Cosmological implications of the group field theory approach to quantum gravity

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MdC,MS: Phys.Lett. B764 (2017) 49-53, ArXiv:1603.01764
MdC,AP,MS: Phys.Rev. D94 (2016) no.6, 064051, ArXiv:1606.00352
MdC,DO,AP,MS: ArXiv: 1709.00994

COST training school: "Quantum Spacetime and Physics Models"
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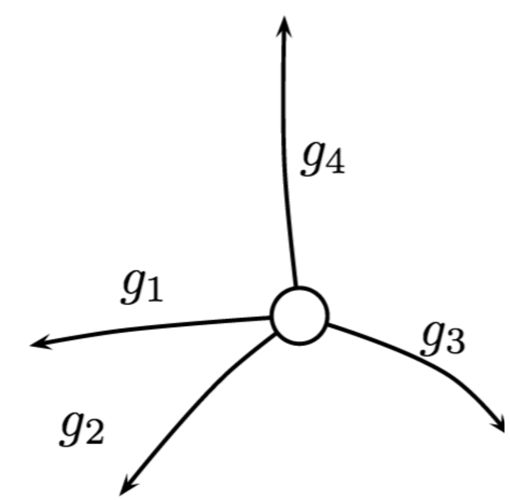
cost Action MP 1405
Quantum Structure of Spacetime

Plan of the Talk

- Emergent cosmology scenario in group field theory (GFT)
    - Some brief remarks on GFT
    - Effective Friedmann dynamics
- Cosmological implications of GFT models
    - No interactions between quanta
    - Interactions are allowed
    - Anisotropies in the EPRL model
- Conclusions

Group Field Theory (GFT)

- GFT is a non-perturbative and background independent approach to Quantum Gravity (Oriti ’11, ’13)

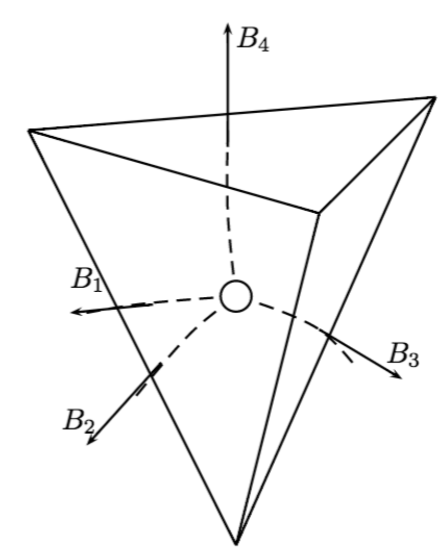


- Fundamental degrees of freedom are discrete objects carrying pre-geometric data (holonomies, fluxes)

- The dynamics of the GFT is obtained from a path-integral

$$\mathcal{Z} = \int \mathrm{e}^{-S[\varphi]} \qquad S = K + V$$

- The interaction potential contains the information about the gluing of quanta to form 4D geometric objects.



Figures: Gielen,Sindoni ‘16

Cosmology from GFT

- Quantum spacetime as a many-body system
- Spacetime is an emergent concept, determined by the collective dynamics of 'quanta' of geometry
- Cosmological background obtained by considering the dynamics of a condensates of such 'quanta' (Gielen,Oriti,Sindoni '13)
- Evolution is defined in a relational sense (matter clock ϕ)

Main advantages

- Cosmology can be obtained from the full theory
- The formalism naturally allows for a varying number of 'quanta'
- It may offer a way to go beyond standard cosmology

Cosmological Background

- mean field $\varphi = \varphi(g_1, g_2, g_3, g_4; \phi)$ $g_i \in \mathrm{SU}(2)$
- spin representation $\varphi^{j_1, j_2, j_3, j_4, \iota}(\phi)$
- Enforce isotropy (tetrahedra have equal faces)

$$\sigma_j(\phi) = \varphi^{j,j,j,j,\iota^\star}(\phi)$$

- Homogeneous and isotropic background can be studied by looking at the expectation value of the volume operator

$$V(\phi) = \sum_j V_j |\sigma_j(\phi)|^2 \qquad V_j \sim j^{3/2} \ell_{Pl}^3$$

Emergent Friedmann dynamics

(Oriti,Sindoni,Wilson-Ewing '16)

- The dynamics of σ_j yields effective equations for the evolution of V

$$\frac{\partial_\phi V}{V} = \frac{2\sum_j V_j \rho_j \partial_\phi \rho_j}{\sum_j V_j \rho_j^2} \qquad \rho_j = |\sigma_j|$$

- Simplification obtained when restricting to one spin j

$$\frac{\partial_\phi V}{V} = 2\frac{\partial_\phi \rho}{\rho} \qquad \frac{\partial_\phi^2 V}{V} = 2\left[\frac{\partial_\phi^2 \rho}{\rho} + \left(\frac{\partial_\phi \rho}{\rho}\right)^2\right]$$

- Last assumption is justified by results concerning the emergence of a low-spin phase (Gielen'16) (Pithis,Sakellariadou,Tomov'16)

Effective Friedmann equation

The non-interacting case

(Oriti,Sindoni,Wilson-Ewing '16)

(MdC,Sakellariadou '16)

- Single spin effective action (index j dropped)

$$S_{eff} = \int \mathrm{d}\phi \, \left(A \, |\partial_\phi \sigma|^2 + \mathcal{V}(\sigma) \right) \qquad \mathcal{V}(\sigma) = B|\sigma(\phi)|^2$$

- Global U(1) conserved charge Q, interpreted as the momentum canonically conjugated to ϕ

$$\pi_\phi = a^3 \dot{\phi}$$

- Polar decomposition of the mean field

$$\sigma = \rho \, \mathrm{e}^{i\theta} \qquad Q \equiv \rho^2 \partial_\phi \theta$$

- Equation of motion of the radial part

$$\partial_\phi^2 \rho - \frac{Q^2}{\rho^3} - \frac{B}{A}\rho = 0 \qquad E = (\partial_\phi \rho)^2 + \frac{Q^2}{\rho^2} - \frac{B}{A}\rho^2$$

Bouncing Universe

(MdC,Sakellariadou '16)

