sim 2\times10^{-13}$	SR 1st gradient
	$-\frac{7}{12}[\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\vec{a}_0 v_n + 2\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\vec{v}_0 a_n - 3k_{\text{eff}}\vec{a}_0\overleftrightarrow{\gamma}_0\vec{v}_0]T^4/c$			
5	$-\frac{1}{4}[(\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\times\vec{n}_0\times\vec{a}_\perp)\cdot\vec{a}_0 + (\vec{k}_{\text{eff}}\times\vec{n}_0\times\vec{a}_\perp)\overleftrightarrow{\gamma}_0\vec{a}_0]T^5/c$	$\gamma_0 T^2\frac{a_0 T}{c}$	$\sim 2\times10^{-13}$	GR 1st gradient
	$+\frac{3}{4}[k_{\text{eff}}\vec{a}_0\overleftrightarrow{\gamma}_0\vec{a}_0 - \vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0\vec{a}_0 a_n]T^5/c$			
6	$(\omega_1-\omega_2)\,[\vec{a}_\perp\cdot\vec{v}_\perp + 2a_n v_n]\,T^2/c^2$	$\frac{\omega_1-\omega_2}{\omega_1+\omega_2}\frac{v_0}{c}$	$\sim 6\times10^{-14}$	SR
7	$(\omega_1-\omega_2)({a_\perp}^2 + 2{a_n}^2)T^3/c^2$	$\frac{\omega_1-\omega_2}{\omega_1+\omega_2}\frac{a_0 T}{c}$	$\sim 6\times10^{-14}$	GR
8	$\frac{1}{3}\,(1-\beta)\,\vec{k}_{\text{eff}}\overleftrightarrow{\gamma}_0(6\vec{v}_0 + 7\vec{a}_0 T)\phi_0 T^3/c^2$	$\gamma_0 T^2\frac{\phi_0}{c^2}$	$\sim 4\times10^{-14}$	Vanish for GR ($\beta=1$)
9	$\left\{-(1+\gamma)\vec{k}_{\text{eff}}\cdot\vec{a}_n(v_n)^2 + (\vec{k}_{\text{eff}}\cdot\vec{a}_0)[(1-\gamma)(v_0)^2 + \frac{1}{2}(v_n)^2]\right\}T^2/c^2$	$(v/c)^2$	$\sim 2\times10^{-15}$	GR
	$+(\vec{k}_{\text{eff}}\cdot\vec{v}_0)\,\{[3+(2\gamma+1)]\vec{a}_0\cdot\vec{v}_0 + 2a_n v_n\}\,T^2/c^2$			
10	$-[\gamma(\vec{k}_{\text{eff}}\cdot\vec{a}_n)(\vec{a}_\perp - \gamma\vec{a}_n)\cdot\vec{v}_0 + 3(1+\gamma)(\vec{k}_{\text{eff}}\cdot\vec{a}_n)a_n v_n]T^3/c^2$	$a_0 v_0 T/c^2$	$\sim 2\times10^{-15}$	GR
	$+(\vec{k}_{\text{eff}}\cdot\vec{a}_0)\,[2(\gamma+\beta+3)\vec{a}_0\cdot\vec{v}_0 + 4a_n v_n]\,T^3/c^2$			
	$+(\vec{k}_{\text{eff}}\cdot\vec{v}_0)[(2\gamma+4)a_0^2 + 2{a_n}^2]T^3/c^2$			
11	$\frac{7}{12}\left[(2\beta+4\gamma+10)(\vec{k}_{\text{eff}}\cdot\vec{a}_0){a_0}^2 + 6(\vec{k}_{\text{eff}}\cdot\vec{a}_0){a_n}^2\right]T^4/c^2$	$(a_0 T/c)^2$	$\sim 2\times10^{-15}$	GR
	$-\frac{7}{12}\vec{k}_{\text{eff}}\cdot\vec{a}_n\left[\gamma({a_\perp}^2 - \gamma{a_n}^2) + 3(1+\gamma){a_n}^2\right]T^4/c^2$			

More general expression !

3. An analytical study method

TABLE III: A list of the earth rotation-dependence relativistic phase shifts $\Delta\phi^{\Omega-\text{dependent}}_{\text{relativistic}}$ of the three-pulse interferometer (with $\omega_1-\omega_2 = 6.8$ GHz).

