$B-L$ Higgs Inflation in Supergravity with Several Consequences

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Based on:

- *C.P.*, *J. Cosmol. Astropart. Phys.* **16**, no. 10, 037 (2016) [arXiv:1606.09607].
- *C.P.*, *To Appear*.

Outline

Corfu Summer Institute 2017: Workshop on the Standard Model and Beyond

SUGRA (i.e. Supergravity) Potential

- The General Einstein Frame Action For The Scalar Fields z^α Plus Gravity In Four Dimensional, $\mathcal{N}=1$ SUGRA is:

$$\mathcal{S}=\int d^4x\sqrt{-\widehat{g}}\left(-\frac{1}{2}\widehat{\mathcal{R}}+K_{\alpha\bar\beta}\widehat{g}^{\mu\nu}D_\mu z^\alpha D_\nu z^{*\bar\beta}-\widehat{V}\right)$$ Where We Use Units With $m_{\rm P}$=1.

Also K is The **Kähler Potential** With $K_{\alpha\bar\beta}=\dfrac{\partial^2K}{\partial z^\alpha\partial z^{*\bar\beta}}>0$ and $K^{\bar\beta\alpha}K_{\alpha\bar\gamma}=\delta^{\bar\beta}_{\bar\gamma}$; $D_\mu z^\alpha=\partial_\mu z^\alpha+igA^a_\mu T^a_{\alpha\beta}z^\beta$, Where A^a_μ is The Vector Gauge Fields and T_a are the Generators of the Gauge Transformations Of z^α; Finally, $\widehat{V}=\widehat{V}_{\rm F}+\widehat{V}_{\rm D}$ With $\widehat{V}_{\rm F}=e^K\left(K^{\alpha\bar\beta}{\rm F}_\alpha{\rm F}^*_{\bar\beta}-3|W|^2\right)$ With W The **Superpotential** and ${\rm F}_\alpha=W_{,z^\alpha}+K_{,z^\alpha}W$; $\widehat{V}_{\rm D}=\dfrac{1}{2}g^2{\rm D}_a^2$ with ${\rm D}_a=z_\alpha\left(T_a\right)^\alpha_\beta K_{,z^\beta}$.

- We Concentrate on **Higgs Inflation** (**HI**) Driven by $\widehat{V}_{\rm F}$ Since We Can Easily Assure $\widehat{V}_{\rm D}=0$ During HI.

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- Complementarily, From Models of **non-Minimal Chaotic Inflation** (**nMI**) in SUGRA We know that $\widehat{V}_{\rm F}$ is Sufficiently Flat, If We Adopt $K=-{\bf N}\ln\left(1+c_{\mathcal{R}}(\Phi^{\bf n}+\Phi^{*{\bf n}})\right)+\cdots$ and Tune $N>0$ and n With The Exponent m of Φ in $W=\lambda S\Phi^{\bf m}$. E.g.,

If we Select $W=\lambda S\Phi^2$ and $K=-2\ln\left(1+2c_{\mathcal{R}}(\Phi^2+\Phi^{*2})\right)-(\Phi-\Phi^*)^2/2+|S|^2$

We Obtain $\widehat{V}_{\rm F}=e^K K^{SS^*}\left|W_{,S}\right|^2=\lambda\phi^4/4(1+c_{\mathcal{R}}\phi^2)^2\sim{\rm const}$ for $c_{\mathcal{R}}\gg 1$.

How We can Apply These General Ideas to HI?

Selecting Conveniently the Superpotential and Kähler Potentials

- We Use 3 Superfields $z^1=\Phi$, $z^2=\bar\Phi$, **Charged** Under a Local Symmetry, e.g. $U(1)_{B-L}$, and $z^3=S$ (**"Stabilizer" Field**).
  - **Superpotential** $W=\lambda S\left(\bar\Phi\Phi-M^2/4\right)$
  - W Is Uniquely Determined Using $U(1)_{B-L}$ and an R Symmetry and Leads to a **Grand Unified Theory (GUT)** Phase Transition

Charge Assignments

Superfields:	S	Φ	$\bar\Phi$
$U(1)_R$	1	0	0
$U(1)_{B-L}$	0	1	-1

At The SUSY Vacuum $\langle S\rangle=0, |\langle\Phi\rangle|=|\langle\bar\Phi\rangle|=M/2$,
Since in The SUSY Limit, After HI, We Get

$$V_{\rm SUSY}=\lambda^2\left|\bar\Phi\Phi-M^2/4\right|^2+\frac{1}{c_-(1-Nr_\pm)}\lambda^2|S|^2\left(|\Phi|^2+|\bar\Phi|^2\right)+{\rm D-terms}\quad (N, c_-\ \text{and}\ r_\pm\ \text{are Defined Below})$$

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- **Possible Kähler Potentials – Softly Broken Shift Symmetry For Higgs Fields**
- The Shift Symmetry Can Be Formulated By **The Functions** $F_\pm=\left|\Phi\pm\bar\Phi^*\right|^2$ With Coefficients c_+ and c_-, $c_+\leq c_-$.
- HI can be Obtained Selecting the Following K's Which Are **Quadratic and Invariant** Under $U(1)_{B-L}$ and R Symmetries:

$K_1=-N\ln\left(1+c_+F_+\right)+c_-F_-+F_{1S}(|S|^2),\quad K_2=-N\ln\left(1+c_+F_+\right)+F_{2S}(F_-,|S|^2)$ Where We Choose The Functions[1]

$$F_{1S}=\begin{cases}N_S\ln(1+|S|^2/N_S)\\ -N_S\left(e^{-|S|^2/N_S}-1\right)\end{cases}\quad\text{And}\quad F_{2S}=\begin{cases}N_S\ln(1+c_-F_-/N_S+|S|^2/N_S)\\ -N_S\left(e^{-(c_-F_-/N_S+|S|^2/N_S)}-1\right)\end{cases}\quad\text{With } N,N_S>0$$

Since the Simplest Kinetic Term for S, $|S|^2$, Leads to $m_S^2<0$ or $m_S^2<\widehat{H}^2_{\rm HI}$ Along the Inflationary Path.

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$$\bar{\Phi} \to \bar{\Phi} + c^*,\ \Phi \to \Phi + c\ \ (c\in\mathbb{C}) \ \text{ and }\ S \to e^{i\alpha}S\,, \ \text{ in the Limits }\ c_+ \to 0\ \&\ \lambda \to 0\,.$$

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- The **Free Parameters**, For Fixed N, are $r_\pm = c_+/c_-$ **and** λ/c_- (not c_+, c_- and λ) Since If We Perform the Rescalings

$$\Phi \to \Phi/\sqrt{c_-},\ \bar\Phi \to \bar\Phi/\sqrt{c_-},\quad \text{and}\quad S \to S,$$
we see That W Depends on λ/c_- and K on $r_\pm$.