- The effective Friedmann equation can be recast in the form

$$H^2 = \frac{8\pi G_{eff}}{3}\varepsilon, \quad \varepsilon = \frac{\dot{\phi}^2}{2} \qquad (a = V^{1/3})$$

- The effective gravitational constant can be computed exactly in the free model

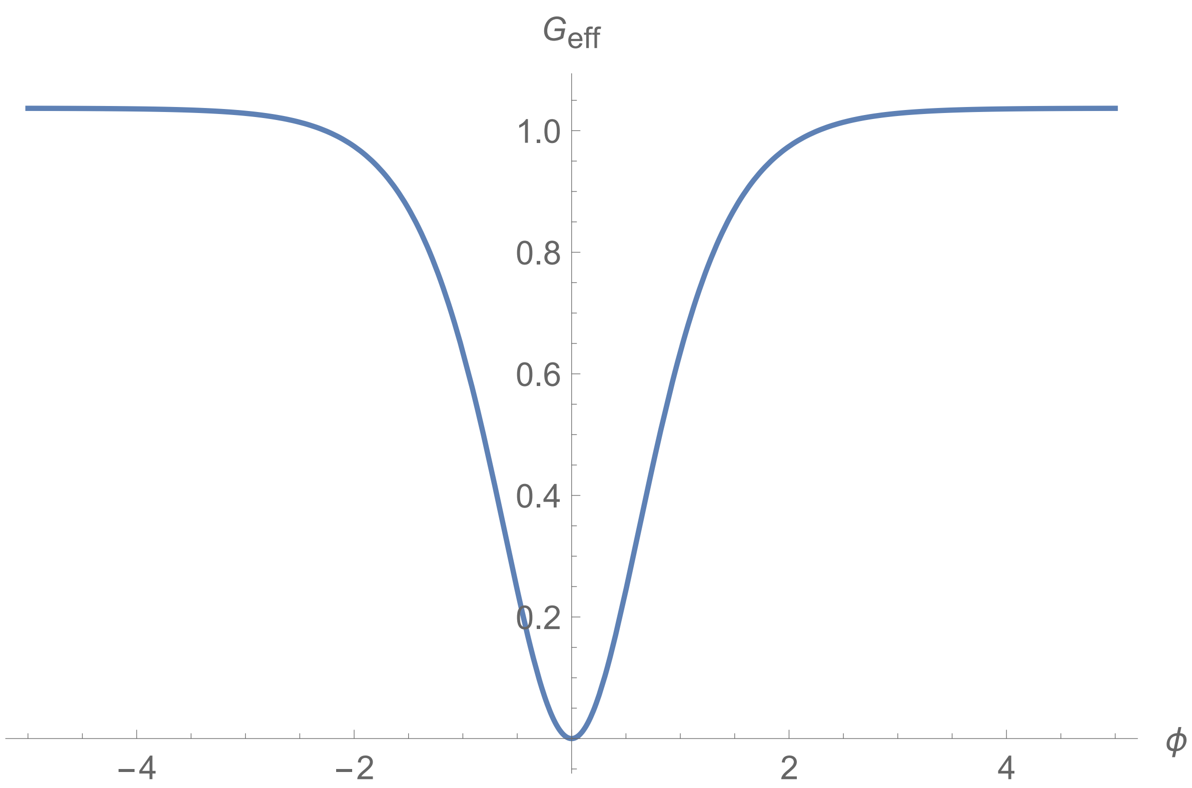
$$G_{eff}(\phi) = \frac{G\left(E^2 + 12\pi G Q^2\right)\sinh^2\left(2\sqrt{3\pi G}(\phi - \Phi)\right)}{\left(E - \sqrt{E^2 + 12\pi G Q^2}\cosh\left(2\sqrt{3\pi G}(\phi - \Phi)\right)\right)^2}.$$

- In the large volume limit $\phi \to \infty$ one recovers the ordinary Friedmann evolution

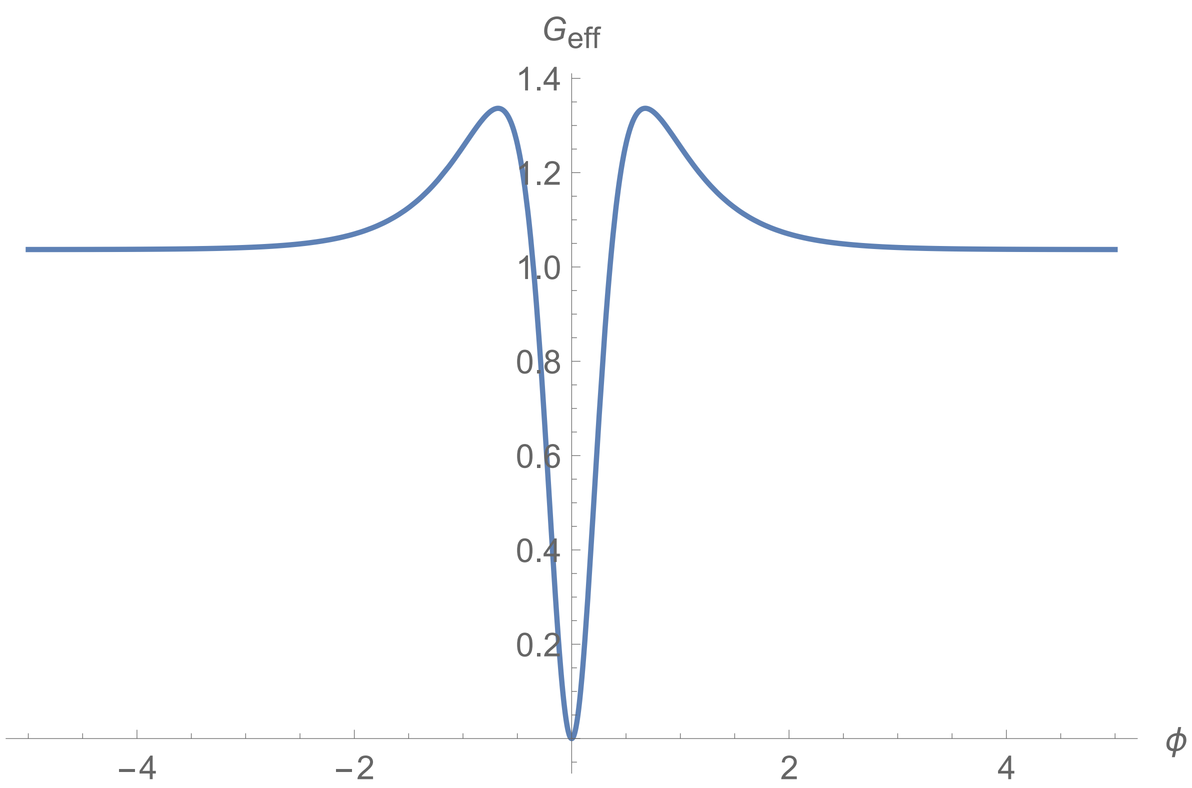
- There is a bounce when $H^2 = 0$. This takes place when

