Some Aspects of Non-linear SUSY

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Introduction

Non-linear SUSY appears as low effective energy of spontaneous broken SUSY.

Why study non-linear supersymmetry?

- Academic: SQFT very useful, teach us many things (ex: AdS/CFT, brane worlds ...). Master QFTs with non-linear supersymmetry.
- Supersymmetry not seen: may be it is there at high energies. Any implications for observable world?

Spontaneous breaking of SUSY leads to a Goldstone fermion: the goldstino.

Suppose it originates from the superfield X

$$X = \phi + \sqrt{2}\theta\psi + \theta\theta F,$$

Wiith the supersymmetry transformation algebra:

$$\begin{aligned}
\delta_\epsilon \phi &= \epsilon\psi, \\
\delta_\epsilon \psi_\alpha &= -i(\sigma^\mu \overline{\epsilon})_\alpha \partial_\mu \phi + \epsilon_\alpha F, \\
\delta_\epsilon F &= -i\overline{\epsilon}\overline{\sigma}^\mu \partial_\mu \psi.
\end{aligned}$$

Here ϕ is called the sgoldstino

We want to decouple the heavy complex scalar at low energy
We want to write ϕ as a function of ψ and F i.e.

$$\phi \to \phi(\psi, F)$$

Then

$$\begin{aligned} \delta_\epsilon \phi(\psi, F) &= \frac{\partial \phi}{\partial \psi_\alpha} \delta_\epsilon \psi_\alpha + \frac{\partial \phi}{\partial F} \delta_\epsilon F \\ \epsilon \psi &= \frac{\partial \phi}{\partial \psi_\alpha} [-i(\sigma^\mu \overline{\epsilon})_\alpha \partial_\mu \phi + \epsilon_\alpha F] - \frac{\partial \phi}{\partial F} (i\overline{\epsilon \overline{\sigma}}^\mu \partial_\mu \psi). \end{aligned}$$

Solution:

$$\phi = \frac{\psi\psi}{2F}.$$

The chiral multiplet can be written as:

$$X = \frac{\psi\psi}{2F} + \sqrt{2}\theta\psi + \theta\theta F,$$

One constraint ⟶ One component projected : X is nilpotent

$$\boxed{X^2 = 0}$$

We can also get rid of the F-term by

$$\boxed{\bar{X} D^2 X \propto \bar{X}}$$

One can use couplings of X to describe the couplings of goldstino

Because of the nilpotent constraint, the general form for the Lagrangian without supersymmetric covariant derivatives for this superfield is:

$$\begin{aligned}\mathcal{L}_X &= \int d^4\theta \overline{X} X + (\int d^2\theta f X + h.c.) \\ &\to \mathcal{L}_{AV}\end{aligned}$$

This recovers Volkov-Akulov action after field-redefinition.

Rocek, Lindstrom - Rocek, ... ’82

Komargodski - Seiberg '09

The goldstino X controls the non-conservation of the Ferrara-Zumino supercurrent $\mathcal{J}_{\alpha\dot{\alpha}}$:

$$\overline{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = DX$$

One constraint ⟶ One d.o.f. projected : X is nilpotent

$$\boxed{X^2 = 0}$$

We can also ~~get rid of~~ keep the F-term

$$\boxed{\cancel{\bar{X}D^2X \propto \bar{X}}}$$

$$\begin{aligned}\mathcal{L}_X &= \int d^4\theta\overline{X}X + (\int d^2\theta f X + h.c.)\\ &\to \mathcal{L}_{AV}\end{aligned}$$

One can use couplings of X to describe the couplings of goldstino

One can use X to impose constraint on superfields in order to:

- project complex scalar of a chiral multiplet Q

$$\overline{X}XQ = 0$$

- project fermions of a gauge W_α and a matter multiplet $\mathcal{H}$

$$XW_\alpha = 0;$$
$$X\overline{D}_{\dot{\alpha}}\overline{\mathcal{H}} = 0$$

- project the fermion, one real scalar and the auxiliary field of a chiral multiplet $\mathcal{A}$

$$X(\mathcal{A} + \overline{\mathcal{A}}) = 0.$$

Which microscopic theory gives rise to these constraints?

With X one can be construct a real (vector) superfield V_{NL}

$$V_{NL} = \frac{\overline{X}X}{|F|^2}$$

It satisfies

$$\begin{aligned} V_{NL} &= V_{NL}^{\dagger} \\ V_{NL}^2 &= 0 \\ V_{NL}\,D\overline{D}^2 D\,V_{NL} + V_{NL}\,\overline{D}D^2\overline{D}V_{NL} &\propto V_{NL} \end{aligned}$$

Rocek - Lindstrom '79 - Samuel - Wess '83

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It satisfies

$$\begin{aligned} V_{NL} &= V_{NL}^{\dagger} \\ V_{NL}^2 &= 0 \end{aligned}$$

$$\boxed{V_{NL} D \overline{D}^2 D V_{NL} + V_{NL} \overline{D} D^2 \overline{D} V_{NL} \propto V_{NL}}$$

Note that the last condition uses the assumption

$$\boxed{\bar{X} D^2 X \propto \bar{X}}$$

Non-linear FI

First: components

Fayet-Iliopoulos model: $m^2 > \frac{1}{2}g\xi$

A $U(1)$ vector superfield and two chiral superfields $\Phi_\pm$

$$\mathcal{L} = \int d^2\theta(\frac{1}{4}W^\alpha W_\alpha + m\Phi_+\Phi_-) + h.c. + \int d^4\theta(\overline{\Phi}_+ e^{2gV}\Phi_+ + \overline{\Phi}_- e^{-2gV}\Phi_+ 2\xi V)$$

When $m^2 > \frac{1}{2}g\xi$

- Supersymmetry is spontaneously broken.
- Massless: gauge boson and gaugino = goldstino

Samuel-Wess '82: $V \to V_{NL}$ allows to read the soft terms.