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A $B-L$ Extension of MSSM

Promoting To Local The Already Existing $U(1)_{B-L}$ Global Symmetry of the MSSM, We Obtain:

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- **Generation of Masses for the light Neutrinos.** Through The Type I Seesaw Mechanism Which can be Realized by the Terms

$$W_{\rm RHN} = \lambda_{ijN^c}\bar\Phi N^c_i N^c_j + h_{\nu ij} N^c_i L_j H_u\,.$$

Note that the Three RHNs, N^c_i, Are Necessary To Cancel the $B-L$ Gauge Anomaly.

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- **A Motivation for the Origin of the μ Term.** This can be Explained If We Combine $W_{\rm HI}$ With[2]

$$W_\mu = \lambda_\mu S H_u H_d\,.\quad \textbf{(: I)}$$

The Part Of The Scalar Potential Which Includes The Soft Susy Breaking Terms Corresponding to $W_{\rm HI}+W_\mu$

$$V_{\rm soft} = \left(\lambda A_\lambda S\bar\Phi\Phi + \lambda_\mu A_\mu S H_u H_d - {\rm a}_S S\lambda M^2/4 + {\rm h.c.}\right) + m^2_{\bar\alpha}\left|z^{\bar\alpha}\right|^2 \quad \text{with}\quad z^{\bar\alpha} = \Phi,\bar\Phi,S,H_u,H_d$$

where $m_\alpha, A_\lambda, A_\mu$ and ${\rm a}_S$ are Soft SUSY Breaking Mass Parameters. **Minimizing** $V_{\rm tot} = V_{\rm SUSY} + V_{\rm soft}$ and Substituting in $V_{\rm soft}$ the SUSY v.e.vs of Φ and $\bar\Phi$ we get

$$\langle V_{\rm tot}(S)\rangle = \lambda^2 M^2 S^2/2c_-(1-Nr_\pm) - \lambda{\rm a}_\mu M^2 S,\quad \text{where}\quad m_S \ll M \quad\text{and}\quad (|A_\lambda|+|{\rm a}_S|) = 2{\rm a}_\mu m_{3/2}$$

where $m_{3/2}$ is the Gravitino Mass. The Minimized $\langle V_{\rm tot}(S)\rangle$ w.r.t S leads to a non Vanishing $\langle S\rangle$ as Follows:

$$d\langle V_{\rm tot}(S)\rangle/dS = 0 \;\Rightarrow\; \langle S\rangle \simeq {\rm a}_\mu c_-(1-Nr_\pm)m_{3/2}/\lambda \simeq 10^5{\rm a}_\mu m_{3/2}\mathcal{F}(N,r_\pm).$$

Therefore, the Generated μ Parameter From Eq. (I) is $\mu = \lambda_\mu\langle S\rangle \simeq \lambda_\mu m_{3/2}{\rm a}_\mu c_-(1-Nr_\pm)/\lambda \simeq 10^5 m_{3/2}\lambda_\mu\mathcal{F}(N,r_\pm)$.
Successful HI Needs $\lambda_\mu \leq 9\cdot 10^{-6}$ and so the Prefactor is Absorbed. Therefore, **$\mu \simeq 1$ TeV Implies $m_{3/2} \gtrsim 1$ TeV**.

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The Relevant Super- & Kähler Potentials

- We Focus on a Superpotential Invariant under the $G_{\rm SM}\times U(1)_{B-L}$ Gauge Group:

$$W = \lambda S\left(\bar\Phi\Phi - M^2/4\right)$$

to Achieve HI & Break $U(1)_{B-L}$

$$+\ \lambda_\mu S H_u H_d$$

to Generate $\mu \simeq 10^5\lambda_\mu m_{3/2}\mathcal{F}(N,r_\pm)\sim 1$ TeV

$$+\ \lambda_{ij\nu}\bar\Phi N_i^c N_j^c$$

to Generate Majorana Masses for Neutrinos & Ensure The Inflaton Decay

$$+\ h_{ijN} N_i^c L_j H_u$$

to Generate Dirac Masses for Neutrinos

+ W of MSSM with $\mu = 0$

Super-fields	Representations under $G_{\rm SM}\times U(1)_{B-L}$	Global Symmetries R	B	L
	Matter Fields			
e_i^c	$(\mathbf{1},\mathbf{1},1,1)$	0	0	-1
N_i^c	$(\mathbf{1},\mathbf{1},0,1)$	0	0	-1
L_i	$(\mathbf{1},\mathbf{1},-1/2,-1)$	2	0	1
u_i^c	$(\mathbf{3},\mathbf{2},-2/3,-1/3)$	1	$-1/3$	0
d_i^c	$(\mathbf{3},\mathbf{2},1/3,-1/3)$	1	$-1/3$	0
Q_i	$(\bar{\mathbf{3}},\mathbf{2},1/6,-1/3)$	1	$1/3$	0
	Higgs Fields			
H_d	$(\mathbf{1},\mathbf{2},-1/2,0)$	0	0	0
H_u	$(\mathbf{1},\mathbf{2},1/2,0)$	0	0	0
S	$(\mathbf{1},\mathbf{1},0,0)$	4	0	0
$\bar\Phi$	$(\mathbf{1},\mathbf{1},0,2)$	0	0	-2
Φ	$(\mathbf{1},\mathbf{1},0,-2)$	0	0	2

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$$+\ \lambda_\mu S H_u H_d$$
to Generate $\mu \simeq 10^5\lambda_\mu m_{3/2}\mathcal{F}(N,r_\pm)\sim 1$ TeV
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to Generate Majorana Masses for Neutrinos & Ensure The Inflaton Decay
$$+\ h_{ijN}N^c_i L_j H_u$$
to Generate Dirac Masses for Neutrinos
$$+\ \text{W of MSSM with } \mu=0$$