$$G_{eff} = 0 \quad (\text{at } \phi = \Phi)$$

Bouncing Universe



E<0



E>0

profile of G_{eff} for opposite signs of the conserved charge E.
The behaviour is generic for $Q \neq 0$

Accelerated expansion (geometric inflation)

- An early era of accelerated expansion is usually assumed in order to solve the classic cosmological puzzles.

- We seek a relational definition of the acceleration

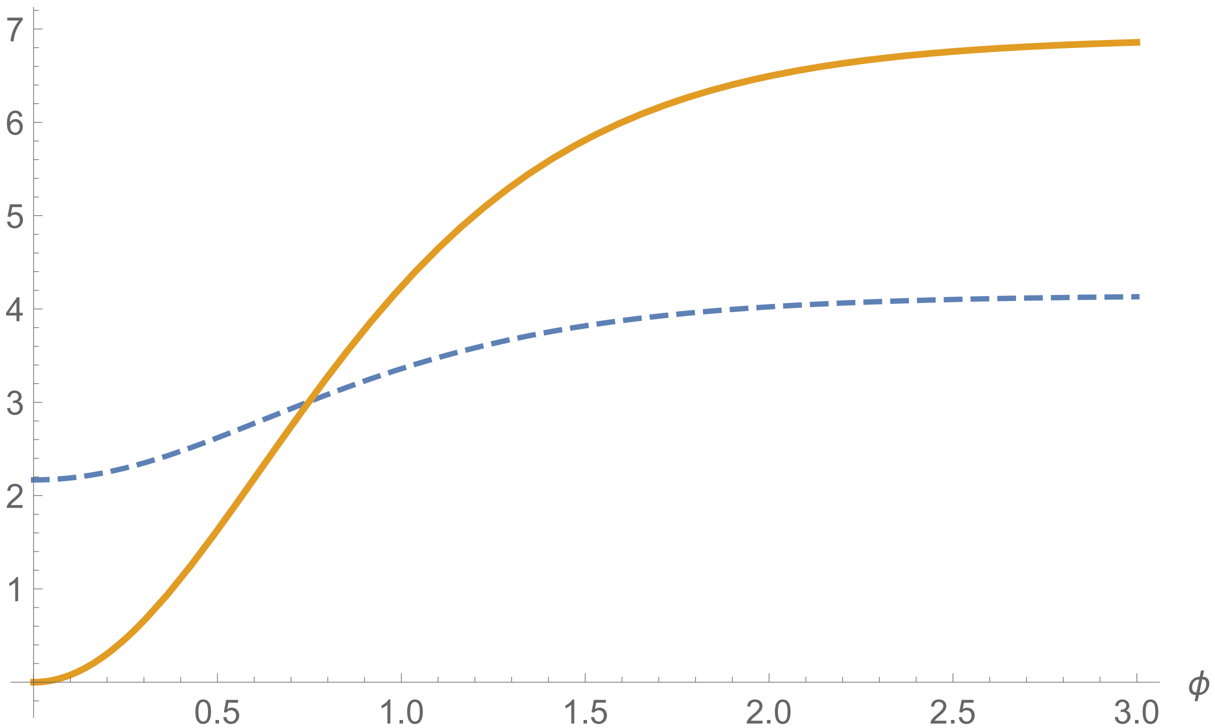
- Classically, for a minimally coupled scalar field, we have:

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left(\frac{\pi_\phi}{V}\right)^2 \left[\frac{\partial_\phi^2 V}{V} - \frac{5}{3}\left(\frac{\partial_\phi V}{V}\right)^2\right]$$

