Probing violations of CPT with B_d mesons

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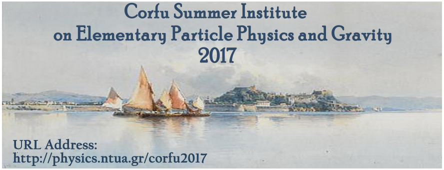

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J. Bernabéu & F.J. Botella (IFIC – U. Valencia)

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+ N. Mavromatos (K.C. London)

arXiv:1612.05652

Disclaimer

- Some (unavoidable) overlap with the (kaon) talk by

  A. di Domenico, but...
- *Never be first, try to be second*

  E. Fermi
- *Never underestimate the joy people derive from hearing something they already know*

  E. Fermi

Introduction

- Effective hamiltonian
- Entanglement
- Time evolution
- Double decay rates

Effective Hamiltonian

The effective hamiltonian ruling the B_d^0–$\bar{B}_d^0$ system is $\mathbf{H}$

$$\mathbf{H} = \mathbf{M} - i\mathbf{\Gamma}/2 \quad \text{with} \quad \mathbf{M}^\dagger = \mathbf{M},\ \mathbf{\Gamma}^\dagger = \mathbf{\Gamma}$$

Eigenvectors[1]:

$$\begin{aligned}
\mathbf{H}|B_H\rangle &= \mu_H|B_H\rangle, \qquad & |B_H\rangle &= p_H|B_d^0\rangle + q_H|\bar{B}_d^0\rangle, \\
\mathbf{H}|B_L\rangle &= \mu_L|B_L\rangle, \qquad & |B_L\rangle &= p_L|B_d^0\rangle - q_L|\bar{B}_d^0\rangle.
\end{aligned}$$

Eigenvalues: $\mu_{H,L} = M_{H,L} - \frac{i}{2}\Gamma_{H,L}$

$$\mu = \mu_H + \mu_L \equiv M - \frac{i}{2}\Gamma, \quad \Delta\mu = \mu_H - \mu_L \equiv \Delta M - \frac{i}{2}\Delta\Gamma,$$

Weisskopf & Wigner, Z.Phys. 65 (1930)

Lee, Oehme & Yang, PR 106 (1957)

[1] N.B. "H" and "L" correspond to the "heavy" and "light" states respectively, and thus $\Delta M > 0$ while the sign of $\Delta\Gamma$ is not a matter of convention

Mixing parameters $\theta, q/p \in \mathbb{C}$:

$$\frac{q_H}{p_H} = \frac{q}{p}\sqrt{\frac{1+\theta}{1-\theta}}, \qquad \frac{q_L}{p_L} = \frac{q}{p}\sqrt{\frac{1-\theta}{1+\theta}}, \qquad \delta = \frac{1-|q/p|^2}{1+|q/p|^2}.$$

$$\theta = \frac{\mathbf{H}_{22}-\mathbf{H}_{11}}{\Delta\mu}, \quad \left(\frac{q}{p}\right)^2 = \frac{\mathbf{H}_{21}}{\mathbf{H}_{12}}.$$

- θ is CP and CPT violating,
- δ is CP and T violating.

Hamiltonian:

$$\mathbf{H} = \begin{pmatrix} \mu - \frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} \\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu + \frac{\Delta\mu}{2}\theta \end{pmatrix}.$$

Silva, PRD 62 (2000)

Entanglement

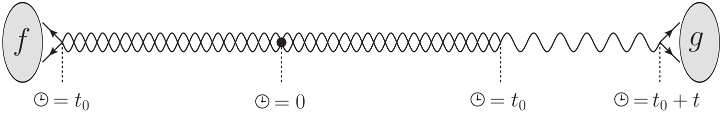
- $e^+e^- \to \Upsilon(4S) \to B\bar{B}$ production:

$$|\Psi_0\rangle = \Big(|B_d^0\rangle|\bar{B}_d^0\rangle - |\bar{B}_d^0\rangle|B_d^0\rangle\Big)/\sqrt{2}$$
$$= \Big(|B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle\Big)/\left[\sqrt{2}(p_Lq_H + p_Hq_L)\right]$$

- Amplitude for decay of the first state into $|f\rangle$ at time t_0, and of the second state into $|g\rangle$ at time $t + t_0 > t_0$:

$$\langle f, t_0; g, t{+}t_0|T|\Psi_0\rangle = \frac{e^{-i(\mu_H+\mu_L)t_0}}{\sqrt{2}(p_Lq_H + p_Hq_L)}\Big(e^{-i\mu_H t}\mathcal{A}_f^L\mathcal{A}_g^H - e^{-i\mu_L t}\mathcal{A}_f^H\mathcal{A}_g^L\Big)$$

with $\mathcal{A}_f^{H,L} \equiv \langle f|T|B_{H,L}\rangle$



- Double decay rate $I(f,g;t)$

$$I(f,g;t)=\int_0^\infty dt_0\ \left|\langle f,t_0;g,t+t_0|T|\Psi_0\rangle\right|^2=$$
$$\frac{e^{-\Gamma\, t}}{4\Gamma|p_Lq_H+p_Hq_L|^2}\times$$
$$\left|e^{i\Delta M\,t/2}e^{\Delta\Gamma\,t/4}\mathcal{A}_f^H\mathcal{A}_g^L-e^{-i\Delta M\,t/2}e^{-\Delta\Gamma\,t/4}\mathcal{A}_f^L\mathcal{A}_g^H\right|^2$$

- Expanded ($\Delta\Gamma=0$ and $\langle\Gamma_f\rangle=\frac{1}{2}(|A_f|^2+|\bar{A}_f|^2)$):

$$I(f,g;t)=e^{-\Gamma\,t}\frac{\langle\Gamma_f\rangle\langle\Gamma_g\rangle}{\Gamma}\times$$
$$\left\{\mathscr{C}_h[f,g]+\mathscr{C}_c[f,g]\cos(\Delta M\,t)+\mathscr{S}_c[f,g]\sin(\Delta M\,t)\right\},$$

Reduced intensity: $\hat{\mathrm{I}}(f,g;t)\equiv\Gamma\langle\Gamma_f\rangle^{-1}\langle\Gamma_g\rangle^{-1}I(f,g;t)$

- Combined transformation $t\mapsto -t$ and $f\leftrightarrows g$ implies

$$\mathscr{C}_h[f,g]=\mathscr{C}_h[g,f],\quad \mathscr{C}_c[f,g]=\mathscr{C}_c[g,f],\quad \mathscr{S}_c[f,g]=-\mathscr{S}_c[g,f]$$

- As usual

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad S_f \equiv \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}, \quad R_f \equiv \frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}$$

with $\langle f|T|B_d^0\rangle \equiv A_f$, $\langle f|T|\bar{B}_d^0\rangle \equiv \bar{A}_f$ (N.B. $C_f^2 + S_f^2 + R_f^2 = 1$)

- For flavour specific channels $f = \ell^\pm + X$ (“$f = \ell^\pm$”), with no wrong lepton charge sign decays (i.e. $\Delta F = \Delta Q$)

$$C_{\ell^\pm} = \pm 1, \quad R_{\ell^\pm} = S_{\ell^\pm} = 0$$

- Complete expressions, $N_{[\pm,g]} = \frac{1\pm\delta}{1-\delta C_g}$:

$$\mathscr{C}_h[\ell^\pm, g] = N_{[\pm,g]} \begin{Bmatrix} (1+|\theta|^2)(1\mp C_g) \pm 2\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right) R_g \\ +|1-\theta^2|(1\pm C_g) + 2\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right) S_g \end{Bmatrix}$$

$$\mathscr{C}_c[\ell^\pm, g] = N_{[\pm,g]} \begin{Bmatrix} (1-|\theta|^2)(1\mp C_g) \mp 2\mathrm{Re}\left(\theta^*\sqrt{1-\theta^2}\right) R_g \\ -|1-\theta^2|(1\pm C_g) - 2\mathrm{Im}\left(\theta^*\sqrt{1-\theta^2}\right) S_g \end{Bmatrix}$$

$$\mathscr{S}_c[\ell^\pm, g] = 2N_{[\pm,g]} \begin{Bmatrix} \mp\mathrm{Re}\left(\sqrt{1-\theta^2}\right) S_g + \mathrm{Im}(\theta)\,(\pm 1 - C_g) \\ +\mathrm{Im}\left(\sqrt{1-\theta^2}\right) R_g \end{Bmatrix}$$

Motion reversal and time reversal asymmetries

- Conditions for motion reversal asymmetries
- States
- Decays as *filters*
- Channels

- Original proposal by Bañuls & Bernabéu to observe T independent of CP

  PLB464 (1999), NPB590 (2000)

- Followed by Bernabéu, Martínez-Vidal & Villanueva

  JHEP08 (2012)

- Implemented by BaBar

  PRL109 (2012)