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L_i	$(\mathbf{1},\mathbf{1},-1/2,-1)$	2	0	1
u^c_i	$(\mathbf{3},\mathbf{2},-2/3,-1/3)$	1	$-1/3$	0
d^c_i	$(\mathbf{3},\mathbf{2},1/3,-1/3)$	1	$-1/3$	0
Q_i	$(\bar{\mathbf{3}},\mathbf{2},1/6,-1/3)$	1	$1/3$	0
	Higgs Fields			
H_d	$(\mathbf{1},\mathbf{2},-1/2,0)$	0	0	0
H_u	$(\mathbf{1},\mathbf{2},1/2,0)$	0	0	0
S	$(\mathbf{1},\mathbf{1},0,0)$	4	0	0
$\bar\Phi$	$(\mathbf{1},\mathbf{1},0,2)$	0	0	-2
Φ	$(\mathbf{1},\mathbf{1},0,-2)$	0	0	2

- The Above W May Cooperate With The Following Kähler Potential Potentials Which Respect the Imposed Symmetries

$$K_1 = -N\ln\left(1+c_+F_+\right)+c_-F_- + F_{1X}(|X|^2),\quad K_2 = -N\ln\left(1+c_+F_+\right)+F_{2X}(F_-,|X|^2)\quad \text{Where}$$

$$F_{1S} = \begin{cases} N_X\ln\left(1+X^\alpha X_\alpha/N_X\right)\\ -N_X\left(e^{-X^\alpha X_\alpha/N_X}-1\right)\end{cases}\ \text{And}\ F_{2S} = \begin{cases} N_X\ln\left(1+X^\alpha X_\alpha/N_X+c_-F_-/N_X\right)\\ -N_X\left(e^{-(c_-F_-/N_X+X^\alpha X_\alpha/N_X)}-1\right)\end{cases}\ \text{With } N,N_X>0$$

and $X^\alpha=S,H_u,H_d,N^c_i$ – Placing $X^\alpha X_\alpha$ Inside the Argument of ln, We Obtain Tighter Restrictions on λ_μ.

The Inflationary Potential

- If We Use The Parametrization:

$$\Phi = \phi e^{i\theta}\cos\theta_\Phi/\sqrt{2} \quad \text{and} \quad \bar\Phi = \phi e^{i\bar\theta}\sin\theta_\Phi/\sqrt{2} \quad \text{with} \quad 0\leq\theta_\Phi\leq\pi/2 \quad \text{and} \quad X^\beta = \left(x^\beta + i\bar x^\beta\right)/\sqrt{2},$$

Where $X^\beta = S, H_u, H_d, N^c_i$, We Can Show That A D-Flat Direction Is $\theta=\bar\theta=x^\beta=\bar x^\beta=0$, and $\theta_\Phi=\pi/4$ (: I)

- The Only **Surviving Term** of $\widehat{V}_{\rm F}$ Along the Path in Eq. (I) is

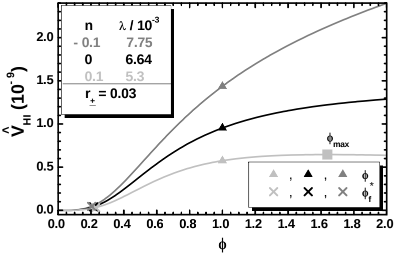
$$\widehat{V}_{\rm HI} = e^K K^{SS^*}|W_{,S}|^2 = \frac{\lambda^2(\phi^2-M^2)^2}{16 f_{\mathcal{R}}^{2(1+n)}} \quad \text{With} \quad f_{\mathcal{R}} = 1 + c_+\phi^2$$

Playing The Role Of A **Non-Minimal Coupling to Gravity**. Also,

$$n = N/2 - 1 \quad \text{and} \quad K^{\beta\bar\beta} = 1$$

- For $n>0$, $\widehat{V}_{\rm HI}$ Develops A Local **Maximum**

$$\widehat{V}_{\rm HI}(\phi_{\max}) = \frac{\lambda^2 n^{2n}}{16c_+^2(1+n)^{2(1+n)}} \quad \text{at} \quad \phi_{\max} = \frac{1}{\sqrt{c_+ n}}$$



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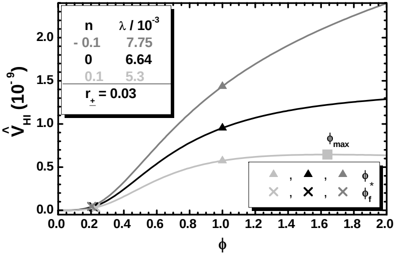

- The EF Canonically Normalized Fields, Which Are Denoted By Hat, Can Be Obtained As Follows:

$$\frac{d\widehat\phi}{d\phi} = J = \sqrt{\kappa_+},\ \widehat\theta_+ = \frac{J\phi\theta_+}{\sqrt{2}},\ \widehat\theta_- = \sqrt{\frac{\kappa_-}{2}}\phi\theta_-, \quad \text{and} \quad \widehat\theta_\Phi = \phi\sqrt{\kappa_-}\left(\theta_\Phi - \frac{\pi}{4}\right),\ \left(\widehat{x}^\beta, \widehat{\bar{x}}^\beta\right) = \left(x^\beta, \bar{x}^\beta\right),$$

Where $\theta_\pm = (\theta \pm \bar\theta)/\sqrt{2}$, $\kappa_+ = c_-\left(1 + Nr_\pm(c_+\phi^2 - 1)/f_{\mathcal{R}}^2\right) \simeq c_-$ and $\kappa_- = c_-\left(1 - Nr_\pm/f_{\mathcal{R}}\right) > 0 \quad \Rightarrow \quad \mathbf{r_\pm < 1/N}.$

- We Can Check the Stability of the Trajectory in Eq. (I) w.r.t the Fluctuations Of The Various Fields, i.e.

$$\left.\frac{\partial V}{\partial \widehat{z}^\alpha}\right|_{\text{Eq. (I)}} = 0 \quad \text{and} \quad \widehat{m}^2_{z^\alpha} > 0 \quad \text{Where} \quad \widehat{m}^2_{z^\alpha} = \text{Egv}\left[\widehat{M}^2_{\alpha\beta}\right] \quad \text{With} \quad \widehat{M}^2_{\alpha\beta} = \left.\frac{\partial^2 V}{\partial\widehat{z}^\alpha\partial\widehat{z}^\beta}\right|_{\text{Eq. (I)}} \quad \text{and} \quad z^\alpha = \theta_-, \theta_+, \theta_\Phi, x^\beta, \bar{x}^\beta.$$