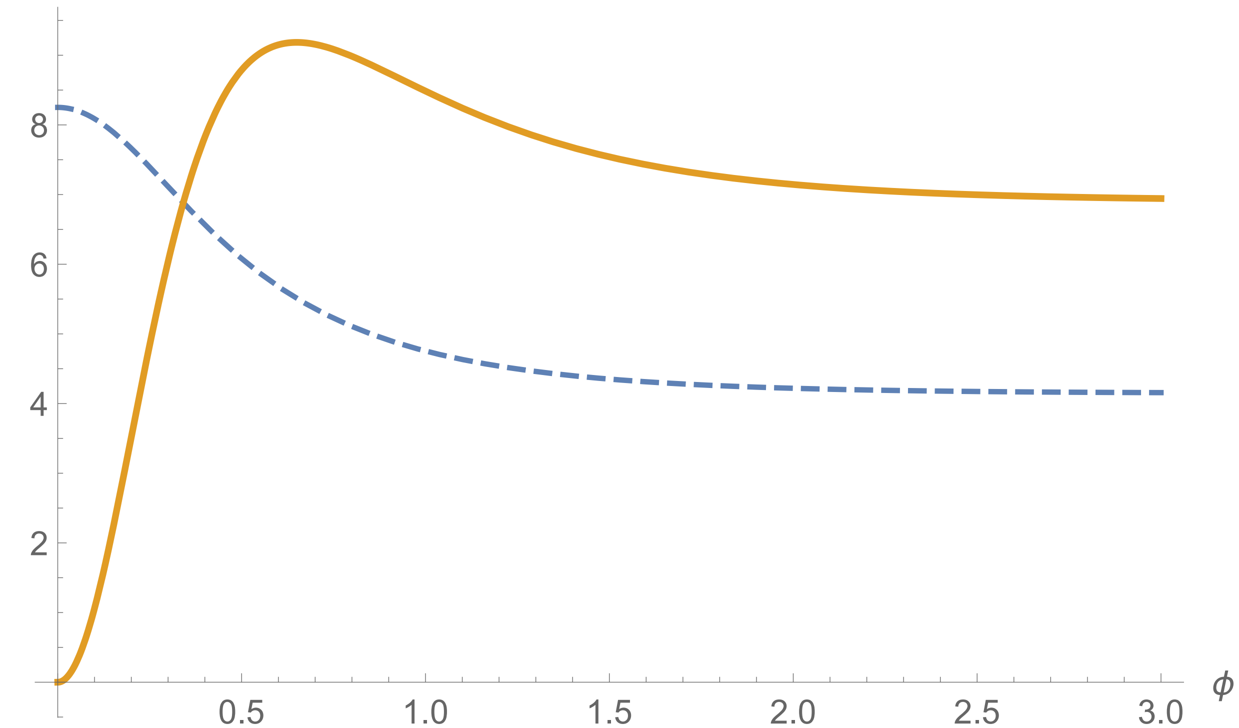
$(a = V^{1/3})$

Accelerated expansion

(MdC,Sakellariadou '16)



E<0



E>0

$$\frac{\ddot{a}}{a} = \frac{1}{3}\left(\frac{\pi_\phi}{V}\right)^2 \left[\frac{\partial_\phi^2 V}{V} - \frac{5}{3}\left(\frac{\partial_\phi V}{V}\right)^2\right]$$

number of e-folds

$$N = \frac{1}{3}\log\left(\frac{V_{\text{end}}}{V_{\text{bounce}}}\right) = \frac{2}{3}\log\left(\frac{\rho_{\text{end}}}{\rho_{\text{bounce}}}\right)$$

too small in the free case

$$N \sim 0.1$$

The impact of interactions

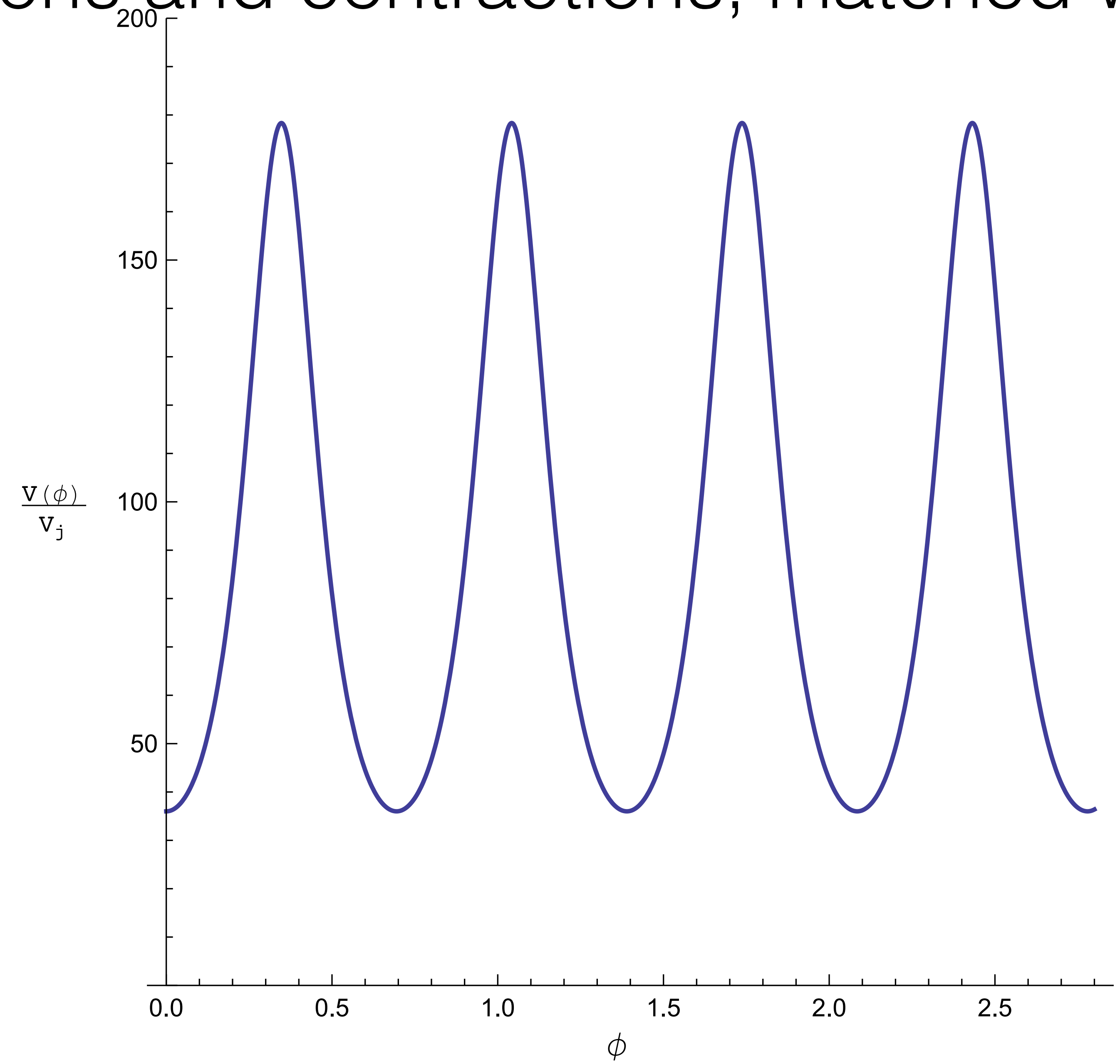
- The role of interactions in GFT is crucial for two main reasons:
  - Interactions prescribe the gluing of quanta to form 4D geometric objects
  - They have important consequences for cosmology
- Phenomenological approach $S_{eff} = \int \mathrm{d}\phi \, \left(A \, |\partial_\phi \sigma|^2 + \mathcal{V}(\sigma) \right)$

$$\mathcal{V}(\sigma) = B|\sigma(\phi)|^2 + \frac{2}{n} w |\sigma|^n + \frac{2}{n'} w' |\sigma|^{n'} \qquad w' > 0, \quad A, B < 0$$