Three ingredients

1. Time reversal in the B_d^0–$\bar{B}_d^0$ Hilbert space
   - Reference transition $B_1 \to B_2(t)$ *among meson states* compared to $B_2 \to B_1(t)$
   - Probability $\mathrm{P}_{12}(t) = |\langle B_2|U(t,0)|B_1\rangle|^2$
   - Proposed T violating asymmetry $\mathrm{P}_{12}(t) - \mathrm{P}_{21}(t)$
2. Going beyond $B_{1,2} = B_d^0, \bar{B}_d^0$ ($\Rightarrow$ independent of CP)
   - Reference $B_d^0 \to B_+$ with a defined $CP = +$ decay channel $f_{CP=+}$
   - How to measure the reverse transition?
3. Importance of entangled nature of the initial state
   - To connect meson transitions to double decay rates
   - To identify the reverse transition:
     - *assume* that observing a $f_{CP=-}$ decay one *filters* a B_-
     - due to entanglement, one tags the orthogonal state in the opposite side, which should be B_+

Starting with the initial antisymmetric entangled state,

- if at time t_0 we observe a decay product f in one side, the still living meson state in the opposite side is

$$|B_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2 + |\bar{A}_f|^2}} \left(\bar{A}_f |B_d^0\rangle - A_f |\bar{B}_d^0\rangle\right)$$

- The orthogonal state $\langle B_{\nrightarrow f}^{\perp} | B_{\nrightarrow f}\rangle = 0$ is

$$|B_{\nrightarrow f}^{\perp}\rangle = \frac{1}{\sqrt{|A_f|^2 + |\bar{A}_f|^2}} \left(A_f^* |B_d^0\rangle + \bar{A}_f^* |\bar{B}_d^0\rangle\right)$$

This is the state filtered by a decay f

- The *filtering identity*

$$\left|\langle B_{\nrightarrow f}^{\perp} | B_1\rangle\right|^2 = \frac{\left|\langle f|T|B_1\rangle\right|^2}{|A_f|^2 + |\bar{A}_f|^2}$$

Bernabéu, Botella & MN, PLB728 (2014)

relates "probabilities to decay rates"

- For $B_1 = B_{\not\to g}(t)$, this is exactly the reduced intensity $\hat{\mathrm{I}}(g,f;t)$:

$$\hat{\mathrm{I}}(g,f;t) = \frac{\left|\langle f|T|B_{\not\to g}(t)\rangle\right|^2}{|A_f|^2 + |\bar{A}_f|^2} = \left|\langle B^{\perp}_{\not\to f}|B_{\not\to g}(t)\rangle\right|^2 .$$

This is the precise connection between
meson transition probabilities and double decay rates

- Measuring $\hat{\mathrm{I}}(f_1,f_2;t)$, "decays (f_1,f_2)", we access the transition probability $\mathrm{P}_{12}(t)$ between meson states $(B_1,B_2) = (B_{\not\to f_1}, B^{\perp}_{\not\to f_2})$
- To compare with the reverse $\mathrm{P}_{21}(t)$ we would need $(B^{\perp}_{\not\to f_2}, B_{\not\to f_1})$:
this is not what we access experimentally
- How do we bypass this?

- Reference: (f_1, f_2) gives transition probability for $(B_{\nrightarrow f_1}, B^{\perp}_{\nrightarrow f_2})$;
we now need $(B^{\perp}_{\nrightarrow f_2}, B_{\nrightarrow f_1})$
- Two new decay channels (f'_2, f'_1) give $(B_{\nrightarrow f'_2}, B^{\perp}_{\nrightarrow f'_1})$;
provided they fulfill

$$|B_{\nrightarrow f'_i}\rangle = |B^{\perp}_{\nrightarrow f_i}\rangle,$$

this new transition (f'_2, f'_1) gives the reversed meson transition
- For flavour specific decay channels, with no wrong lepton charge sign decays,

$$|B^0_d\rangle = |B_{\nrightarrow \ell^-}\rangle \quad \text{and} \quad |\bar{B}^0_d\rangle = |B_{\nrightarrow \ell^+}\rangle$$

- The identity is obviously $|\bar{B}^0_d\rangle = |(B^0_d)^{\perp}\rangle$:
if $f_1 = X\ell^+\nu_\ell$, then $f'_1 = X'\ell^-\bar{\nu}_\ell$

- For the CP decay channel the condition is

$$\lambda_{f_2}\lambda^*_{f_2'} = -\left|\frac{q}{p}\right|^2$$

the original proposal used $f_2 = J/\psi K_+$ and $f_2' = J/\psi K_-$.
From now on, K_S for K_+ and K_L for K_-, which is accurate up to CP violation in the kaon system.

- Considering that

$$\lambda_{J/\psi K_S} \equiv \lambda_{K_S} \sim \left|\frac{q}{p}\right| e^{-i2\beta} \text{ and } \lambda_{J/\psi K_L} \equiv \lambda_{K_L} \sim -\left|\frac{q}{p}\right| e^{-i2\beta}$$

we parameterise

$$\lambda_{K_S} = \left|\frac{q}{p}\right| \rho\,(1+\epsilon_\rho)\,e^{-i(2\beta+\epsilon_\beta)}\,, \quad \lambda_{K_L} = -\left|\frac{q}{p}\right| \frac{1}{\rho}\,(1+\epsilon_\rho)\,e^{-i(2\beta-\epsilon_\beta)}$$

with $\{\rho, \beta, \epsilon_\rho, \epsilon_\beta\}$ real *to control deviations from the requirement*

- Recapitulating: if $\epsilon_\rho = 0$ and $\epsilon_\beta = 0$, the considered channels allow to compare $B_1 \to B_2(t)$ with the reversed transition $B_2 \to B_1(t)$ (even if $\rho \neq 1$)
- At last, for that motion reversal asymmetry to be truly a time reversal asymmetry, one needs decay channels f such that, in the limit of T invariance, $S_f = 0$

Bernabéu, Botella & MN, PLB728 (2014)

- For CP eigenstates this amounts to no CP violation in the decay in the limit of T invariance, the additional condition is

$$\rho = 1$$

- Overall, deviations from

$$\epsilon_\rho = \epsilon_\beta = 0,\ \rho = 1$$

$$C_{K_S} = C_{K_L} = \delta\,,\ S_{K_S} + S_{K_L} = 0\,,\ R_{K_S} + R_{K_L} = 0$$

will be a source of fake T violation

N.B. In the absence of wrong flavour decays in $B_d^0 \to J/\psi K^0$ and $\bar{B}_d^0 \to J/\psi \bar{K}^0$

$$\lambda_{K_S} + \lambda_{K_L} = 0 \quad \Leftrightarrow \quad \rho = 1 \;\&\; \epsilon_\beta = 0$$

Grossman, Kagan & Ligeti, PLB538 (2002)

Observables, experiment & theory

- BaBar observables, independent asymmetries
- Genuine parameters
- Genuine vs. fake T-asymmetries

- Out of experimental convenience, BaBar fixed the normalization of the constant term and used the normalized decay intensity

$$\mathbf{g}_{f,g}(t) \propto e^{-\Gamma\, t}\,\{1 + \mathrm{C}[f,g]\cos(\Delta M\, t) + \mathrm{S}[f,g]\sin(\Delta M\, t)\}$$

BaBar, PRL 109 (2012)

- Two quantities,

$$\mathrm{C}[f,g] = \frac{\mathscr{C}_c[f,g]}{\mathscr{C}_h[f,g]}, \quad \mathrm{S}[f,g] = \frac{\mathscr{S}_c[f,g]}{\mathscr{C}_h[f,g]}$$

are measured for each pair (f, g)

- Notice that

$$\mathrm{C}[f,g] = \mathrm{C}[g,f], \quad \mathrm{S}[f,g] = -\mathrm{S}[g,f]$$

	Transition	$\mathbf{g}_{f,g}(t)$	$\mathbf{g}_{g,f}(t)$	Transition	
Ref.	$\bar{B}_d^0 \to B_-$	(ℓ^+, K_S)	(K_S, ℓ^+)	$B_+ \to B_d^0$	Ref.
T(Ref.)	$B_- \to \bar{B}_d^0$	(K_L, ℓ^-)	(ℓ^-, K_L)	$B_d^0 \to B_-$	T(Ref.)
CP(Ref.)	$B_d^0 \to B_-$	(ℓ^-, K_S)	(K_S, ℓ^-)	$B_+ \to \bar{B}_d^0$	CP(Ref.)
CPT(Ref.)	$B_- \to B_d^0$	(K_L, ℓ^+)	(ℓ^+, K_L)	$\bar{B}_d^0 \to B_+$	CPT(Ref.)