	Relativistic phase shift	Relative size	Interpretation
1	$2\vec{k}_{\text{eff}}\cdot(\Omega\times\vec{v}_0)v_n T^2/c$	$\frac{\Omega v_0}{a_0}\frac{v_0}{c}\sim 4\times10^{-12}$	SR coriolis acceleration
2	$\vec{k}_{\text{eff}}\cdot\left\{(\vec{a}_\perp-\gamma\vec{a}_n)(\vec{n}_0\times\vec{\Omega})+(\vec{n}_0\times\vec{\Omega})(\vec{a}_\perp-\gamma\vec{a}_n)\right\}\cdot\vec{v}_0T^3/c$	$\Omega T\frac{v_0}{c}\sim 4\times10^{-12}$	SR earth's rotation
	$+[\vec{k}_{\text{eff}}\times\vec{n}_0\times\vec{a}_\perp-k_{\text{eff}}(\vec{a}_\perp-\gamma\vec{a}_n)]\cdot(\vec{\Omega}\times\vec{v}_0)T^3/c$		
	$+2[\vec{k}_{\text{eff}}\cdot\vec{\Omega}\times(\vec{a}_\perp-\gamma\vec{a}_n)]v_nT^3/c$		
3	$-\frac{7}{12}[\gamma(\vec{k}_{\text{eff}}\cdot\vec{a}_n)(\vec{n}_0\times\vec{\Omega})\cdot\vec{a}_0-2\vec{k}_{\text{eff}}\cdot\vec{\Omega}\times(\vec{a}_\perp-\gamma\vec{a}_n)a_n]T^4/c$	$\Omega T\frac{a_0T}{c}\sim 4\times10^{-12}$	GR earth's rotation
	$+\frac{7}{12}[\vec{k}_{\text{eff}}\times\vec{n}_0\times\vec{a}_\perp-k_{\text{eff}}(\vec{a}_\perp-\gamma\vec{a}_n)]\cdot(\vec{\Omega}\times\vec{a}_0)T^4/c$		
4	$[-(\gamma+1)\vec{k}_{\text{eff}}\cdot\vec{a}_0(\vec{\Omega}\times\vec{R}_e)^2+(\vec{k}_{\text{eff}}\cdot\vec{\Omega}\times\vec{R}_e)(\vec{\Omega}\times\vec{R}_e)\cdot\vec{a}_0]T^2/c^2$	$(\frac{\Omega R_e}{c})^2\sim 2\times10^{-12}$	SR earth's rotation
5	$-2\vec{k}_{\text{eff}}\cdot(\Omega\times\vec{v}_0)\phi_0T^2/c^2$	$\frac{\Omega v_0}{a_0}\frac{\phi_0}{c^2}\sim 7\times10^{-14}$	GR coriolis acceleration
6	$-4(1-\beta)\vec{k}_{\text{eff}}\cdot(\vec{\Omega}\times\vec{a}_0)\phi_0T^3/c^2$	$\Omega T\frac{\phi_0}{c^2}\sim 7\times10^{-14}$	Vanish for GR ($\beta=1$)
7	$-(2\gamma+1)\vec{k}_{\text{eff}}\times(\vec{\Omega}\times\vec{R}_e)\times\vec{a}_0\cdot\vec{v}_0T^2/c^2$	$\frac{\Omega R_e v_0}{c^2}\sim 7\times10^{-14}$	SR earth's rotation
	$+(\vec{k}_{\text{eff}}\times\vec{a}_0\times\vec{n}_0)\cdot\vec{v}_0-3(\vec{k}_{\text{eff}}\cdot\vec{a}_0)v_n(\vec{\Omega}\times\vec{R}_e\cdot\vec{n}_0)T^2/c^2$		
	$+\left\{[\vec{k}_{\text{eff}}\times(\vec{\Omega}\times\vec{R}_e)\times\vec{a}_0]\cdot\vec{v}_0+3(\vec{k}_{\text{eff}}\cdot\vec{a}_0)[\vec{v}_0\cdot(\vec{\Omega}\times\vec{R}_e)]\right\}T^2/c$		
8	$-(\vec{\Omega}\times\vec{R}_e\cdot\vec{n}_0)(\vec{k}_{\text{eff}}\times\vec{n}_0\times\vec{a}_\perp)\cdot\vec{a}_0T^3/c^2$	$\frac{\Omega R_e a_0 T}{c^2}\sim 7\times10^{-14}$	GR earth's rotation
	$+\left\{[\vec{k}_{\text{eff}}\times(\vec{\Omega}\times\vec{R}_e)\times\vec{a}_0]\cdot\vec{a}_0+3(\vec{k}_{\text{eff}}\cdot\vec{a}_0)(\vec{a}_0\cdot\vec{\Omega}\times\vec{R}_e)\right\}T^3/c^2$		
	$+[(\vec{k}_{\text{eff}}\times\vec{a}_0\times\vec{n}_0)\cdot\vec{a}_0-3(\vec{k}_{\text{eff}}\cdot\vec{a}_0)a_n](\vec{\Omega}\times\vec{R}_e\cdot\vec{n}_0)T^3/c^2$		
	$-\left\{2(\gamma+1)[\vec{k}_{\text{eff}}\times(\vec{\Omega}\times\vec{R}_e)\times\vec{a}_0]\cdot\vec{a}_0-3(\vec{k}_{\text{eff}}\cdot\vec{a}_0)[\vec{a}_0\cdot(\vec{\Omega}\times\vec{R}_e)]\right\}T^3/c^2$		

New results！

4. Conclusion

- Developed an analytical study method, in which a coordinate transformation is performed to simplify the calculation!

- Based on the method, we studied FSL effect in atom gravimeters, and further proposed a preliminary experimental design to test this effect.

$$g = g_0\left[1 + 3\frac{\vec{v}(T)\cdot\vec{e}_k}{c} - 2\frac{\vec{v}(T)\cdot\vec{e}_k}{c}\frac{\alpha_1-\alpha_2}{\vec{k}_{\text{eff}}\cdot\vec{g}_0} + \frac{2\vec{v}(T)}{c}\cdot\frac{\alpha_1\vec{n}_1-\alpha_2\vec{n}_2}{\vec{k}_{\text{eff}}\cdot\vec{g}_0}\right] + \ldots$$

- Extending the analysis on FSL effect, we established a more complete relativistic model for atom gravimeters!

Thanks for your attention!