(MdC,Pithis,Sakellariadou '16)

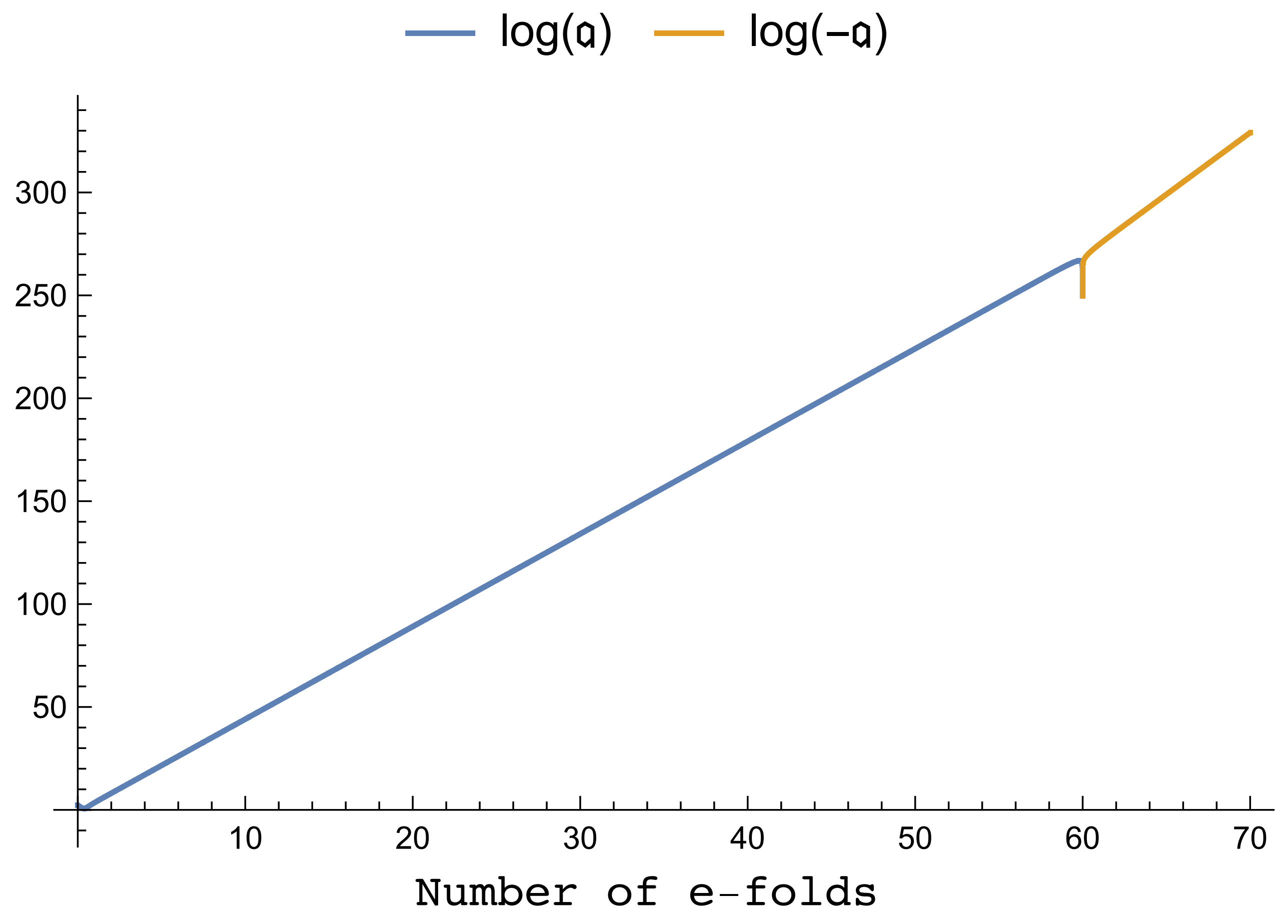
Cyclic Universe

Cosmological evolution is periodic. Endless sequence of expansions and contractions, matched with bounces



Number of e-folds: switching on the interactions

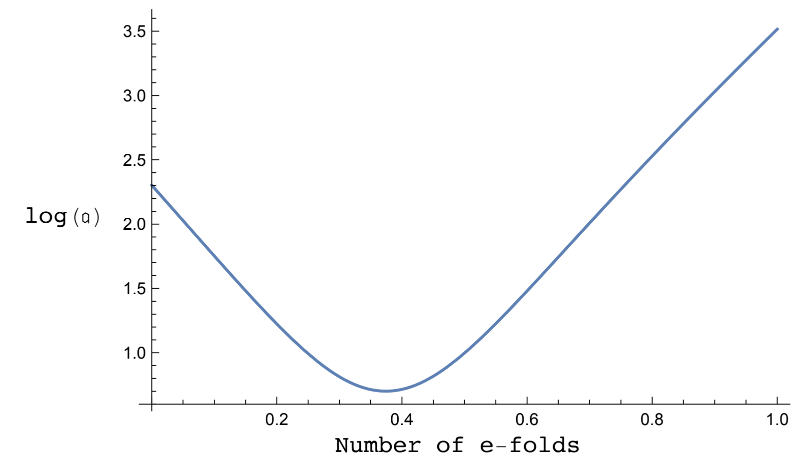
(MdC,Pithis,Sakellariadou '16)

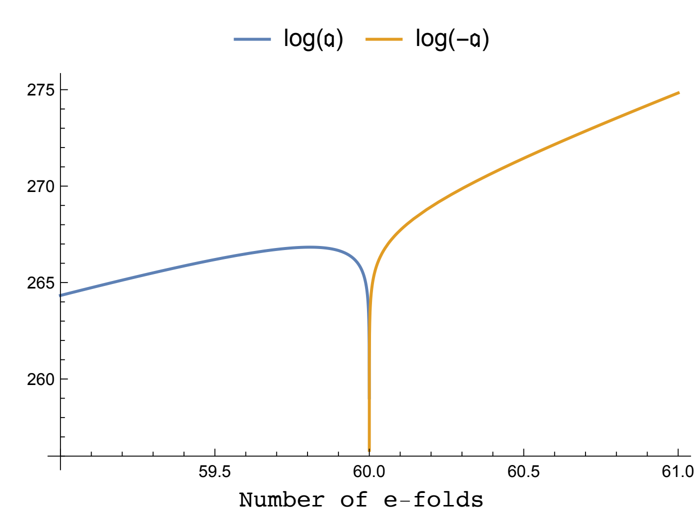


$$N = \frac{2}{3}\log\left(\frac{\rho_{\rm end}}{\rho_{\rm bounce}}\right)$$

$$\mathcal{V}(\sigma) = B|\sigma(\phi)|^2 + \frac{2}{n}w|\sigma|^n + \frac{2}{n'}w'|\sigma|^{n'}$$