Decay channels, corresponding *filtered* meson *states* and transformed transitions

- 16 experimentally independent measurements
- theoretically only 8 are independent

- BaBar asymmetries

$$A_{\mathrm{T}}(t) = \mathbf{g}_{K_L,\ell^-}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$
$$A_{\mathrm{CP}}(t) = \mathbf{g}_{\ell^-,K_S}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$
$$A_{\mathrm{CPT}}(t) = \mathbf{g}_{K_L,\ell^+}(t) - \mathbf{g}_{\ell^+,K_S}(t)$$

$$A_{\mathrm{S}}(t) = e^{-\Gamma t}\left\{\Delta C_{\mathrm{S}}[\ell^+, K_S]\cos(\Delta M\, t) + \Delta S_{\mathrm{S}}[\ell^+, K_S]\sin(\Delta M\, t)\right\}$$
$$\mathrm{S} = \mathrm{T}, \mathrm{CP}, \mathrm{CPT}$$

where

$$\Delta C^+_{\mathrm{T}} \equiv \Delta C_{\mathrm{T}}[\ell^+, K_S] = \mathrm{C}[K_L, \ell^-] - \mathrm{C}[\ell^+, K_S]$$
$$\Delta C^+_{\mathrm{CP}} \equiv \Delta C_{\mathrm{CP}}[\ell^+, K_S] = \mathrm{C}[\ell^-, K_S] - \mathrm{C}[\ell^+, K_S]$$
$$\Delta C^+_{\mathrm{CPT}} \equiv \Delta C_{\mathrm{CPT}}[\ell^+, K_S] = \mathrm{C}[K_L, \ell^+] - \mathrm{C}[\ell^+, K_S]$$

$$\Delta S^+_{\mathrm{T}} \equiv \Delta S_{\mathrm{T}}[\ell^+, K_S] = \mathrm{S}[K_L, \ell^-] - \mathrm{S}[\ell^+, K_S]$$
$$\Delta S^+_{\mathrm{CP}} \equiv \Delta S_{\mathrm{CP}}[\ell^+, K_S] = \mathrm{S}[\ell^-, K_S] - \mathrm{S}[\ell^+, K_S]$$
$$\Delta S^+_{\mathrm{CPT}} \equiv \Delta S_{\mathrm{CPT}}[\ell^+, K_S] = \mathrm{S}[K_L, \ell^+] - \mathrm{S}[\ell^+, K_S]$$

To linear order in θ

$$\begin{aligned}
\Delta S_{\mathrm{T}}^{+} &\simeq S_{K_S} - S_{K_L} - \mathrm{Re}(\theta)\,(S_{K_S}R_{K_S} + S_{K_L}R_{K_L}) \\
&\qquad + \mathrm{Im}(\theta)\,(S_{K_S}^2 - S_{K_L}^2 + C_{K_S} + C_{K_L}) \\
\Delta S_{\mathrm{CP}}^{+} &\simeq 2S_{K_S} + 2\mathrm{Im}(\theta)\,(S_{K_S}^2 - 1) \\
\Delta S_{\mathrm{CPT}}^{+} &\simeq S_{K_L} + S_{K_S} - \mathrm{Re}(\theta)\,(S_{K_L}R_{K_L} + S_{K_S}R_{K_S}) \\
&\qquad + \mathrm{Im}(\theta)\,(-2 + S_{K_S}^2 + S_{K_L}^2 + C_{K_S} + C_{K_L}) \\
\Delta C_{\mathrm{T}}^{+} &\simeq C_{K_S} + C_{K_L} + \mathrm{Re}(\theta)\,(R_{K_S}(1 - C_{K_S}) + R_{K_L}(1 + C_{K_L})) \\
&\qquad + \mathrm{Im}(\theta)\,(S_{K_L}(1 + C_{K_L}) - S_{K_S}(1 - C_{K_S})) \\
\Delta C_{\mathrm{CP}}^{+} &\simeq 2C_{K_S} + 2\mathrm{Re}(\theta)\,R_{K_S} + 2\mathrm{Im}(\theta)\,S_{K_S}C_{K_S} \\
\Delta C_{\mathrm{CPT}}^{+} &\simeq C_{K_S} - C_{K_L} + \mathrm{Re}(\theta)\,(R_{K_S}(1 - C_{K_S}) - R_{K_L}(1 - C_{K_L})) \\
&\qquad + \mathrm{Im}(\theta)\,(S_{K_L}(1 - C_{K_L}) - S_{K_S}(1 - C_{K_S}))
\end{aligned}$$

Important: $\Delta S_{\mathrm{T}}^{+} \neq \Delta S_{\mathrm{CP}}^{+}$ & $\Delta C_{\mathrm{T}}^{+} \neq \Delta C_{\mathrm{CP}}^{+}$

Why

$$e^{-\Gamma t}\Big\{\mathscr{C}_h[f,g] + \mathscr{C}_c[f,g]\cos(\Delta M\, t) + \mathscr{S}_c[f,g]\sin(\Delta M\, t)\Big\}$$

vs.

$$e^{-\Gamma t}\Big\{1 + \mathrm{C}[f,g]\cos(\Delta M\, t) + \mathrm{S}[f,g]\sin(\Delta M\, t)\Big\}\ ?$$

- Although experimentally C & S are more appropriate, it is more desirable to access $\mathscr{C}_h$, $\mathscr{C}_c$, $\mathscr{S}_c$.

  For example
  - Asymmetry in ratios implies symmetry violation,
  - and yet no asymmetry in the ratios may still come from a symmetry violation

  Is it possible to go from data on C, S to all $\mathscr{C}_h$, $\mathscr{C}_c$, $\mathscr{S}_c$?

Yes, through

$$\mathscr{C}_h[\ell^\pm, K_{S,L}] + \mathscr{C}_c[\ell^\pm, K_{S,L}] = \frac{(1 \pm \delta)(1 \mp C_{K_{S,L}})}{2(1 - \delta C_{K_{S,L}})} = \mathscr{C}_h[\ell^\pm, K_{S,L}] \left(1 + \mathrm{C}[\ell^\pm, K_{S,L}]\right)$$

- $\mathrm{C}[\ell^\pm, K_{S,L}]$ and $C_{K_{S,L}}$ constrained/extracted from the BaBar data
- if we add information on δ,
  - compute $\mathscr{C}_h[\ell^\pm, K_{S,L}]$
  - compute then $\mathscr{C}_c[\ell^\pm, K_{S,L}]$ and from $\mathrm{S}[\ell^\pm, K_{S,L}]$, $\mathscr{S}_c[\ell^\pm, K_{S,L}]$

Genuine asymmetry parameters

$$\mathscr{A}_{\rm T}(t) = \hat{\rm I}(K_L, \ell^-; t) - \hat{\rm I}(\ell^+, K_S; t)$$
$$\mathscr{A}_{\rm CP}(t) = \hat{\rm I}(\ell^-, K_S; t) - \hat{\rm I}(\ell^+, K_S; t)$$
$$\mathscr{A}_{\rm CPT}(t) = \hat{\rm I}(K_L, \ell^+; t) - \hat{\rm I}(\ell^+, K_S; t)$$

which can also be expanded as

$$\mathscr{A}_{\rm S}(t) = e^{-\Gamma t}\left\{\Delta\mathscr{C}_h^{\rm S} + \Delta\mathscr{C}_c^{\rm S}\,\cos(\Delta M\,t) + \Delta\mathscr{S}_c^{\rm S}\,\sin(\Delta M\,t)\right\}$$
$$\mathrm{S} = \mathrm{T}, \mathrm{CP}, \mathrm{CPT}$$

To linear order in θ, δ

$$\mathscr{C}_h[\ell^\pm, g] = \frac{1}{2}\left\{1 + \delta(C_g \pm 1) \pm \mathrm{Re}(\theta)\,R_g - \mathrm{Im}(\theta)\,S_g\right\}$$
$$\mathscr{C}_c[\ell^\pm, g] = \frac{1}{2}\left\{\mp C_g + \delta C_g(\mp C_g - 1) \mp \mathrm{Re}(\theta)\,R_g + \mathrm{Im}(\theta)\,S_g\right\}$$
$$\mathscr{S}_c[\ell^\pm, g] = \frac{1}{2}\left\{\mp S_g + \delta S_g(\mp C_g - 1) + \mathrm{Im}(\theta)\,(\pm 1 - C_g)\right\}$$

Genuine asymmetry parameters in $\mathscr{C}_h$, up to linear order in θ and δ:

$$\Delta\mathscr{C}_h^{\mathrm{T}} \equiv \mathscr{C}_h[K_L,\ell^-] - \mathscr{C}_h[\ell^+,K_S] = \frac{1}{2}\Big\{\delta(C_{K_L} - C_{K_S} - 2) - \mathrm{Re}(\theta)\,(R_{K_L} + R_{K_S}) + \mathrm{Im}(\theta)\,(S_{K_S} - S_{K_L})\Big\}$$

$$\Delta\mathscr{C}_h^{\mathrm{CP}} \equiv \mathscr{C}_h[\ell^-,K_S] - \mathscr{C}_h[\ell^+,K_S] = -\Big\{\delta + \mathrm{Re}(\theta)\,R_{K_S}\Big\}$$

$$\Delta\mathscr{C}_h^{\mathrm{CPT}} \equiv \mathscr{C}_h[K_L,\ell^+] - \mathscr{C}_h[\ell^+,K_S] = \frac{1}{2}\Big\{\delta(C_{K_L} - C_{K_S}) + \mathrm{Re}(\theta)\,(R_{K_L} - R_{K_S}) + \mathrm{Im}(\theta)\,(S_{K_S} - S_{K_L})\Big\}$$

