

# CP violation in extended Higgs sectors

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(and ~~work in progress~~)

# Consider the potential of a 3HDM

$$V = Y_{ab} \Phi_a^\dagger \Phi_b + \frac{1}{2} Z_{abcd} (\Phi_a^\dagger \Phi_b) (\Phi_c^\dagger \Phi_d)$$

$a, b, c, d$  run over values 1,2,3

3 diagonal (real)  $Y$ 's:  $Y_{11}, Y_{22}, Y_{33}$

3 off-diagonal ones (complex)

lots of  $Z_{abcd}$ , some real, some complex

$$Z_{badc}^* = Z_{abcd}, \quad Z_{cdab} = Z_{abcd}, \quad Y_{ba}^* = Y_{ab}$$

All counted: 54 parameters

(not all independent)

# Consider the potential of a 3HDM

$$V = Y_{ab} \Phi_a^\dagger \Phi_b + \frac{1}{2} Z_{abcd} (\Phi_a^\dagger \Phi_b) (\Phi_c^\dagger \Phi_d)$$

All counted: 54 parameters

we may rotate:

$$\begin{pmatrix} \Phi'_1 \\ \Phi'_2 \\ \Phi'_3 \end{pmatrix} = U \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix}$$

1. diagonalize bilinear part (- 6 parameters)
2. remove 2 relative phases (- 2 parameters)

Remaining: **46 parameters** (linearly independent)

Olaussen et al, 2011 (general formula)

Compare 2HDM: **11 parameters**

# How do we identify CP violation?

If all coefficients and all vevs are real, then CP is conserved

First guess: otherwise violated

But phases of vevs may be modified by  
a phase rotation on the field.

The coefficients in the potential would pick up such phases.

The most general CP transformation allows  
(Branco, Lavoura, Silva, 1999)

$$\Phi_i \xrightarrow{\text{CP}} U_{ij} \Phi_j^*$$

$U$  unitary, arb.

# How do we identify CP violation?

Certain reparametrization invariants should vanish,  
for CP to be conserved

In the **2HDM**, conditions expressed in terms of **3 invariants**

Branco, Rebelo, Silva-Marcos, 2005

Gunion, Haber, 2005

in terms of  
potential parameters

These conditions can also be written in terms of  
masses and physical couplings.

Lavoura & Silva, 1994; Botella & Silva 1995

Grzadkowski et al, 2014

In the 3HDM, more invariants

# Spontaneous CP violation?

Assuming there is no explicit CP violation.  
Could there be spontaneous CPV?

Branco, Gerard, Grimus, 1984:

If a unitary matrix  $U$  exists, that is a symmetry of the Lagrangian,  
satisfying also

$$U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \Phi_i | 0 \rangle$$

then the vacuum is CP invariant

No spontaneous CP violation either

# 2HDM case

Suppose CPV is established

Is it explicit or spontaneous?

Gunion & Haber defined 4 invariants (in terms of potential parameters) that can be used to exclude spontaneous CPV

Equivalently (Grzadkowski et al, 2016):

1. Charged Higgs mass takes a particular value, in terms of neutral Higgs masses and couplings
2. Quartic Charged Higgs coupling takes a particular value, in terms of neutral Higgs masses and couplings

**If satisfied, CPV is spontaneous**

# Back to the NHDM

In some cases, finding (or excluding!)  
a transformation  $U$  satisfying

$$U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \Phi_i | 0 \rangle$$

may be difficult

## Other approach:

Transform to “the Higgs basis”:

Only one doublet has non-vanishing vev

illustrated for 3 doublets...



# $S_3$ symmetric 3HDM

Consider the potential  $V = V_2 + V_4$

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2),$$

$$\begin{aligned} V_4 = & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2. \end{aligned}$$

$h_S$       singlet under  $S_3$

$h_1, h_2$       doublet under  $S_3$

Term with  $\lambda_4$  plays  
a special role

16 different **complex** vacua with vevs

$$(w_1, w_2, w_S)$$

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	$w_1, w_2, w_S$	$\rho_1, \rho_2, \rho_3$
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm \frac{2\pi i}{3}}, xe^{\mp \frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x + iy, x - iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon \hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2 \sigma_1)}{1+9\tan^2 \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\pm \hat{w}_2 e^{-i \arctan(3 \tan \sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$
C-IV-c	$\sqrt{1 + 2 \cos^2 \sigma_2} \hat{w}_2,$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1 + 2 \cos^2 \rho)} + x,$ $re^{i\rho} - r\sqrt{3(1 + 2 \cos^2 \rho)} + x, -2re^{i\rho} + x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm \hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1} \xi + x, re^{i\rho_2} - re^{i\rho_1} \xi + x,$ $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2} \psi + x,$ $re^{i\rho_1} - re^{i\rho_2} \psi + x, -2re^{i\rho_1} + x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