No intermediate deceleration stage





Number of e-folds: switching on the interactions

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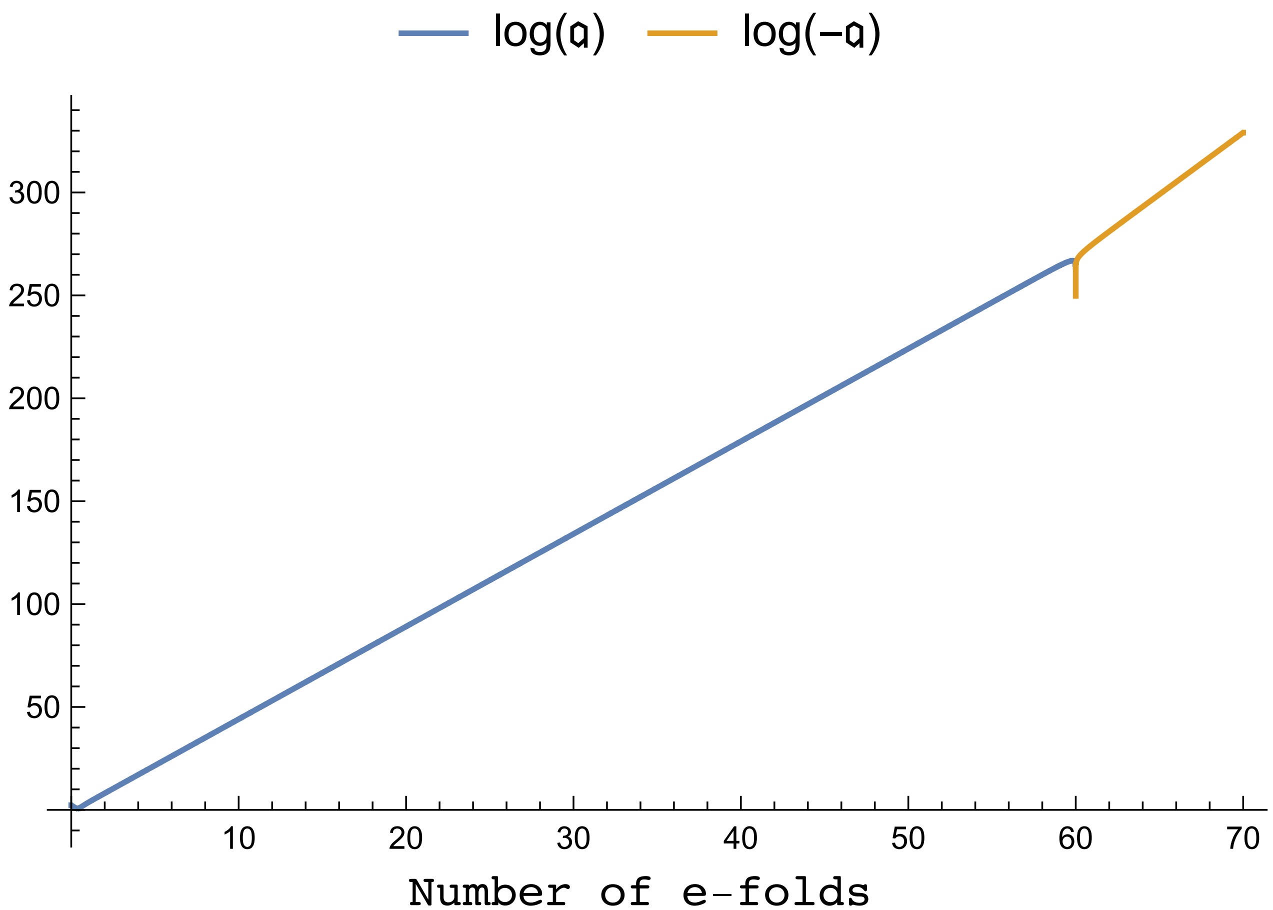


$$N = \frac{2}{3}\log\left(\frac{\rho_{\rm end}}{\rho_{\rm bounce}}\right)$$

$$\mathcal{V}(\sigma) = B|\sigma(\phi)|^2 + \frac{2}{n}w|\sigma|^n + \frac{2}{n'}w'|\sigma|^{n'}$$

Constraints:

$$n' > n$$

$$n \geq 5$$

$$w < 0$$

Bonus

N can be arbitrarily large!