Stability and Radiative Corrections

The Mass Spectrum Along The Inflationary Trajectory

Fields	Eingestates	Masses Squared		
			$K = K_1$	$K = K_2$
2 Real Scalars	$\widehat{\theta}_+$	$\widehat{m}^2_{\theta+}$	$6\widehat{H}^2_{\rm HI}$	$6(1+1/N_X)\widehat{H}^2_{\rm HI}$
	$\widehat{\theta}_\Phi$	$\widehat{m}^2_{\theta_\Phi}$	$M^2_{BL}+6\widehat{H}^2_{\rm HI}$	$M^2_{BL}+6(1+1/N_X)\widehat{H}^2_{\rm HI}$
1 Complex Scalars	$\widehat{s}, \widehat{\bar{s}}$	$\widehat{m}^2_s$	$6\widehat{H}^2_{\rm HI}/N_X$	
4 Complex Scalars	$h_\pm, \bar{h}_\pm$	$\widehat{m}^2_{h\pm}$	$3\widehat{H}^2_{\rm HI}(1+1/N_X \pm 4\lambda_\mu/\lambda\phi^2)$	
3 Complex Scalars	$\tilde{\nu}^c_i, \bar{\tilde{\nu}}^c_i$	$m^2_{i\tilde{\nu}^c}$	$3\widehat{H}^2_{\rm HI}(1+1/N_X+16\lambda_{iN^c}/\lambda^2\phi^2)$	
1 Gauge boson	A_{BL}	M^2_{BL}	$g^2 c_-\,(1-Nr_\pm/f_{\mathcal{R}})\,\phi^2$	
4 Weyl Spinors	$\widehat{\psi}_\pm$	$\widehat{m}^2_{\psi\pm}$	$24\widehat{H}^2_{\rm HI}/c_-\phi^2 f^2_{\mathcal{R}}$	
	ψ_{iN^c}	$\widehat{m}^2_{\psi_{iN^c}}$	$48\lambda^2_{iN^c}\widehat{H}^2_{\rm HI}/\lambda^2\phi^2$	
	$\lambda_{BL}, \widehat{\psi}_{\Phi-}$	M^2_{BL}	$g^2 c_-\,(1-Nr_\pm/f_{\mathcal{R}})\,\phi^2$	

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha} > 0$. Especially

$$\widehat{m}^2_s > 0 \ \Leftrightarrow\ N_X < 6 \ \text{and}\ \widehat{m}^2_{H-} > 0 \ \Leftrightarrow\ \lambda_\mu \le \lambda(1+1/N_X)\phi_{\rm f}/4 \ (\text{E.g. } \lambda_\mu < 9\cdot 10^{-6} \text{ for } r_\pm = 0.03)\,.$$

- We can Obtain $\forall\,\alpha,\ \widehat{m}^2_{\chi^\alpha} > \widehat{H}^2_{\rm HI}$ and So Any Inflationary Perturbations Of The Fields Other Than ϕ Are Safely Eliminated;
- $M_{BL} \neq 0$ Signals the Fact that That $U(1)_{B-L}$ Is Broken and so, **no Topological Defects** are Produced.
- The One-Loop **Radiative Corrections** à la Coleman-Weinberg to $\widehat{V}_{\rm HI}$ Can Be Kept Under Control Provided that
  - $M^2_{BL} > m^2_{\rm P}$ and $\widehat{m}^2_{\theta_\Phi} > m^2_{\rm P}$ Are not Taken Into Account.
  - The Renormalization Group Mass Scale Λ Is Determined By Requiring $\Delta\widehat{V}_{\rm HI}(\phi_\star) = 0$ or $\Delta\widehat{V}_{\rm HI}(\phi_{\rm f}) = 0$.

Approximating the Inflationary Dynamics

- The **Slow-Roll Parameters** Are Determined Using the Standard Formulae Employing The Canonically Normalized $\widehat{\phi}$:

$$\widehat{\epsilon} = \frac{1}{2}\left(\frac{\widehat{V}_{\rm HI,\widehat{\phi}}}{\widehat{V}_{\rm HI}}\right)^2 \simeq \frac{8(1-nc_+\phi^2)^2}{c_-\phi^2 f_{\mathcal{R}}^2} \quad \text{AND} \quad \widehat{\eta} = \frac{\widehat{V}_{\rm HI,\widehat{\phi}\widehat{\phi}}}{\widehat{V}_{\rm HI}} = 4\,\frac{3-(3+9n)c_+\phi^2+n(1+4n)c_+^2\phi^4}{c_-\phi^2 f_{\mathcal{R}}^2}\,.$$

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- The **Number of e-Foldings** That $k_\star = 0.05$ Mpc Experiences During HI Is Calculated to be

$$\widehat{N}_\star = \int_{\widehat{\phi}_{\rm f}}^{\widehat{\phi}_\star} d\widehat{\phi}\,\frac{\widehat{V}_{\rm HI}}{\widehat{V}_{\rm HI,\widehat{\phi}}} \simeq \begin{cases} \left((1+c_+\phi_\star^2)^2-1\right)/16r_\pm & \text{for } n=0 \\ -\left(nc_+\phi_\star^2+(1+n)\ln(1-nc_+\phi_\star^2)\right)/8n^2r_\pm & \text{for } n\neq 0\,. \end{cases}$$

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$$\text{For } n=0,\quad \phi_\star \leq 1 \quad\Rightarrow\quad c_- \geq (f_{n\star}-1)/r_\pm \simeq 100,\quad \text{with}\quad f_{n\star} = \left(1+16r_\pm\widehat{N}_\star\right)^{1/2} \quad \text{and}\quad \widehat{N}_\star \simeq 58.$$

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- A Clear **Dependence of The Observables** (Spectral Index $n_{\rm s}$ and Tensor-To-Scalar Ratio, r) On $r_\pm$ and n Arises, I.e.,

$$n_{\rm s}=1-6\widehat{\epsilon}_\star+2\widehat{\eta}_\star\simeq 1-4n^2r_\pm-2n\frac{r_\pm^{1/2}}{\widehat{N}_\star^{1/2}}-\frac{3-2n}{2\widehat{N}_\star}-\frac{3-n}{8(\widehat{N}_\star^3r_\pm)^{1/2}}\,,\quad r=16\widehat{\epsilon}_\star\simeq-\frac{8n}{\widehat{N}_\star}+\frac{3+2n}{6\widehat{N}_\star^2r_\pm}+\frac{6-n}{3(\widehat{N}_\star^3r_\pm)^{1/2}}+\frac{8n^2r_\pm^{1/2}}{\widehat{N}_\star^{1/2}}\,,$$

With Negligible $n_{\rm s}$ Running, $\alpha_{\rm s}$. The Variables With Subscript $\star$ Are Evaluated at $\widehat{\phi}=\widehat{\phi}_\star$.