**Table 2.** Complex vacua. Notation:  $\epsilon = 1$  and  $-1$  for C-III-d and C-III-e, respectively;  $\xi = \sqrt{-3 \sin 2\rho_1 / \sin 2\rho_2}$ ,  $\psi = \sqrt{[3 + 3 \cos(\rho_2 - 2\rho_1)] / (2 \cos \rho_2)}$ . With the constraints of table 4 the vacua labelled with an asterisk (\*) are in fact real.

## **Important:**

↑ 1 constraint

↑ 3 constraints

Each vacuum, C-I-a, C-III-a, etc, is accompanied by a set of constraints on the coefficients of the potential

These arise from the minimisation conditions,  
and their mutual consistency

# 1st example

Consider

	C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm \frac{2\pi i}{3}}, xe^{\mp \frac{2\pi i}{3}}$
	C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
	C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x + iy, x - iy, x$
	C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
	C-III-d,e	$\pm i\hat{w}_1, \epsilon \hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
	C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
	C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
	C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
	C-III-i	$\sqrt{\frac{3(1+\tan^2 \sigma_1)}{1+9\tan^2 \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\pm \hat{w}_2 e^{-i \arctan(3 \tan \sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
	C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
	C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, re^{i\rho} + re^{-i\rho} + x$
	C-IV-c	$\sqrt{1 + 2 \cos^2 \sigma_2} \hat{w}_2,$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1 + 2 \cos^2 \rho)} + x,$ $re^{i\rho} - r\sqrt{3(1 + 2 \cos^2 \rho)} + x, -2re^{i\rho} + x$
	C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm \hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
	C-IV-e	$\sqrt{\frac{\sin 2\sigma_2}{\sin 2\sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1} \xi + x, re^{i\rho_2} - re^{i\rho_1} \xi + x,$ $-2re^{i\rho_2} + x$
	C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2} \psi + x,$ $re^{i\rho_1} - re^{i\rho_2} \psi + x, -2re^{i\rho_1} + x$
	C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

requires

$$\lambda_4 = 0$$

**Table 2.** Complex vacua. Notation:  $\epsilon = 1$  and  $-1$  for C-III-d and C-III-e, respectively;  $\xi = \sqrt{-3 \sin 2\rho_1 / \sin 2\rho_2}$ ,  $\psi = \sqrt{[3 + 3 \cos(\rho_2 - 2\rho_1)] / (2 \cos \rho_2)}$ . With the constraints of table 4 the vacua labelled with an asterisk (\*) are in fact real.

# 1st example

Consider C-III-c  $(w_1, w_2, w_S) = (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$

Real potential, complex vevs

Is CP violated (spontaneously) or not?

It is **not** violated

But there is no simple way to find the matrix U

Go to the **Higgs basis**:

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_S \end{pmatrix} = \frac{1}{v} \begin{pmatrix} \hat{w}_1 & \hat{w}_2 & 0 \\ \hat{w}_2 & -\hat{w}_1 & 0 \\ 0 & 0 & v \end{pmatrix} \begin{pmatrix} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix}$$

$$v^2 = (\hat{w}_1^2 + \hat{w}_2^2) \quad \text{normalisation}$$

The coefficients of the potential remain real

**So CP is conserved** (check via invariants)

# 1st example

Construct the matrix  $U$  in this form:

$$U = e^{i(\delta_1 + \delta_2)} \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Choose  $\theta$  such that vevs rotate into

$(ae^{i\delta_1}, ae^{i\delta_2}, 0)$  same modulus!

A further rotation by  $\exp[-i(\delta_1 + \delta_2)/2]$

and we obtain the vevs in the form

$$(ae^{i\delta}, ae^{-i\delta}, 0)$$

Under a CP transformation (complex conjugation, accompanied by  $h_1 \leftrightarrow h_2$ ) the potential is invariant!