Genuine asymmetry parameters in $\mathscr{C}_c$, up to linear order in θ and δ:

$$\begin{aligned}
\Delta\mathscr{C}_c^{\mathrm{T}} &\equiv \mathscr{C}_c[K_L,\ell^-]-\mathscr{C}_c[\ell^+,K_S]= \\
&\frac{1}{2}\Big\{C_{K_S}+C_{K_L}+\delta(C_{K_L}(C_{K_L}-1)+C_{K_S}(C_{K_S}+1)) \\
&+\mathrm{Re}(\theta)\,(R_{K_S}+R_{K_L})+\mathrm{Im}(\theta)\,(S_{K_L}-S_{K_S})\Big\} \\
\Delta\mathscr{C}_c^{\mathrm{CP}} &\equiv \mathscr{C}_c[\ell^-,K_S]-\mathscr{C}_c[\ell^+,K_S]=\Big\{C_{K_S}+\delta C_{K_S}^2+\mathrm{Re}(\theta)\,R_{K_S}\Big\} \\
\Delta\mathscr{C}_c^{\mathrm{CPT}} &\equiv \mathscr{C}_c[K_L,\ell^+]-\mathscr{C}_c[\ell^+,K_S]= \\
&\frac{1}{2}\Big\{C_{K_S}-C_{K_L}+\delta(C_{K_S}(C_{K_S}+1)-C_{K_L}(C_{K_L}+1)) \\
&+\mathrm{Re}(\theta)\,(R_{K_S}-R_{K_L})+\mathrm{Im}(\theta)\,(S_{K_L}-S_{K_S})\Big\}
\end{aligned}$$

Genuine asymmetry parameters in $\mathscr{S}_c$, up to linear order in θ and δ:

$$\Delta\mathscr{S}_c^{\mathrm{T}} \equiv \mathscr{S}_c[K_L,\ell^-] - \mathscr{S}_c[\ell^+,K_S] = \frac{1}{2}\Big\{S_{K_S} - S_{K_L} + \delta(S_{K_S}(1+C_{K_S}) + S_{K_L}(1-C_{K_L})) + \mathrm{Im}(\theta)\,(C_{K_S}+C_{K_L})\Big\}$$

$$\Delta\mathscr{S}_c^{\mathrm{CP}} \equiv \mathscr{S}_c[\ell^-,K_S] - \mathscr{S}_c[\ell^+,K_S] = \Big\{S_{K_S} + \delta S_{K_S}C_{K_S} - \mathrm{Im}(\theta)\Big\}$$

$$\Delta\mathscr{S}_c^{\mathrm{CPT}} \equiv \mathscr{S}_c[K_L,\ell^+] - \mathscr{S}_c[\ell^+,K_S] = \frac{1}{2}\Big\{S_{K_S} + S_{K_L} + \delta(S_{K_S}(C_{K_S}+1) + S_{K_L}(C_{K_L}+1)) - \mathrm{Im}(\theta)\,(2+C_{K_S}-C_{K_L})\Big\}$$

Interlude

- To reconstruct the genuine asymmetry parameters, we used

$$\hat{\mathrm{I}}(f,g;0) = \mathscr{C}_h[f,g] + \mathscr{C}_c[f,g]$$

- Has a straightforward physical interpretation, the reduced intensity prior to any time evolution is simply the overlap between $|B^{\perp}_{\not\to g}\rangle$ and $|B_{\not\to f}\rangle$:

$$\hat{\mathrm{I}}(f,g;0) = \left|\langle B^{\perp}_{\not\to g}|B_{\not\to f}\rangle\right|^2 = \frac{\left|\bar{A}_f A_g - A_f \bar{A}_g\right|^2}{(|A_f|^2 + |\bar{A}_f|^2)(|A_g|^2 + |\bar{A}_g|^2)}\,.$$

- Nicely, if the conditions for a Motion Reversal measurement are verified, $\mathscr{A}_{\mathrm{T}}(0) = 0$

Interlude

With

$$\begin{aligned}\mathscr{A}_{\mathrm{T}}(0) &= \hat{\mathrm{I}}(K_L,\ell^-;0) - \hat{\mathrm{I}}(\ell^+,K_S;0)\\ &= \left|\langle B^{\perp}_{\nrightarrow K_L}|B_{\nrightarrow \ell^-}\rangle\right|^2 - \left|\langle B^{\perp}_{\nrightarrow \ell^+}|B_{\nrightarrow K_S}\rangle\right|^2\\ &= \frac{|A_{K_L}|^2}{|A_{K_L}|^2+|\bar{A}_{K_L}|^2} - \frac{|\bar{A}_{K_S}|^2}{|A_{K_S}|^2+|\bar{A}_{K_S}|^2}\end{aligned}$$

we have

$$\mathscr{A}_{\mathrm{T}}(0) = 0 \Leftrightarrow \frac{|\bar{A}_{K_L}|^2}{|A_{K_L}|^2} = \frac{|A_{K_S}|^2}{|\bar{A}_{K_S}|^2}$$

$$\text{while } \lambda_{K_L}\lambda^*_{K_S} = -\left|\frac{q}{p}\right|^2 \Leftrightarrow \frac{\bar{A}_{K_L}}{A_{K_L}} = -\frac{A^*_{K_S}}{\bar{A}^*_{K_S}}$$

Consistent with the intuitive requirement that a genuine Motion Reversal asymmetry cannot be already present at $t=0$, i.e. in the absence of time evolution.

Genuine T-reverse and fake asymmetries

- As discussed, candidate T-asymmetries can be "contaminated", they can receive contributions not truly T-violating
- This occurs when $\lambda_{K_S}\lambda^*_{K_L} = -|q/p|^2$ is not fulfilled
- How can we disentangle *fake* effects from *true* violations in T and CPT asymmetries?

Take for example $\Delta\mathscr{S}_c^{\mathrm{T}}$:

- In terms of the parameters in the problem, $\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\epsilon_\rho,\epsilon_\beta,\delta,\theta)$
- $\Delta\mathscr{S}_c^{\mathrm{T}}$ would be a *true* T-violation asymmetry if

$$\epsilon_\rho=\epsilon_\beta=0 \text{ and } \rho=1$$

- Separate

$$\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\epsilon_\rho,\epsilon_\beta,\delta,\theta)=\left[\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\epsilon_\rho,\epsilon_\beta,\delta,\theta)-\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)\right] + \Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)$$

- Term $[\ldots]$ has exactly the desired properties for the *fake* contribution: independently of β, δ and θ, $[\ldots]=0$ when the conditions are fulfilled.
- The *true* T asymmetry is $\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)$
- In terms of $\{\delta,\rho,\beta,\epsilon_\rho,\epsilon_\beta\}$,

$$\left\{\begin{matrix}C_{K_S}\\ C_{K_L}\end{matrix}\right\}\to\delta,\ \left\{\begin{matrix}S_{K_S}\\ -S_{K_L}\end{matrix}\right\}\to-\sqrt{1-\delta^2}\sin2\beta,\ \left\{\begin{matrix}R_{K_S}\\ -R_{K_L}\end{matrix}\right\}\to\sqrt{1-\delta^2}\cos2\beta\,.$$

Results

- With everything now clear
  1. take experimental data (input)
  2. fit for the parameters $\{\rho, \beta, \epsilon_\rho, \epsilon_\beta, \mathrm{Re}(\theta)\,, \mathrm{Im}(\theta)\}$

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

TABLE II: Measured values of the $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ coefficients. The first uncertainty is statistical and the second systematic. The indices $\alpha = \ell^-, \ell^+$ and $\beta = K^0_S, K^0_L$ stand for reconstructed final states that identify the B meson as $\overline{B}^0$, B^0 and B_-, B_+, respectively.

Transition		Parameter	Result
$B_- \to \overline{B}^0$	$(J/\psi K^0_L, \ell^- X)$	$S^-_{\ell^-,K^0_L}$	$-0.83 \pm 0.11 \pm 0.06$
$B^0 \to B_-$	$(\ell^- X, c\bar{c}K^0_S)$	$S^+_{\ell^-,K^0_S}$	$-0.76 \pm 0.06 \pm 0.04$
$B_- \to B^0$	$(J/\psi K^0_L, \ell^+ X)$	$S^-_{\ell^+,K^0_L}$	$0.70 \pm 0.19 \pm 0.12$
$\overline{B}^0 \to B_-$	$(\ell^+ X, c\bar{c}K^0_S)$	$S^+_{\ell^+,K^0_S}$	$0.55 \pm 0.09 \pm 0.06$
$B^0 \to B_+$	$(\ell^- X, J/\psi K^0_L)$	$S^+_{\ell^-,K^0_L}$	$0.51 \pm 0.17 \pm 0.11$
$B_+ \to \overline{B}^0$	$(c\bar{c}K^0_S, \ell^- X)$	$S^-_{\ell^-,K^0_S}$	$0.67 \pm 0.10 \pm 0.08$
$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K^0_L)$	$S^+_{\ell^+,K^0_L}$	$-0.69 \pm 0.11 \pm 0.04$
$B_+ \to B^0$	$(c\bar{c}K^0_S, \ell^+ X)$	$S^-_{\ell^+,K^0_S}$	$-0.66 \pm 0.06 \pm 0.04$
$B_- \to \overline{B}^0$	$(J/\psi K^0_L, \ell^- X)$	$C^-_{\ell^-,K^0_L}$	$0.11 \pm 0.12 \pm 0.08$
$B^0 \to B_-$	$(\ell^- X, c\bar{c}K^0_S)$	$C^+_{\ell^-,K^0_S}$	$0.08 \pm 0.06 \pm 0.06$
$B_- \to B^0$	$(J/\psi K^0_L, \ell^+ X)$	$C^-_{\ell^+,K^0_L}$	$0.16 \pm 0.13 \pm 0.06$
$\overline{B}^0 \to B_-$	$(\ell^+ X, c\bar{c}K^0_S)$	$C^+_{\ell^+,K^0_S}$	$0.01 \pm 0.07 \pm 0.05$
$B^0 \to B_+$	$(\ell^- X, J/\psi K^0_L)$	$C^+_{\ell^-,K^0_L}$	$-0.01 \pm 0.13 \pm 0.08$
$B_+ \to \overline{B}^0$	$(c\bar{c}K^0_S, \ell^- X)$	$C^-_{\ell^-,K^0_S}$	$0.03 \pm 0.07 \pm 0.04$
$\overline{B}^0 \to B_+$	$(\ell^+ X, J/\psi K^0_L)$	$C^+_{\ell^+,K^0_L}$	$-0.02 \pm 0.11 \pm 0.08$
$B_+ \to B^0$	$(c\bar{c}K^0_S, \ell^+ X)$	$C^-_{\ell^+,K^0_S}$	$-0.05 \pm 0.06 \pm 0.03$

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

TABLE III: Statistical correlation coefficients for the vector of $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ measurements given in the same order as in Table II. Only lower off-diagonal terms are written, in %.