Testing Against Observations

- The **Combined** Bicep2/*Keck Array* **and *Planck* Results**[3] Although Do Not Exclude Inflationary Models With Negligible r's, They **Seem to Favor** Those With r's of Order 0.01 Which Imply **Observable Gravitational Waves**.

$$\text{Current Data:}\quad r=0.028^{+0.026}_{-0.025}\quad\Rightarrow\quad 0.003\lesssim r\lesssim 0.054\quad\text{at 68\% c.l. And}\quad r\leq 0.07\quad\text{at 95\% c.l.}$$

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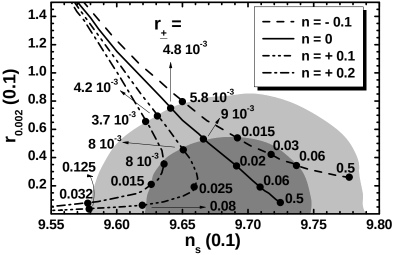

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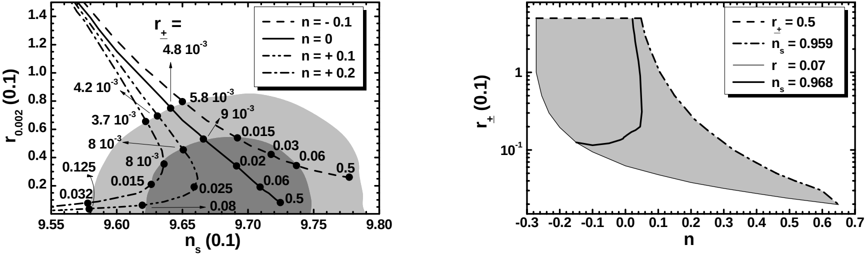

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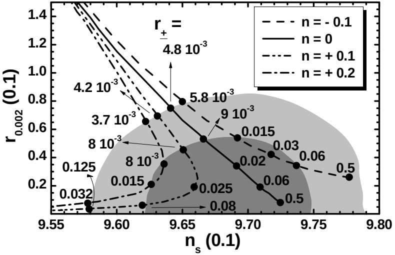

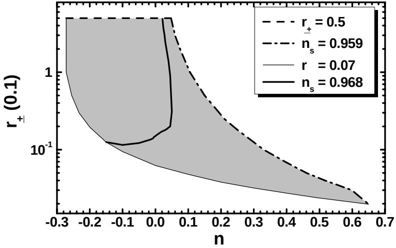

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- Fixing $n_s = 0.968$ and Let n Vary We Find the **Allowed Ranges** of the Parameters and the Required (Mild) **Tuning**:

$$-1.21 \lesssim n/0.1 \lesssim 0.215, \quad 0.12 \lesssim r_\pm/0.1 \lesssim 5, \quad 0.4 \lesssim r/0.01 \lesssim 7 \ \text{ and }\ \Delta_{\max\star} = (\phi_{\max} - \phi_\star)/\phi_{\max} \gtrsim 0.4.$$

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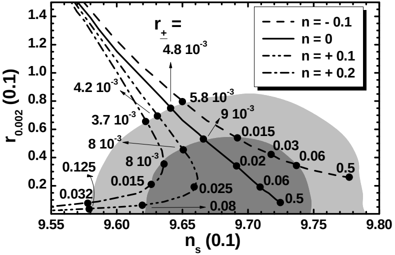

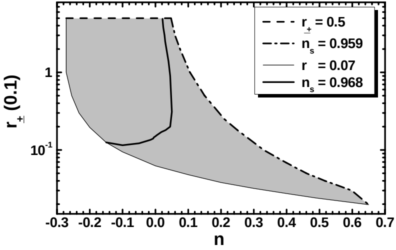

- For $n > 0$ [$n < 0$] The Curves Move To The Left [Right] of the Curve Obtained for $n = 0$. Therefore the 1-σ Observationally Favored Range **Can Be Covered** For Quite Natural $r_\pm$'s – e.g. $0.0029 \lesssim r_\pm \lesssim 0.5$.
- **Positivity** of κ_- Provides an **Upper Bound** on $r_\pm$ Which is Translated to a **Lower Bound** on r.
- Fixing $n_s = 0.968$ and Let n Vary We Find the **Allowed Ranges** of the Parameters and the Required (Mild) **Tuning**:

$$-1.21 \lesssim n/0.1 \lesssim 0.215, \quad 0.12 \lesssim r_\pm/0.1 \lesssim 5, \quad 0.4 \lesssim r/0.01 \lesssim 7 \quad \text{and} \quad \Delta_{\max\star} = (\phi_{\max} - \phi_\star)/\phi_{\max} \gtrsim 0.4.$$

- **Special Cases:** $(n, r_\pm) = (\mathbf{0}, 0.015) \Rightarrow (n_s, r) = (\mathbf{0.968}, 0.043)$ and $(n, r_\pm) = (0.042, 0.025) \Rightarrow (n_s, r) = (\mathbf{0.968}, \mathbf{0.028})$.

[3] *Planck Collaboration (2015);* Bicep2/*Keck Array and Planck Collaborations (2015)*

Ultraviolet (UV) Cut-off Scale ($\Lambda_{\rm UV}$)

- The Implementation Of Our Inflationary Model With $\phi \leq 1$ Requires **Relatively Large c_-'s**. Therefore, We Have To Check If the Resulting Effective Theory Respects Perturbative unitarity up to $m_{\rm P}=1$, Analyzing The Small-Field Behavior[4] Of the Theory. I.e., We Expand About $\langle\phi\rangle = 0$ the Action $\mathcal{S}$ Along The Inflationary Path

$$\mathcal{S} = \int d^4x\sqrt{-\widehat{\mathfrak{g}}}\left(-\frac{1}{2}\widehat{\mathcal{R}} + \frac{1}{2}J^2\dot{\phi}^2 - \widehat{V}_{\rm HI0} + \cdots\right).$$

- In Particular, We Find $\langle J\rangle$ as Follows

$$J^2 = \left(\frac{d\widehat{\phi}}{d\phi}\right)^2 = \kappa_+ = \frac{f_{\rm K}}{f_{\mathcal{R}}} + \frac{Nc_+(c_+\phi^2-1)}{f_{\mathcal{R}}^2} \quad\Rightarrow\quad \langle J\rangle \simeq c_- \neq 1, \quad \text{Where}\quad f_{\rm K} = c_- f_{\mathcal{R}} \ \text{ and } \ \langle f_{\mathcal{R}}\rangle \simeq 1.$$

I.E., The First Term Includes the **a non-Canonical Kinetic Mixing** Whereas the Second One is Due to the non-Minimal Coupling $f_{\mathcal{R}}$. For this Reason, we Call This Model **Kinetically Modified non-Minimal** HI.