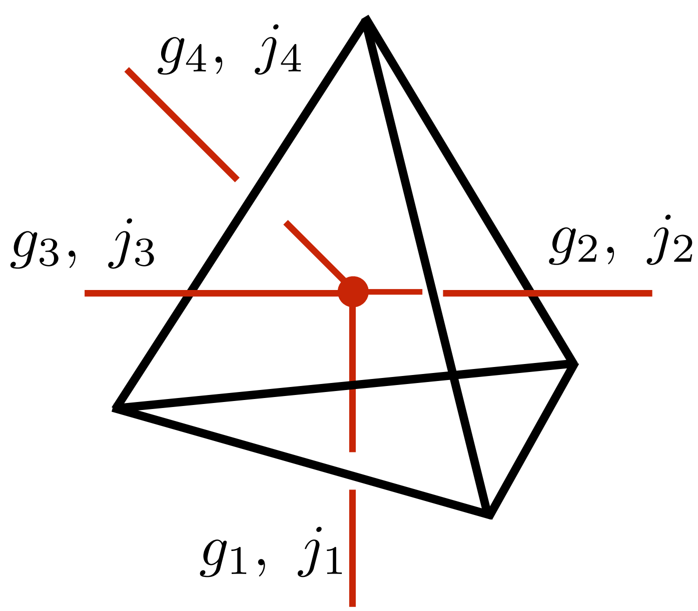
Microscopic anisotropies

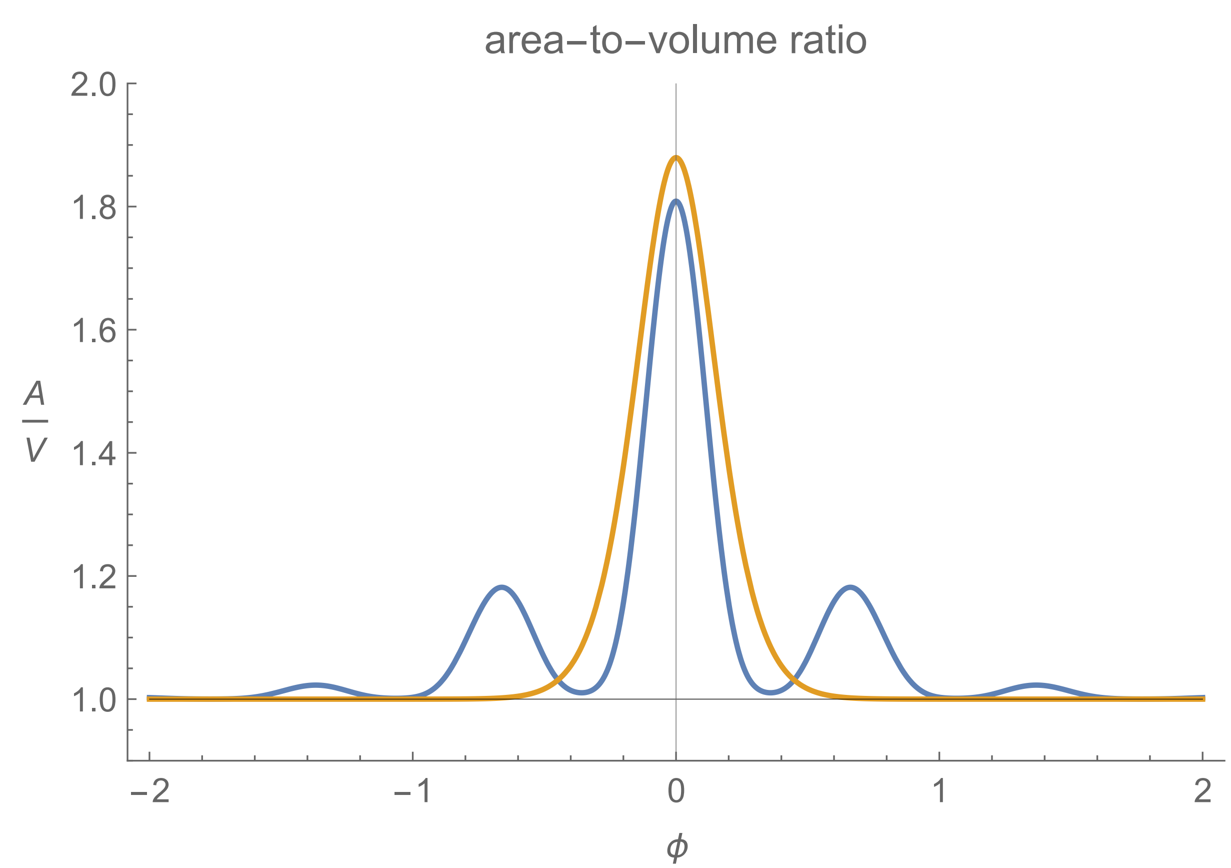
(MdC,Oriti,Pithis,Sakellariadou '17)

- GFT for the EPRL model $S = K + V_5 + \overline{V}_5$

$$V = \frac{1}{5}\int \mathrm{d}\phi \sum_{j_i,m_i,\iota_a} \varphi^{j_1 j_2 j_3 j_4 \iota_1}_{m_1 m_2 m_3 m_4} \varphi^{j_4 j_5 j_6 j_7 \iota_2}_{-m_4 m_5 m_6 m_7} \varphi^{j_7 j_3 j_8 j_9 \iota_3}_{-m_7 -m_3 m_8 m_9} \varphi^{j_9 j_6 j_2 j_{10} \iota_4}_{-m_9 -m_6 -m_2 m_{10}} \varphi^{j_{10} j_8 j_5 j_1 \iota_5}_{-m_{10} -m_8 -m_5 -m_1}$$
$$\times \prod_{i=1}^{10} (-1)^{j_i - m_i} \; \mathcal{V}_5(j_1, \ldots, j_{10}; \iota_1, \ldots \iota_5)$$

- Perturb the isotropic background $\varphi = \varphi_0 + \delta\varphi$





Conclusions

Within the GFT framework, homogeneous and isotropic cosmologies are described as coherent states of basic building blocks.

Our results:

- Bouncing cosmologies are a generic feature of the theory
- Interactions between quanta lead to a cyclic Universe
- Suitable choices of the interactions can make the early Universe accelerate for an arbitrarily large number of e-folds
- Anisotropies decay away from the bounce in a region of parameter space