100															
0	100														
-14	0	100													
2	-6	0	100												
8	0	41	0	100											
0	18	0	38	0	100										
6	0	19	0	-7	0	100									
0	10	0	16	0	-9	1	100								
-45	0	38	-1	31	0	9	0	100							
0	-33	0	31	0	28	0	6	0	100						
27	0	-9	0	23	0	18	0	-14	0	100					
0	28	0	-14	0	23	0	18	1	-15	0	100				
15	0	21	0	-21	0	27	0	-16	0	22	0	100			
0	18	0	21	0	-18	0	29	0	-16	0	21	0	100		
1	0	25	0	31	0	-37	0	22	0	-15	0	-20	0	100	
0	7	0	23	0	31	0	-41	0	20	0	-17	0	-20	0	100

Input from BaBar, PRL 109 (2012) 211801 [arXiv:1207.5832]

TABLE IV: Systematic correlation coefficients for the vector of $(S^{\pm}_{\alpha,\beta}, C^{\pm}_{\alpha,\beta})$ measurements given in the same order as in Table II. Only lower off-diagonal terms are written, in %.

100															
6	100														
18	-14	100													
44	3	66	100												
16	-4	57	58	100											
37	-19	67	66	44	100										
-5	-5	10	8	-4	-3	100									
30	-19	57	59	10	58	6	100								
-28	-10	39	13	43	21	-8	-1	100							
42	-20	60	68	57	72	-6	47	30	100						
-31	0	23	17	20	8	11	6	58	18	100					
41	-27	70	66	46	64	0	71	32	81	20	100				
31	-16	63	63	39	67	-23	59	39	63	24	73	100			
1	-1	15	7	2	2	-31	5	23	7	5	18	49	100		
28	-23	73	72	52	61	-1	64	43	69	28	84	83	39	100	
-14	-13	12	-6	-34	11	2	34	23	0	31	17	26	15	15	100

Results – Fit summary

WWA Parameters			
$\text{Re}(\theta)$	$\pm(5.92 \pm 3.03)10^{-2}$	$\text{Im}(\theta)$	$(0.22 \pm 1.90)10^{-2}$
ρ	1.021 ± 0.032	β	0.380 ± 0.020
ϵ_ρ	-0.023 ± 0.013	ϵ_β	0.013 ± 0.040
S_{K_S}	-0.679 ± 0.022	R_{K_S}	$\pm(0.734 \pm 0.020)$
C_{K_S}	$(9.4 \pm 3.22)\,10^{-4}$		
$S_{K_S} + S_{K_L}$	$(1.9 \pm 4.5)10^{-2}$	$R_{K_S} + R_{K_L}$	$(-1.9 \pm 3.9)10^{-2}$
$C_{K_S} - C_{K_L}$	$(-4.3 \pm 6.0)10^{-2}$		

$$\left\{\begin{matrix} \mathrm{Re}(\theta) = \pm(5.92 \pm 3.03) \times 10^{-2} \\ \mathrm{Im}(\theta) = (0.22 \pm 1.90) \times 10^{-2} \end{matrix}\right\}$$

$$\text{and } \left\{\begin{matrix} \mathrm{Re}(\theta) = \pm(3.92 \pm 1.43) \times 10^{-2} \\ \mathrm{Im}(\theta) = (-0.22 \pm 1.64) \times 10^{-2} \end{matrix}\right\} \text{ with } \lambda_{K_S} + \lambda_{K_L} = 0,$$

Significant improvement on the uncertainty of $\mathrm{Re}(\theta)$ quoted by the Particle Data Group

$$\mathrm{Re}(\theta)_{\mathrm{PDG}} = \pm(1.9 \pm 3.7 \pm 3.3) \times 10^{-2}, \quad \mathrm{Im}(\theta)_{\mathrm{PDG}} = (-0.8 \pm 0.4) \times 10^{-2}.$$

BaBar, PRD 70 (2004), PRL 96 (2006); Belle, PRD 85 (2012)

With $\Delta\Gamma = 0$

$$\left\{\begin{matrix} \mathbf{M}_{22} - \mathbf{M}_{11} = \pm(2.0 \pm 1.0) \\ \mathbf{\Gamma}_{22} - \mathbf{\Gamma}_{11} = -0.1 \pm 1.3 \end{matrix}\right\} 10^{-5}\mathrm{eV}$$

$$\text{and } \left\{\begin{matrix} \mathbf{M}_{22} - \mathbf{M}_{11} = \pm(1.3 \pm 0.5) \\ \mathbf{\Gamma}_{22} - \mathbf{\Gamma}_{11} = 0.1 \pm 1.1 \end{matrix}\right\} 10^{-5}\mathrm{eV} \text{ with } \lambda_{K_S} + \lambda_{K_L} = 0.$$

Similar result: BaBar, PRD 94 (2016)

(BaBar's preprint `1605.04545`, our `1605.03925`)

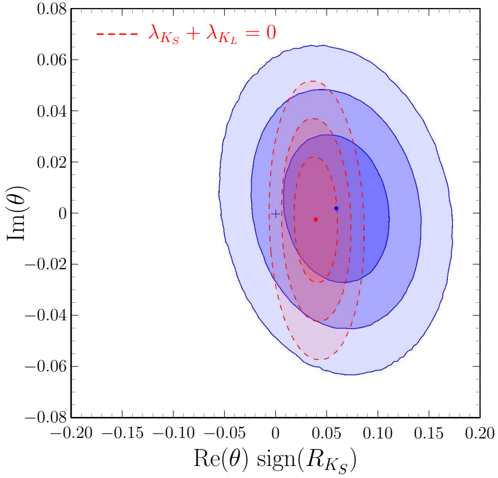
Results – Fit summary

BaBar Asymmetries			
ΔS^+_{T}	-1.317 ± 0.050	ΔS^+_{CP}	-1.360 ± 0.038
$\Delta S^+_{\mathrm{CPT}}$	$(7.6 \pm 4.8)10^{-2}$		
ΔC^+_{T}	$(4.7 \pm 3.7)10^{-2}$	ΔC^+_{CP}	$(8.9 \pm 3.2)10^{-2}$
$\Delta C^+_{\mathrm{CPT}}$	$(4.4 \pm 3.6)10^{-2}$		
Genuine T-reverse		Fake	
ΔS^+_{T} g.	-1.318 ± 0.050	ΔS^+_{T} f.	$(0.9 \pm 2.0)10^{-3}$
$\Delta S^+_{\mathrm{CPT}}$ g.	$(5.6 \pm 4.3)10^{-2}$	$\Delta S^+_{\mathrm{CPT}}$ f.	$(1.9 \pm 4.7)10^{-2}$
ΔC^+_{T} g.	$(0.2 \pm 2.5)10^{-2}$	ΔC^+_{T} f.	$(4.5 \pm 2.6)10^{-2}$
$\Delta C^+_{\mathrm{CPT}}$ g.	$(8.9 \pm 5.2)10^{-2}$	$\Delta C^+_{\mathrm{CPT}}$ f.	$(-4.5 \pm 6.2)10^{-2}$