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• **For $r_\pm\leq 1$**, We obtain $\underline{\Lambda_{\rm UV}=m_{\rm P}}$ Since The Expansions Abound $\langle\phi\rangle=0$ Are Just **$r_\pm$ (not c_- or c_+) Dependent**:

$$J^2\dot{\phi}^2\simeq\left(1+3Nr_\pm^2\widehat{\phi}^2-5Nr_\pm^3\widehat{\phi}^4+\cdots\right)\dot{\widehat{\phi}}^2\ \ \text{and}\ \ \widehat{V}_{\rm HI}\simeq\frac{\lambda^2\widehat{\phi}^4}{16c_-^2}\left(1-2(1+n)r_\pm\widehat{\phi}^2+(3+5n)r_\pm^2\widehat{\phi}^4-\cdots\right).$$

Consequently, **No Problem** With The Perturbative Unitarity Emerges for $r_\pm\leq 1$, Even If c_+ and c_- Are Large.

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Consequently, **No Problem** With The Perturbative Unitarity Emerges for $r_\pm \leq 1$, Even If c_+ and c_- Are Large.

• This Has to Be Contrasted to the Situation in **Standard non-Minimal HI** Which is Defined for

$$f_{\rm K} = 1 \text{ and } f_{\mathcal{R}} = 1 + c_{\mathcal{R}}\phi^2 \text{ Leading to } \langle J\rangle = 1\,.$$

This Results to $\Lambda_{\rm UV} = m_{\rm P}/c_{\mathcal{R}} \ll m_{\rm P}$ For $c_{\mathcal{R}} > 1$ Since The Expansions About $\langle\phi\rangle \simeq 0$ Are $c_{\mathcal{R}}$ Dependent, I.E.,

$$J^2\dot\phi^2 = \left(1 - c_{\mathcal{R}}\widehat\phi^2 + 6c_{\mathcal{R}}^2\widehat\phi^2 + c_{\mathcal{R}}^2\widehat\phi^4 + \cdots\right)\dot{\widehat\phi}^2 \text{ and } \widehat{V}_{\rm HI} = \frac{\lambda^2\widehat\phi^4}{2}\left(1 - 2c_{\mathcal{R}}\widehat\phi^2 + 3c_{\mathcal{R}}^2\widehat\phi^4 - 4c_{\mathcal{R}}^3\widehat\phi^6 + \cdots\right).$$

Where The Term Which Yields The Smallest Denominator For $c_{\mathcal{R}} > 1$ is **$6c_{\mathcal{R}}^2\widehat\phi^2$**.

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Perturbative Reheating

- At the SUSY Vacuum, The Inflaton And The RHNs, N_i^c, Acquire Masses $\widehat{m}_{\delta\phi}$ and M_{iN^c} Respectively Given by

$$\widehat{m}_{\delta\phi} \simeq \frac{\lambda M}{\sqrt{2c_-(1-Nr_\pm)}} \;\; (\text{E.g. } 9\cdot 10^{10}\ \mathrm{GeV} \text{ for } r_\pm = 0.03) \;\; \text{and} \;\; M_{iN^c} = \lambda_{iN^c} M\,,$$

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- The Inflaton Can Decay Perturbatively Into:
  - **A Pair of RHNs** (N_j^c) With Majorana Masses M_{jN^c} Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to N_i^c} = \frac{\lambda_{iN^c}^2}{16\pi}\widehat{m}_{\delta\phi}\left(1-\frac{4M_{iN^c}^2}{\widehat{m}_{\delta\phi}^2}\right)^{3/2} \;\; \text{With} \;\; \lambda_{iN^c} = \frac{M_{iN^c}}{2\langle J\rangle M}\left(1-3c_+\frac{N}{2}\frac{M^2}{m_{\rm P}^2}\right) \;\; \text{Arising from} \;\; \mathcal{L}_{\widehat{\delta\phi}\to N_i^c} = \lambda_{iN^c}\widehat{\delta\phi}\, N_i^c N_i^c\,.$$

  - **H_u and H_d** Through The Following Decay Width

$$\widehat{\Gamma}_{\delta\phi\to H} = \frac{2}{8\pi}\lambda_H^2\widehat{m}_{\delta\phi} \;\; \text{with} \;\; \lambda_H = \frac{\lambda_\mu}{\sqrt{2}}\left(1-2c_+(n+1)\frac{M^2}{m_{\rm P}^2}\right) \;\; \text{Arising from} \;\; \mathcal{L}_{\widehat{\delta\phi}\to H_uH_d} = -\lambda_H\widehat{m}_{\delta\phi}\widehat{\delta\phi}H_u^*H_d^*\,.$$

  - **MSSM (s)-Particles XYZ** Through The Following c_+-Dependent 3-Body Decay Width

$$\widehat{\Gamma}_{\delta\phi\to XYZ} = \lambda_y^2\frac{14n_{\rm f}}{512\pi^3}\frac{\widehat{m}_{\delta\phi}^3}{m_{\rm P}^2} \;\; \text{With} \;\; \lambda_y = Ny_3c_+\frac{M}{\langle J\rangle m_{\rm P}} \;\; \text{and} \;\; y_3 = h_{t,b,\tau}(\widehat{m}_{\delta\phi}) \simeq 0.5\,.$$

  This Decay Arises From $\mathcal{L}_{\widehat{\delta\phi}\to XYZ} = -\lambda_y(\widehat{\delta\phi}/m_{\rm P})\,(X\psi_Y\psi_Z + Y\psi_X\psi_Z + Z\psi_X\psi_Y) + \text{h.c.}$

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- The Reheating Temperature, $T_{\rm rh}$, is given by

$$T_{\rm rh} = \left(72/5\pi^2 g_*\right)^{1/4}\widehat{\Gamma}_{\delta\phi}^{1/2}m_{\rm P}^{1/2} \;\; \text{with} \;\; \widehat{\Gamma}_{\delta\phi} = \widehat{\Gamma}_{\delta\phi\to N_i^c} + \widehat{\Gamma}_{\delta\phi\to H} + \widehat{\Gamma}_{\delta\phi\to XYZ}, \;\; \text{with} \;\; g_* \simeq 228.75\,.$$