References

- MdC,Sakellariadou'16: "Accelerated expansion of the Universe without an inflaton and resolution of the initial singularity from Group Field Theory condensates", arXiv:1603.01764
- MdC,Pithis,Sakellariadou'16: "Cosmological implications of interacting Group Field Theory models: cyclic Universe and accelerated expansion", arXiv:1606.00352
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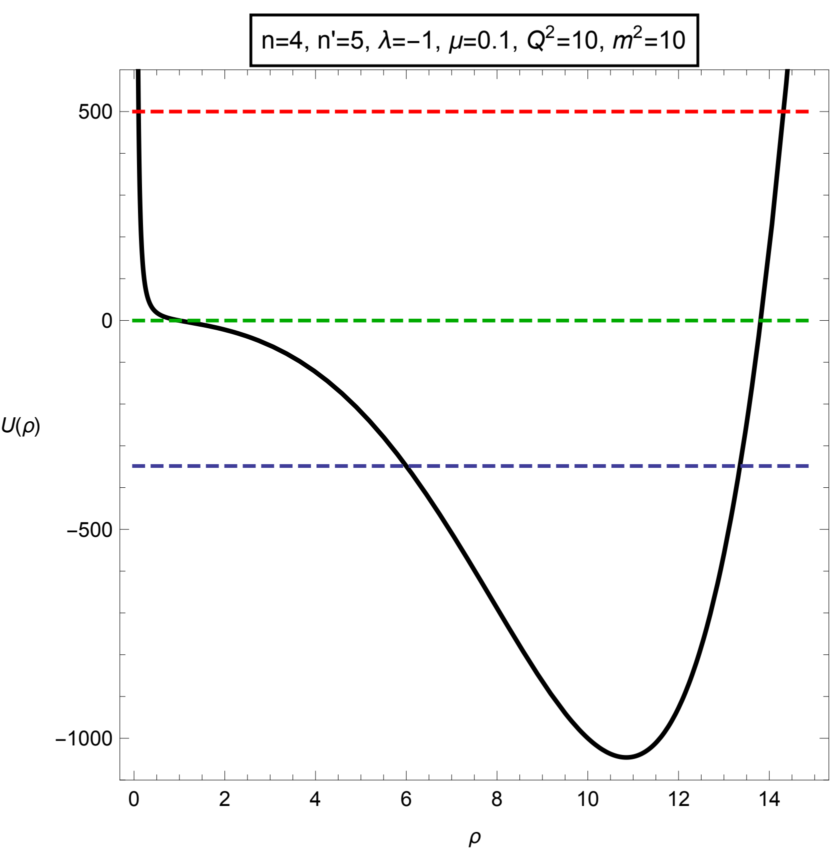
- Oriti'11: "The microscopic dynamics of quantum space as a group field theory", arXiv:1110.5606
- Oriti'13: "Group field theory as the 2nd quantization of Loop Quantum Gravity", arXiv:1310.7786
- Gielen,Oriti,Sindoni'13: "Cosmology from Group Field Theory Formalism for Quantum Gravity", arXiv:1303.3576
- Oriti,Sindoni,Wilson-Ewing'16: "Bouncing cosmologies from quantum gravity condensates", arXiv:1602.08271
- Gielen'16: "Emergence of a low spin phase in group field theory condensates", arXiv:1604.06023

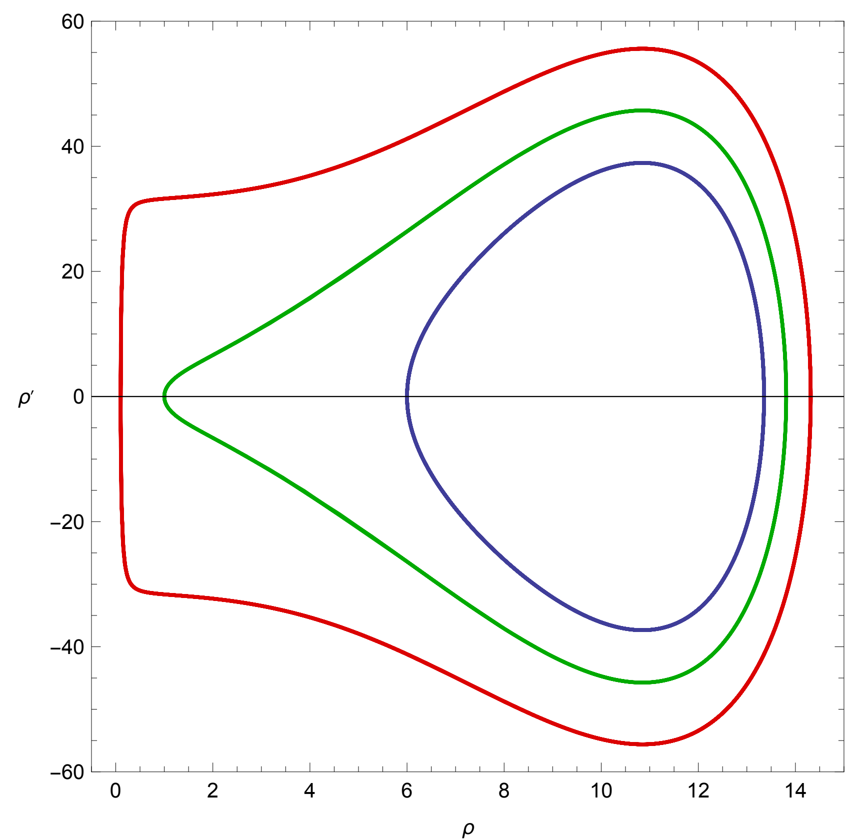
EXTRAS

Dynamics in the interacting model

- Polar decomposition: $\sigma = \rho\, \mathrm{e}^{i\theta}$ $\qquad Q \equiv \rho^2 \partial_\phi \theta$
- Dynamics of the radial part: $\partial_\phi^2 \rho = -\partial_\rho U$

$$U(\rho) = -\frac{1}{2} m^2 \rho^2 + \frac{Q^2}{2\rho^2} + \frac{\lambda}{n}\rho^n + \frac{\mu}{n'}\rho^{n'}$$ with: $\lambda = -\frac{w}{A}$ $\qquad \mu = -\frac{w'}{A} > 0$





$$E = \frac{1}{2}(\partial_\phi \rho)^2 + U(\rho)$$

E and Q
conserved quantities

Effective Friedmann equation

(including interaction terms)

$$H^2 = \frac{8Q^2}{9}\left[\frac{\varepsilon_m}{a^6} + \frac{\varepsilon_E}{a^9} + \frac{\varepsilon_Q}{a^{12}} + \frac{\varepsilon_\mu}{a^{9-\frac{3}{2}n}}\right]$$

$$\varepsilon_m = \frac{B}{2A} \qquad \varepsilon_Q = -\frac{Q^2}{2}V_j^2 \qquad \varepsilon_E = V_j E \qquad \varepsilon_\mu = -\frac{w}{n\,A}V_j^{1-n/2}$$

- Quantum gravity corrections
- Compare with ekpyrotic models