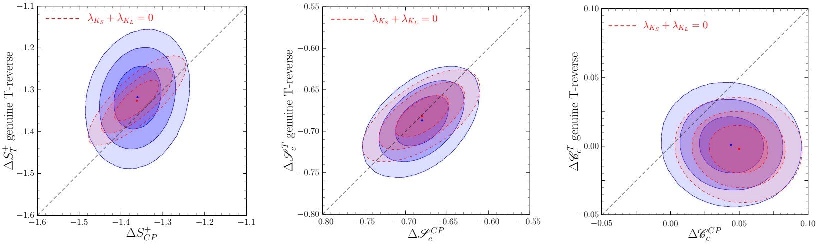
Results – Fit summary

Genuine Asymmetry Parameters			
$\Delta\mathscr{S}_c^{\mathrm{T}}$	-0.687 ± 0.020	$\Delta\mathscr{S}_c^{\mathrm{CP}}$	-0.680 ± 0.021
$\Delta\mathscr{S}_c^{\mathrm{CPT}}$	$(0.7 \pm 2.0)10^{-2}$		
$\Delta\mathscr{C}_c^{\mathrm{T}}$	$(2.4 \pm 2.0)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{CP}}$	$(4.4 \pm 1.6)10^{-2}$
$\Delta\mathscr{C}_c^{\mathrm{CPT}}$	$(2.3 \pm 1.8)10^{-2}$		
$\Delta\mathscr{C}_h^{\mathrm{T}}$	$(-0.2 \pm 1.4)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{CP}}$	$(-4.3 \pm 2.5)10^{-2}$
$\Delta\mathscr{C}_h^{\mathrm{CPT}}$	$(-4.4 \pm 2.6)10^{-2}$		
Genuine T-reverse		Fake	
$\Delta\mathscr{S}_c^{\mathrm{T}}$ g.	-0.687 ± 0.020	$\Delta\mathscr{S}_c^{\mathrm{T}}$ f.	$(0.4 \pm 1.2)10^{-3}$
$\Delta\mathscr{S}_c^{\mathrm{CPT}}$ g.	$(-0.2 \pm 1.9)10^{-2}$	$\Delta\mathscr{S}_c^{\mathrm{CPT}}$ f.	$(1.0 \pm 2.4)10^{-2}$
$\Delta\mathscr{C}_c^{\mathrm{T}}$ g.	$(0.1 \pm 1.4)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{T}}$ f.	$(2.3 \pm 1.3)10^{-2}$
$\Delta\mathscr{C}_c^{\mathrm{CPT}}$ g.	$(2.4 \pm 2.6)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{CPT}}$ f.	$(-2.1 \pm 3.2)10^{-2}$
$\Delta\mathscr{C}_h^{\mathrm{T}}$ g.	$(-0.1 \pm 1.4)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{T}}$ f.	$(-0.5 \pm 1.6)10^{-3}$
$\Delta\mathscr{C}_h^{\mathrm{CPT}}$ g.	$(-4.4 \pm 2.7)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{CPT}}$ f.	$(0.6 \pm 5.0)10^{-4}$

Results – Plots

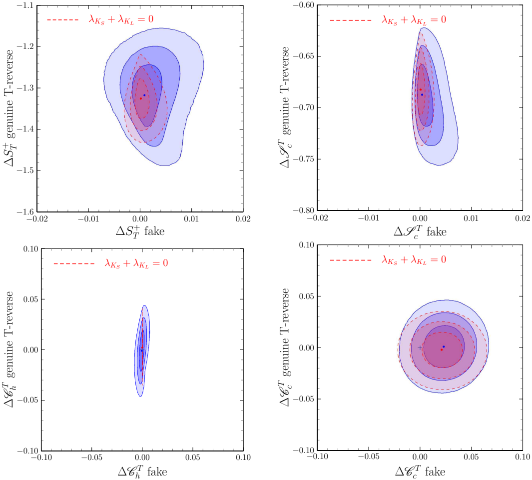


Results – Plots



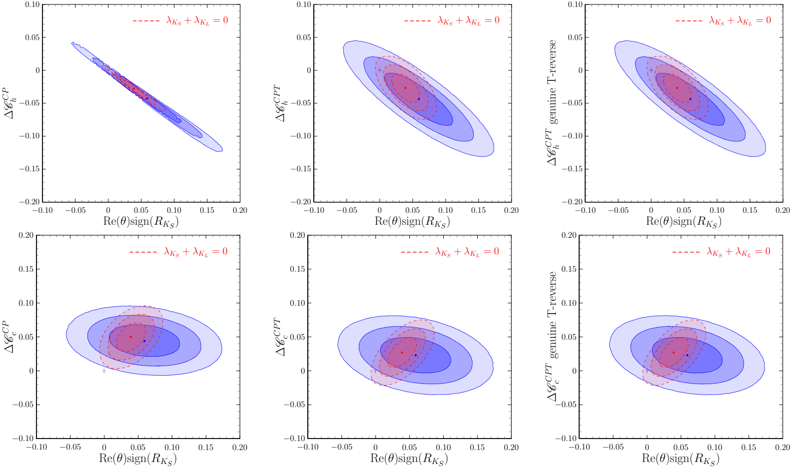
Genuine T-reverse vs. CP asymmetries

Results – Plots



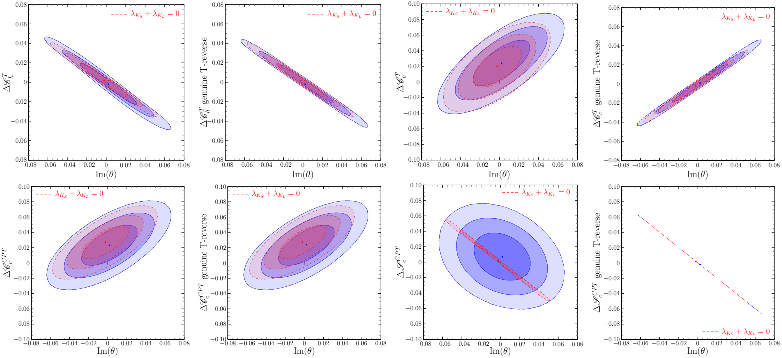
Genuine T-reverse vs. fake contributions

Results – Plots



Correlations with $\mathrm{Re}(\theta)\mathrm{sign}(R_{K_S})$.

Results – Plots



Correlations with $\text{Im}(\theta)$.

ω effect

- Initial state and time evolution
- Symmetry properties and sensitivity
- Analysis, results: robustness of bounds on θ, extraction of ω

- Ill-defined CPT operator implies modified initial state in $e^+e^- \to \Upsilon(4S) \to B\bar{B}$:

$$|\Psi_0'\rangle \propto \Big(|B_d^0\rangle|\bar{B}_d^0\rangle - |\bar{B}_d^0\rangle|B_d^0\rangle\Big) + \omega\Big(|B_d^0\rangle|\bar{B}_d^0\rangle + |\bar{B}_d^0\rangle|B_d^0\rangle\Big),$$

Bernabéu, Mavromatos & Papavassiliou, PRL 92 (2004)

$$|\Psi_0'\rangle \propto |B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle + \omega\theta\Big[|B_H\rangle|B_L\rangle + |B_L\rangle|B_H\rangle\Big] + \omega\Big\{(1-\theta)\frac{p_L}{p_H}|B_H\rangle|B_H\rangle - (1+\theta)\frac{p_H}{p_L}|B_L\rangle|B_L\rangle\Big\}.$$

One can expect

- Modified time dependence
- Terms in ω break the symmetry/antisymmetry properties of the coefficients obtained when the initial state is the entangled antisymmetric one

Time evolution with ω

Time evolution of two-meson flavour states:

$$\begin{pmatrix} |\mathrm{A}(t)\rangle \\ |B_d^0(t)\rangle|B_d^0(t)\rangle \\ |\mathrm{S}(t)\rangle \\ |\bar{B}_d^0(t)\rangle|\bar{B}_d^0(t)\rangle \end{pmatrix} = e^{-(\Gamma+i2M)\,t} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \mathrm{C}+\mathrm{E}_{[+]}e^{i\Delta\mu\,t}+\mathrm{E}_{[-]}e^{-i\Delta\mu\,t} & \\ 0 & & & \end{pmatrix} \begin{pmatrix} |\mathrm{A}\rangle \\ |B_d^0\rangle|B_d^0\rangle \\ |\mathrm{S}\rangle \\ |\bar{B}_d^0\rangle|\bar{B}_d^0\rangle \end{pmatrix}$$

with

$$|\mathrm{A}(t)\rangle = \frac{1}{\sqrt{2}}\left[|B_d^0(t)\rangle|\bar{B}_d^0(t)\rangle - |\bar{B}_d^0(t)\rangle|B_d^0(t)\rangle\right],$$
$$|\mathrm{S}(t)\rangle = \frac{1}{\sqrt{2}}\left[|B_d^0(t)\rangle|\bar{B}_d^0(t)\rangle + |\bar{B}_d^0(t)\rangle|B_d^0(t)\rangle\right],$$

and

$$|B_d^0(t)\rangle = e^{-i\mathbf{H}t}\,|B_d^0\rangle, \quad |\bar{B}_d^0(t)\rangle = e^{-i\mathbf{H}t}\,|\bar{B}_d^0\rangle.$$

where

$$\mathrm{C} = \begin{pmatrix} \frac{1}{2}(1-\theta^2) & \frac{1}{\sqrt{2}}\frac{q}{p}\theta\sqrt{1-\theta^2} & -\frac{1}{2}\frac{q^2}{p^2}(1-\theta^2) \\ \frac{1}{\sqrt{2}}\frac{p}{q}\theta\sqrt{1-\theta^2} & \theta^2 & -\frac{1}{\sqrt{2}}\frac{q}{p}\theta\sqrt{1-\theta^2} \\ -\frac{1}{2}\frac{p^2}{q^2}(1-\theta^2) & -\frac{1}{\sqrt{2}}\frac{p}{q}\theta\sqrt{1-\theta^2} & \frac{1}{2}(1-\theta^2) \end{pmatrix}$$