Leptogenesis and $\widetilde{G}$ Abundance

- The Out-Of-Equilibrium Decay of N_i^c can Generate an L Asymmetry Which Can Be Converted to the B **Yield**:

$$Y_B = -0.35\; 2\; \frac{5}{4}\; \frac{T_{\rm rh}}{\widehat{m}_{\delta\phi}}\, \frac{\widehat{\Gamma}_{\delta\phi\to N_i^c}}{\widehat{\Gamma}_{\delta\phi}}\,\varepsilon_i \;\; \text{Where}\;\; \varepsilon_i = \sum_{i\neq j} \frac{\mathrm{Im}\left[(m_{\rm D}^\dagger m_{\rm D})_{ij}^2\right]}{8\pi\langle H_u\rangle^2 (m_{\rm D}^\dagger m_{\rm D})_{ii}} \left(F_{\rm S}\left(x_{ij}, y_i, y_j\right) + F_{\rm V}(x_{ij})\right).$$

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(ii) Constraints on M_{iN^c}. To Avoid Any Erasure Of The Produced Y_L and Ensure That The ϕ Decay To ε_i Is Kinematically Allowed and M_{iN^c} are Theoretically Acceptable, We Have To Impose The Constraints:

$$(a)\ M_{1N^c} \gtrsim 10T_{\rm rh},\ (b)\ \widehat{m}_{\delta\phi} \geq 2M_{1N^c} \ \text{and}\ (c)\ M_{iN^c} \lesssim 7.1M \Leftrightarrow \lambda_{iN^c} \lesssim 3.5.$$

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$$\sqrt{c_-(\langle f_{\mathcal{R}}\rangle - Nr_\pm)}gM/\sqrt{\langle f_{\mathcal{R}}\rangle} = M_{\rm GUT} \;\Rightarrow\; M\simeq M_{\rm GUT}/g\sqrt{c_-\left(1-Nr_\pm\right)}\sim 10^{15}\ {\rm GeV} \quad \text{with} \quad g\simeq 0.7 \ \text{(GUT Gauge Coupling).}$$

(ii) Constraints on M_{iN^c}. To Avoid Any Erasure Of The Produced Y_L and Ensure That The ϕ Decay To ε_i Is Kinematically Allowed and M_{iN^c} are Theoretically Acceptable, We Have To Impose The Constraints:

$$(a)\ M_{1N^c}\gtrsim 10T_{\rm rh},\ (b)\ \widehat{m}_{\delta\phi}\geq 2M_{1N^c}\ \text{and}\ (c)\ M_{iN^c}\lesssim 7.1M \Leftrightarrow \lambda_{iN^c}\lesssim 3.5.$$

(iii) The **Achievement Of Baryogenesis** via non-Thermal Leptogenesis Dictates at 95% c.l. $Y_B = \left(8.64^{+0.15}_{-0.16}\right)\cdot 10^{-11}$.

[5] *M. Kawasaki, K. Kohri, and T. Moroi (2005); J.R. Ellis, K.A. Olive, and E. Vangioni (2005).*

Leptogenesis and $\widetilde{G}$ Abundance

- The Out-Of-Equilibrium Decay of N_i^c can Generate an L Asymmetry Which Can Be Converted to the B **Yield**:

$$Y_B = -0.35\, 2\, \frac{5}{4}\, \frac{T_{\rm rh}}{\widehat{m}_{\delta\phi}}\, \frac{\widehat{\Gamma}_{\delta\phi\to N_i^c}}{\widehat{\Gamma}_{\delta\phi}}\varepsilon_i \quad \text{Where} \quad \varepsilon_i = \sum_{i\neq j} \frac{\mathrm{Im}\left[(m_{\rm D}^\dagger m_{\rm D})_{ij}^2\right]}{8\pi\langle H_u\rangle^2 (m_{\rm D}^\dagger m_{\rm D})_{ii}} \left(F_{\rm S}\left(x_{ij}, y_i, y_j\right) + F_{\rm V}(x_{ij})\right).$$

With $x_{ij} := M_{jN^c}/M_{iN^c}$ and $y_i := \Gamma_{iN^c}/M_{iN^c} = (m_{\rm D}^\dagger m_{\rm D})_{ii}/8\pi\langle H_u\rangle^2$ and $\widehat{m}_{\delta\phi} < 2M_{iN^c}$ For Some i with $i = 1, 2, 3$.

- Here $F_{\rm V}$ and $F_{\rm S}$ Represent, Respectively, The Contributions From **Vertex And Self-Energy** Diagrams

$$F_{\rm V}\,(x) = -x \ln\left(1 + x^{-2}\right) \quad \text{and} \quad F_{\rm S}\,(x, y, z) = -2x(x^2-1)/\left(x^2-1\right)^2 + \left(x^2 z - y\right)^2$$

- The **Thermally Produced $\widetilde{G}$ Yield** At The Onset of BBN Is Estimated To Be: $Y_{\widetilde{G}} \simeq 1.9\cdot 10^{-22} T_{\rm rh}/{\rm GeV}$.

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(iii) The **Achievement Of Baryogenesis** via non-Thermal Leptogenesis Dictates at 95% c.l. $Y_B = \left(8.64^{+0.15}_{-0.16}\right)\cdot 10^{-11}$.

(iv) $\widetilde{G}$ Constraints. Assuming Unstable $\widetilde{G}$, We Impose an Upper Bound[5] on $Y_{\widetilde{G}}$ In Order to Avoid Problems With the SBB Nucleosynthesis:

$$Y_{\widetilde{G}} \lesssim \begin{cases}10^{-14}\\ 10^{-13}\end{cases} \Rightarrow\; T_{\rm rh} \lesssim \begin{cases}5.3\cdot 10^{7}\ {\rm GeV}\\ 5.3\cdot 10^{8}\ {\rm GeV}\end{cases} \text{for } \widetilde{G} \text{ Mass } m_{\widetilde{G}} \simeq \begin{cases}0.69\ {\rm TeV}\\ 10.6\ {\rm TeV}.\end{cases}$$

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Lepton-Number Asymmetry And Light Neutrino Data

- $m_{i\mathrm{D}}$ are the Dirac Masses In a Basis (Called N_i^c-Basis) Where N_i^c Are Mass Eigenstates. In the Weak (primed) Basis

$$U^\dagger m_\mathrm{D} U^{c\dagger} = d_\mathrm{D} = \mathrm{diag}\,(m_{1\mathrm{D}}, m_{2\mathrm{D}}, m_{3\mathrm{D}}) \text{ Where } L' = LU \text{ and } N^{c\prime} = U^c N^c \quad (:\text{I}).$$