↑  
symmetry since  $\lambda_4 = 0$

# 2nd example

Consider

C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm \frac{2\pi i}{3}}, xe^{\mp \frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
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C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2 \sigma_1)}{1+9\tan^2 \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\pm\hat{w}_2 e^{-i \arctan(3 \tan \sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
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C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm\hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{\frac{-\sin 2\sigma_2}{\sin 2\sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1} \xi + x, re^{i\rho_2} - re^{i\rho_1} \xi + x,$ $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2} \psi + x,$ $re^{i\rho_1} - re^{i\rho_2} \psi + x, -2re^{i\rho_1} + x$
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**Table 2.** Complex vacua. Notation:  $\epsilon = 1$  and  $-1$  for C-III-d and C-III-e, respectively;  $\xi = \sqrt{-3 \sin 2\rho_1 / \sin 2\rho_2}$ ,  $\psi = \sqrt{[3 + 3 \cos(\rho_2 - 2\rho_1)] / (2 \cos \rho_2)}$ . With the constraints of table 4 the vacua labelled with an asterisk (\*) are in fact real.

## 2nd example

Important difference wrt 1st example:  $\hat{w}_S \neq 0$

An overall phase rotation would make  $w_S$  complex

Transform to the Higgs basis:

$$\begin{pmatrix} h'_1 \\ h'_2 \\ h'_S \end{pmatrix} = \begin{pmatrix} \frac{1}{N_1}(\hat{w}_1 & \hat{w}_2 & \hat{w}_S) \\ \frac{1}{N_2}(\hat{w}_2 & -\hat{w}_1 & 0) \\ \frac{1}{N_3}(\hat{w}_1 & \hat{w}_2 & X) \end{pmatrix} \begin{pmatrix} e^{-i\sigma_1} & 0 & 0 \\ 0 & e^{-i\sigma_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_S \end{pmatrix}$$

$X$  is chosen to make lines 1 and 3 orthogonal

$N_1, N_2, N_3$  are normalisations



## 2nd example

This vacuum is more constrained (4 constraints)

$\hat{w}_1, \hat{w}_2$  are related

and rotating to  $(be^{i\gamma_1}, be^{i\gamma_2}, \hat{w}_S)$

it follows that  $\gamma_1 + \gamma_2 = 0$

Again by the  $h_1 \leftrightarrow h_2$  symmetry,  
we see that CP is conserved

Alternatively, in the Higgs basis, we can rotate the phases of the fields that have vanishing vevs and note that the potential is real, CP is conserved

# 3rd example, T. D. Lee potential

2HDM with **real** coefficients:

$$\begin{aligned} V(\phi) = & -\lambda_1 \phi_1^\dagger \phi_1 - \lambda_2 \phi_2^\dagger \phi_2 \\ & + A(\phi_1^\dagger \phi_1)^2 + B(\phi_2^\dagger \phi_2)^2 + C(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \bar{C}(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\ & + \frac{1}{2}[(\phi_1^\dagger \phi_2)(D\phi_1^\dagger \phi_2 + E\phi_1^\dagger \phi_1 + F\phi_2^\dagger \phi_2) + \text{h.c.}]. \end{aligned}$$

vevs:  $(\rho_1 e^{i\theta}, \rho_2)$

Higgs basis reached via transformation

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \frac{1}{v} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \begin{pmatrix} \rho_1 & \rho_2 \\ -\rho_2 & \rho_1 \end{pmatrix} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

with (normalisation)  $v^2 = \rho_1^2 + \rho_2^2$

## 3rd example, T. D. Lee potential

Transformation generates  $(\phi_1^\dagger \phi_2)$  and  $(\phi_2^\dagger \phi_1)$  terms proportional to

$$\pm \frac{(\lambda_1 - \lambda_2) \rho_1 \rho_2 \sin \chi}{v^2}$$

Even putting this to zero, coefficients of quartic terms would in general be complex.

**CP is violated (spontaneously)**

# Summary

- Powerful methods exist to check for CP conservation in multi-Higgs-doublet models
- Invariants can be related to physical couplings and masses
- Transforming to a Higgs basis offers a simple way to check for spontaneous CP violation
- The latter approach is useful when vevs are complex

# Summary

- Powerful methods exist to check for CP conservation in multi-Higgs-doublet models  
*see also talk by Ivo Medeiros Varzielas*
- Invariants can be related to physical couplings and masses
- Transforming to a Higgs basis offers a simple way to check for spontaneous CP violation
- The latter approach is useful when vevs are complex