$$\mathrm{E}_{[+]} = \begin{pmatrix} \frac{1}{4}(1+\theta)^2 & -\frac{1}{2\sqrt{2}}\frac{q}{p}(1+\theta)\sqrt{1-\theta^2} & \frac{1}{4}\frac{q^2}{p^2}(1-\theta^2) \\ -\frac{1}{2\sqrt{2}}\frac{p}{q}(1+\theta)\sqrt{1-\theta^2} & \frac{1}{2}(1-\theta^2) & -\frac{1}{2\sqrt{2}}\frac{q}{p}(1-\theta)\sqrt{1-\theta^2} \\ \frac{1}{4}\frac{p^2}{q^2}(1-\theta^2) & -\frac{1}{2\sqrt{2}}\frac{p}{q}(1-\theta)\sqrt{1-\theta^2} & \frac{1}{4}(1-\theta)^2 \end{pmatrix}$$

$$\mathrm{E}_{[-]} = \begin{pmatrix} \frac{1}{4}(1-\theta)^2 & \frac{1}{2\sqrt{2}}\frac{q}{p}(1-\theta)\sqrt{1-\theta^2} & \frac{1}{4}\frac{q^2}{p^2}(1-\theta^2) \\ \frac{1}{2\sqrt{2}}\frac{p}{q}(1-\theta)\sqrt{1-\theta^2} & \frac{1}{2}(1-\theta^2) & \frac{1}{2\sqrt{2}}\frac{q}{p}(1+\theta)\sqrt{1-\theta^2} \\ \frac{1}{4}\frac{p^2}{q^2}(1-\theta^2) & \frac{1}{2\sqrt{2}}\frac{p}{q}(1+\theta)\sqrt{1-\theta^2} & \frac{1}{4}(1+\theta)^2 \end{pmatrix}$$

Double decay rate

$$I(f,g;t)=\int_0^\infty dt_0\,|\langle f,t_0;g,t+t_0|T|\Psi_0\rangle|^2$$

$$=\frac{\langle\Gamma_f\rangle\langle\Gamma_g\rangle}{\Gamma}e^{-\Gamma\,t}\left\{\mathscr{C}_h^\omega[f,g]+\mathscr{C}_c^\omega[f,g]\cos(\Delta M t)+\mathscr{S}_c^\omega[f,g]\sin(\Delta M t)\right\},$$

For $\omega\to 0$,

$$\mathscr{C}_h^\omega[f,g]\to\mathscr{C}_h[f,g],\ \mathscr{C}_c^\omega[f,g]\to\mathscr{C}_c[f,g],\ \mathscr{S}_c^\omega[f,g]\to\mathscr{S}_c[f,g],$$

with

$$\mathscr{C}_h[f,g]=\mathscr{C}_h[g,f],\ \mathscr{C}_c[f,g]=\mathscr{C}_c[g,f],\ \mathscr{S}_c[f,g]=-\mathscr{S}_c[g,f],$$

hence, sensitivity to $\omega\neq 0$ through

$$\mathscr{C}_h^\omega[f,g]-\mathscr{C}_h^\omega[g,f],\quad \mathscr{C}_c^\omega[f,g]-\mathscr{C}_c^\omega[g,f]\quad\text{and}\quad \mathscr{S}_c^\omega[f,g]+\mathscr{S}_c^\omega[g,f],$$

To linear order in θ, ω (N.B. $x = \frac{\Delta M}{\Gamma} \simeq 0.77$):

$$\mathscr{C}_h^\omega[f,g] = N_{[f,g]}\Big[1 - R_f R_g + \mathrm{Re}(\theta)\,(C_g R_f + C_f R_g) - \mathrm{Im}(\theta)\,(S_f + S_g) + \frac{1}{1+(x/2)^2}\big\{(2C_f + xS_f)\mathrm{Re}(\omega) + (xC_f - 2S_f)R_g\mathrm{Im}(\omega)\big\}\Big],$$

$$\mathscr{C}_c^\omega[f,g] = N_{[f,g]}\Big[-(C_f C_g + S_f S_g) - \mathrm{Re}(\theta)\,(C_g R_f + C_f R_g) + \mathrm{Im}(\theta)\,(S_f + S_g) + \frac{1}{1+(x/2)^2}\big\{-(2C_g + xS_g)\mathrm{Re}(\omega) + (-xC_g + 2S_g)R_f\mathrm{Im}(\omega)\big\}\Big],$$

$$\mathscr{S}_c^\omega[f,g] = N_{[f,g]}\Big[(C_g S_f - C_f S_g) + \mathrm{Re}(\theta)\,(R_g S_f - R_f S_g) + \mathrm{Im}(\theta)\,(C_f - C_g) + \frac{1}{1+(x/2)^2}\big\{(xC_g - 2S_g)\mathrm{Re}(\omega) - (2C_g + xS_g)R_f\mathrm{Im}(\omega)\big\}\Big].$$

[Flavour specific decays: $C_{\ell^\pm} = \pm 1$, $R_{\ell^\pm} = S_{\ell^\pm} = 0$]

Experimental observables of the form (BaBar)

$$\mathbf{g}_{f,g}(t) \propto e^{-\Gamma\, t}\left\{1 + \mathrm{C}[f,g]\cos(\Delta M\, t) + \mathrm{S}[f,g]\sin(\Delta M\, t)\right\},$$

as before

$$\mathrm{C}[f,g] = \frac{\mathscr{C}^{\omega}_{c}[f,g]}{\mathscr{C}^{\omega}_{h}[f,g]}\ ,\ \mathrm{S}[f,g] = \frac{\mathscr{S}^{\omega}_{c}[f,g]}{\mathscr{C}^{\omega}_{h}[f,g]},$$

$\cos(\Delta M\, t)$ terms

$$\mathrm{C}[\ell^\pm, f] = \mp C_f + \mathrm{Re}(\theta)\, R_f (C_f \mp 1) + \mathrm{Im}(\theta)\, S_f (1 \mp C_f) + \frac{1}{1+(x/2)^2}\left\{-x S_f \mathrm{Re}(\omega) + x C_f R_f \mathrm{Im}(\omega)\right\}$$

$$\mathrm{C}[f, \ell^\pm] = \mp C_f + \mathrm{Re}(\theta)\, R_f (C_f \mp 1) + \mathrm{Im}(\theta)\, S_f (1 \mp C_f) + \frac{1}{1+(x/2)^2}\left\{\pm(2(C_f^2 - 1) + x C_f S_f)\mathrm{Re}(\omega) \mp x R_f \mathrm{Im}(\omega)\right\}$$

$$\mathrm{C}[\ell^\pm, f] - \mathrm{C}[f, \ell^\pm] = \frac{1}{1+(x/2)^2}\left\{\left[x S_f \mp 2(C_f^2 - 1) \mp x C_f S_f\right] \mathrm{Re}(\omega) + x R_f \left[C_f \pm 1\right] \mathrm{Im}(\omega)\right\}$$

$\sin(\Delta M\, t)$ terms

$$\mathrm{S}[\ell^{\pm},f] = \mp S_f + \mathrm{Re}(\theta)\, S_f R_f + \mathrm{Im}(\theta)\,(\pm 1 - C_f \mp S_f^2) + \frac{1}{1+(x/2)^2}\left\{x C_f \mathrm{Re}(\omega) + x S_f R_f \mathrm{Im}(\omega)\right\}$$

$$\mathrm{S}[f,\ell^{\pm}] = \pm S_f - \mathrm{Re}(\theta)\, S_f R_f + \mathrm{Im}(\theta)\,(\mp 1 + C_f \pm S_f^2) + \frac{1}{1+(x/2)^2}\left\{\pm(x(1-S_f^2) - 2C_f S_f)\mathrm{Re}(\omega) \mp 2R_f \mathrm{Im}(\omega)\right\}$$

$$\mathrm{S}[\ell^{\pm},f] + \mathrm{S}[f,\ell^{\pm}] = \frac{1}{1+(x/2)^2}\left\{\left[x C_f \pm x(1-S_f^2) \mp 2C_f S_f\right]\mathrm{Re}(\omega) + R_f\left[x S_f \mp 2\right]\mathrm{Im}(\omega)\right\}$$

- Analysis
  1. go back to the previous analysis
  2. identify the appropriate modifications

  3. crank the gears again

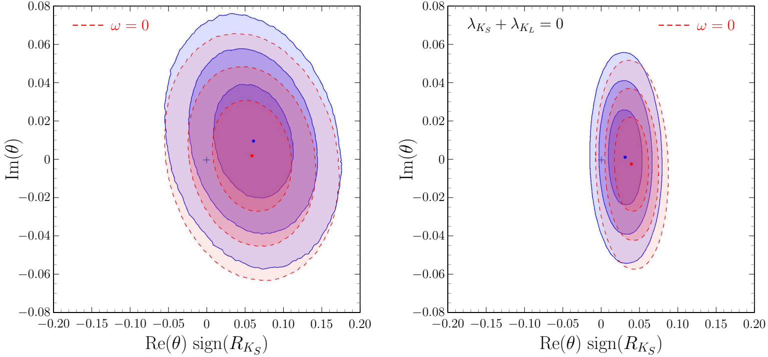
  4. now fitting for the parameters
     $\{\rho, \beta, \epsilon_\rho, \epsilon_\beta, \mathrm{Re}(\theta)\,, \mathrm{Im}(\theta)\,, \mathrm{Re}(\omega), \mathrm{Im}(\omega)\}$