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- Working in the N_i^c-Basis, the **Type I Seesaw** Formula Reads

$$m_\nu = -m_{\mathrm{D}}\, d_{N^c}^{-1}\, m_{\mathrm{D}}^{\mathsf{T}}, \ \ \text{Where}\ \ d_{N^c} = \mathrm{diag}\,(M_{1N^c}, M_{2N^c}, M_{3N^c}) \ \ \text{with}\ M_{1N^c} \leq M_{2N^c} \leq M_{3N^c}\ \text{Real and Positive.}$$

- Replacing m_{D} from Eq. (I) in the Above Equation and We Extract The Mass Matrix of Light Neutrinos In The Weak Basis

$$\bar{m}_\nu = U^\dagger m_\nu U^* = -d_{\mathrm{D}} U^c d_{N^c}^{-1} U^{c\mathsf{T}} d_{\mathrm{D}},$$

Which Can Be Diagonalized by the Unitary **PMNS Matrix** U_ν Parameterized As Follows:

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \begin{pmatrix} e^{-i\varphi_1/2} & & \\ & e^{-i\varphi_2/2} & \\ & & 1 \end{pmatrix},$$

with $c_{ij} := \cos\theta_{ij}$, $s_{ij} := \sin\theta_{ij}$, δ the CP-Violating Dirac Phase and φ_1 and φ_2 the two CP-violating Majorana Phases.

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with $c_{ij} := \cos\theta_{ij}$, $s_{ij} := \sin\theta_{ij}$, δ the CP-Violating Dirac Phase and φ_1 and φ_2 the two CP-violating Majorana Phases.

Parameter	Best Fit $\pm 1\sigma$ Normal Hierarchy	Best Fit $\pm 1\sigma$ Inverted Hierarchy
$\Delta m^2_{21}/10^{-3}\mathrm{eV}^2$	$7.6^{0.19}_{-0.18}$	
$\Delta m^2_{31}/10^{-3}\mathrm{eV}^2$	$2.38^{+0.05}_{-0.07}$	
$\sin^2\theta_{12}/0.1$	3.23 ± 0.16	
$\sin^2\theta_{13}/0.01$	2.26 ± 0.12	2.29 ± 0.12
$\sin^2\theta_{23}/0.1$	$5.67^{+0.32}_{-1.24}$	$5.73^{+0.25}_{-0.39}$
δ/π	$1.41^{+0.55}_{-0.4}$	1.48 ± 0.31

- The Masses, $m_{i\nu}$, of ν_i Are Calculated as Follows:

$$m_{2\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{21}} \text{ and}$$

$$\begin{cases} m_{3\nu} = \sqrt{m^2_{1\nu} + \Delta m^2_{31}}, & \text{for NO } m_\nu\text{'s} \\ \text{or} & \\ m_{1\nu} = \sqrt{m^2_{3\nu} + \left|\Delta m^2_{31}\right|}, & \text{for IO } m_\nu\text{'s} \end{cases}$$

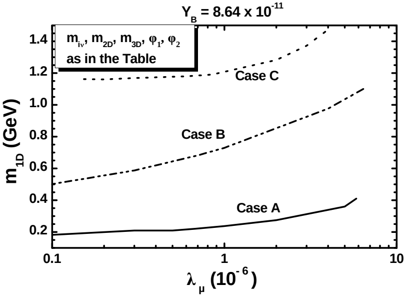
- $\sum_i m_{i\nu} \leq 0.23$ eV at 95% c.l. From *Planck* Data.

Combining Inflationary and Post-Inflationary Requirements

- To Verify The Compatibility of the Post-Inflationary Constraints, We Focus on the Following Central Values of The Parameters of the Inflationary Model

$$(n, r_\pm) = (0.042, 0.025) \;\; \rightarrow \;\; (n_{\rm s}, r) = (0.968, 0.028) \;\;\&\;\; \widehat{m}_{\delta\phi} \simeq 8.6 \cdot 10^{10}\ \mathrm{GeV}.$$

- All the Requirements can be Met Along the lines Presented in the $\lambda_\mu - m_{1\rm D}$ Plane.



CASES :	A	B	C
Hierarchy :	NO	NO	IO
$m_{r\nu}$ / eV	0.01	0.07	0.005
$\Sigma_i m_{i\nu}$ / eV	0.074	0.23	0.104
m_{2D} / GeV	1.3	5	0.9
m_{3D} / GeV	250	250	270
φ_1	$-\pi$	$\pi/2$	π
φ_2	0	$-\pi$	$-\pi/3$
M_{1N^c} / 10^{10} GeV	0.8 - 3	0.4 - 2	2.9 - 3.1
M_{2N^c} / 10^{11} GeV	0.4 - 0.6	13	0.3 - 0.4
M_{3N^c} / 10^{15} GeV	1	0.1	5.2

- We take $m_{r\nu} = m_{1\nu}$ for NO ν_i's and $m_{r\nu} = m_{3\nu}$ for IO ν_i's .
- **The Inflaton Decays into the Lightest and Next-to-Lightest of RHN** Since $2M_{iN^c} > \widehat{m}_{\delta\phi}$ for $i = 3$.
- Y_B Is Equal to its Central Value and **the $\widetilde{G}$ Constraint is Under Control** Even for $m_{3/2} \sim 1$ TeV Since We Obtain

$$0.7 \lesssim Y_{\widetilde{G}}/10^{-15} \lesssim 3 \;\; \text{and} \;\; 0.4 \lesssim T_{\rm rh}/10^7\mathrm{GeV} \lesssim 1.8\,.$$

Conclusions

• We Proposed A Variant of non-Minimal HI (named **Kinetically Modified**) Which Can Safely Accommodate **Observable Gravitational Waves**[6] With **Subplanckian** Inflaton Values and Without Causing Any Problem With The Validity Of the Effective Theory.

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• This Scenario can be Elegantly Implemented Within a $B-L$ SUSY GUT, Adopting A Superpotential Determined by an R-Symmetry and Several **SemiLogarithmic Kähler Potentials** Which Respect a Softly Broken **Shift Symmetry**.

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• Combined Restrictions from **Baryogenesis via nTL, $\widetilde{G}$ Constraints and Neutrino Data** can be Met Even for $m_{3/2} \sim 1$ TeV, With The Inflaton Decaying Mainly to N_1^c and N_2^c – We Obtain M_{iN^c} in the Range $(10^9 - 10^{15})$ GeV.

Thank You!

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