Summary of results

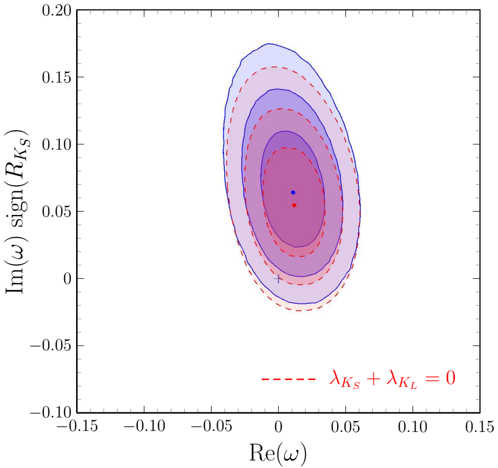
(I) Parameters – General analysis			
$\text{Re}(\theta)$	$\pm(6.11 \pm 3.45)10^{-2}$	$\text{Im}(\theta)$	$(0.99 \pm 1.98)10^{-2}$
$\text{Re}(\omega)$	$(1.09 \pm 1.60)10^{-2}$	$\text{Im}(\omega)$	$\pm(6.40 \pm 2.80)10^{-2}$
S_{K_S}	-0.624 ± 0.030	R_{K_S}	$\pm(0.781 \pm 0.024)$
C_{K_S}	$(-1.44 \pm 3.28)\,10^{-2}$		
$S_{K_S} + S_{K_L}$	$(3.7 \pm 4.9)10^{-2}$	$R_{K_S} + R_{K_L}$	$(-3.27 \pm 4.3)10^{-2}$
$C_{K_S} - C_{K_L}$	$(-6.8 \pm 6.3)10^{-2}$		
(II) Parameters – $\lambda_{K_S} + \lambda_{K_L} = 0$ analysis			
$\text{Re}(\theta)$	$\pm(3.10 \pm 1.51)10^{-2}$	$\text{Im}(\theta)$	$(0.14 \pm 1.67)10^{-2}$
$\text{Re}(\omega)$	$(1.17 \pm 1.59)10^{-2}$	$\text{Im}(\omega)$	$\pm(5.46 \pm 2.70)10^{-2}$
S_{K_S}	-0.640 ± 0.025	R_{K_S}	$\pm(0.769 \pm 0.022)$
C_{K_S}	$(1.61 \pm 1.88)10^{-2}$		

Summary of results

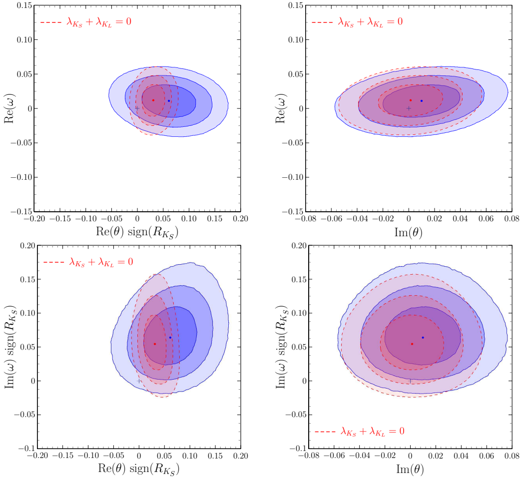
- No significant deviation from $\text{Re}(\omega) = 0$,
  analysis does not improve on $\text{Re}(\omega) = (0.8 \pm 4.6) \times 10^{-3}$
  Álvarez, Bernabéu & MN, JHEP (2006)
- Sensitivity to $\text{Im}(\omega)$ and 2.4σ hint of $\text{Im}(\omega) \neq 0$
- Extraction of θ quite robust
- Effect at the 1σ level on S_{K_S} (affect determination of UT β?)



Robustness of θ extraction



Extraction of ω



Correlations among ω and θ

Conclusions

- Detailed discussion on conditions for T,CPT asymmetries
  - Filtering + connection among meson transition probabilities and double decay rates
  - Conditions for Motion Reversal asymmetries
- BaBar observables vs. theory
- Extraction of genuine asymmetry parameters
- Identification of genuine T, CPT asymmetries
- Best existing limit on $\text{Re}(\theta)$ with intriguing 2σ effect, and $\text{Im}(\theta)$
- Analysis of static ω-effect with decays into flavour & CP eigenstates
  - Robustness of θ extraction
  - First & best limits on $\text{Im}(\omega)$, also intriguing 2σ effect

Thank you!

Backup

Results – Fit summary, $\lambda_{K_S} + \lambda_{K_L} = 0$, $\omega = 0$

WWA Parameters			
$\mathrm{Re}(\theta)$	$\pm(3.92 \pm 1.43)10^{-2}$	$\mathrm{Im}(\theta)$	$(-0.22 \pm 1.64)10^{-2}$
ϵ_ρ	-0.021 ± 0.013	β	0.375 ± 0.016
S_{K_S}	-0.682 ± 0.017	R_{K_S}	$\pm(0.731 \pm 0.016)$
C_{K_S}	$(2.10 \pm 1.31)10^{-2}$		

Results – Fit summary, $\lambda_{K_S} + \lambda_{K_L} = 0$, $\omega = 0$

BaBar Asymmetries			
ΔS^+_{T}	-1.326 ± 0.033	ΔS^+_{CP}	-1.362 ± 0.0358
$\Delta S^+_{\mathrm{CPT}}$	$(4.1 \pm 2.3)10^{-2}$		
ΔC^+_{T}	$(3.8 \pm 3.4)10^{-2}$	ΔC^+_{CP}	0.100 ± 0.029
$\Delta C^+_{\mathrm{CPT}}$	$(5.3 \pm 2.9)10^{-2}$		
Genuine T-reverse		Fake	
ΔS^+_{T} g.	-1.326 ± 0.033	ΔS^+_{T} f.	$(1.9 \pm {}^{+10.0}_{-7.5})10^{-4}$
$\Delta S^+_{\mathrm{CPT}}$ g.	$(4.1 \pm 2.3)10^{-2}$	$\Delta S^+_{\mathrm{CPT}}$ f.	$(-1.1 \pm 8.0)10^{-4}$
ΔC^+_{T} g.	$(0.4 \pm 2.2)10^{-2}$	ΔC^+_{T} f.	$(4.2 \pm 2.6)10^{-2}$
$\Delta C^+_{\mathrm{CPT}}$ g.	$(5.4 \pm 2.9)10^{-2}$	$\Delta C^+_{\mathrm{CPT}}$ f.	$(-1.2 \pm 1.0)10^{-3}$

Results – Fit summary, $\lambda_{K_S} + \lambda_{K_L} = 0$, $\omega = 0$

Genuine Asymmetry Parameters			
$\Delta\mathscr{S}_c^{\mathrm{T}}$	-0.682 ± 0.017	$\Delta\mathscr{S}_c^{\mathrm{CP}}$	-0.680 ± 0.022
$\Delta\mathscr{S}_c^{\mathrm{CPT}}$	$(0.2 \pm 1.6)10^{-2}$		
$\Delta\mathscr{C}_c^{\mathrm{T}}$	$(2.0 \pm 1.8)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{CP}}$	$(5.0 \pm 1.5)10^{-2}$
$\Delta\mathscr{C}_c^{\mathrm{CPT}}$	$(2.7 \pm 1.5)10^{-2}$		
$\Delta\mathscr{C}_h^{\mathrm{T}}$	$(0.2 \pm 1.2)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{CP}}$	$(-2.8 \pm 1.0)10^{-2}$
$\Delta\mathscr{C}_h^{\mathrm{CPT}}$	$(-2.7 \pm 1.5)10^{-2}$		
Genuine T-reverse		Fake	
$\Delta\mathscr{S}_c^{\mathrm{T}}$ g.	-0.682 ± 0.017	$\Delta\mathscr{S}_c^{\mathrm{T}}$ f.	$(1.1 \pm 5.1)10^{-4}$
$\Delta\mathscr{S}_c^{\mathrm{CPT}}$ g.	$(0.2 \pm 1.7)10^{-2}$	$\Delta\mathscr{S}_c^{\mathrm{CPT}}$ f.	$(-0.5 \pm 4.4)10^{-4}$
$\Delta\mathscr{C}_c^{\mathrm{T}}$ g.	$(-0.2 \pm 1.2)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{T}}$ f.	$(2.1 \pm 1.3)10^{-2}$
$\Delta\mathscr{C}_c^{\mathrm{CPT}}$ g.	$(2.7 \pm 1.5)10^{-2}$	$\Delta\mathscr{C}_c^{\mathrm{CPT}}$ f.	$(0.6 \pm 4.0)10^{-5}$
$\Delta\mathscr{C}_h^{\mathrm{T}}$ g.	$(0.2 \pm 1.2)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{T}}$ f.	$(3.3 \pm 4.0)10^{-5}$
$\Delta\mathscr{C}_h^{\mathrm{CPT}}$ g.	$(-2.7 \pm 1.5)10^{-2}$	$\Delta\mathscr{C}_h^{\mathrm{CPT}}$ f.	$(0.6 \pm 2.0)10^{-5}$