Fayet-Iliopoulos model: $m^2 < \frac{1}{2}g\xi$

When $m^2 < \frac{1}{2}g\xi$

$$|\phi_-| = v\,, \qquad \frac{g^2v^2}{2} = \xi g - m^2$$
$$D = -\frac{m^2}{g}\,, \qquad F_+^* = -\frac{mv}{\sqrt{2}}$$

The fermionic eigenstates are

$$\begin{aligned} \psi &= \psi_-, \\ \tilde{\psi} &= \frac{1}{\sqrt{m^2+g^2v^2}}(m\psi_+ - gv\lambda), \\ \tilde{\lambda} &= \frac{1}{\sqrt{m^2+g^2v^2}}(m\lambda + gv\psi_+). \end{aligned}$$

Now only $\tilde{\lambda}$ is massless. It is the goldstino.

Fayet-Iliopoulos integrated out

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We can integrate out the massive state. They become:

$$\begin{aligned}
A_\mu &= -\frac{g}{m^2+g^2v^2}\tilde{\lambda}\sigma^\mu\bar{\tilde{\lambda}} + \dots \\
\phi_+ &= -\frac{g^2 v}{\sqrt{2}m(m^2+g^2v^2)}\tilde{\lambda}\tilde{\lambda} + \mathcal{O}(\tilde{\lambda}^4) \\
\psi_- &= -\frac{g^3 v}{m(m^2+g^2v^2)^{3/2}}\overline{\tilde{\lambda}\tilde{\lambda}}\tilde{\lambda} + \dots \\
\tilde{\psi} &= -\frac{g^3 v}{m(m^2+g^2v^2)^2} i\sigma^\mu\partial_\mu(\tilde{\lambda}\tilde{\lambda}\bar{\tilde{\lambda}}) + \dots
\end{aligned}$$

Fayet-Iliopoulos integrated out

$$\begin{aligned}\lambda =& \frac{1}{\sqrt{m^2+g^2v^2}}(gv\tilde{\lambda}+m\tilde{\psi}) \\ =& \frac{1}{\sqrt{m^2+g^2v^2}}gv\tilde{\lambda}-\frac{g^3v}{(m^2+g^2v^2)^{5/2}}i\sigma^\mu\partial_\mu(\tilde{\lambda}\tilde{\lambda}\bar{\tilde{\lambda}})+\ldots \\ =& \frac{gv}{\sqrt{m^2+g^2v^2}}\left[\tilde{\lambda}+\frac{g^2}{(m^2+g^2v^2)^2}i\sigma^\mu\partial_\mu[(\tilde{\lambda}\tilde{\lambda})\bar{\tilde{\lambda}}]+\ldots\right] \qquad (1)\end{aligned}$$

Fayet-Iliopoulos integrated out

Thus we can write the vector multiplet V in terms of Goldstone fermion $\tilde{\lambda}$:

$$V(\lambda, v^{\mu}, D) \to \frac{m}{\sqrt{m^2 + \frac{1}{2}e^2 v^2}} V(\tilde{\lambda}, \frac{\tilde{\lambda}\sigma^{\mu}\bar{\tilde{\lambda}}}{\tilde{D}}, \tilde{D}).$$

or as a function of the chiral multiplet:

$$\Phi_+(\phi_+, \psi_+, F_+) \to \frac{gv}{\sqrt{m^2 + g^2 v^2}} \hat{\Phi}_+(\frac{\tilde{\lambda}\tilde{\lambda}}{2\tilde{F}}, \tilde{\lambda}, \tilde{F}) \qquad (2)$$

with

$$V(\lambda, v^{\mu}, D) \sim -\frac{\overline{\Phi}_+ \Phi_+}{gv^2}$$

Both Φ_+ can be used to represent the goldstino and it is nilpotent.

Second: superspace

Fayet-Iliopoulos in super-unitary gauge

Let us use the super-unitary gauge.

$$\begin{aligned} V &= V' + i(\Lambda - \Lambda^\dagger) \\ \Phi_+ &= e^{-ie\Lambda}\Phi'_+ \\ \Phi_- &= e^{ie\Lambda}\Phi'_- = \frac{v}{\sqrt{2}} \end{aligned}$$

The Lagrangian becomes:

$$\int d^2\theta(\frac{1}{4}W^\alpha W_\alpha + \frac{1}{\sqrt{2}}mv\Phi_+) + h.c. + \int d^4\theta(\overline{\Phi}_+ e^{2gV}\Phi_+ + \frac{1}{2}v^2 e^{-2gV} + 2\xi V).$$

Using

$$V = \theta^4 \frac{1}{2}(-\xi + \frac{gv^2}{2}) + \hat{V}$$

The equation of motion for V reads:

$$0 = \frac{1}{8}(D^\alpha \overline{D}^2 D_\alpha + h.c.)\hat{V} + 2g\overline{\Phi}_+ e^{2gV}\Phi_+ + gv^2(1 - e^{-2gV})$$

Nilpotent superfield

Solving the equation of motion

$$\begin{aligned} e^{-2gV} =& \frac{1}{2}\left[1+\sqrt{1+\frac{8\overline{\Phi}_+\Phi_+}{v^2}}\right] \\ gV \simeq& -\frac{\overline{\Phi}_+\Phi_+}{v^2} + 3\frac{\overline{\Phi}_+\Phi_+\overline{\Phi}_+\Phi_+}{v^4} + \dots \end{aligned} \tag{3}$$

Putting back in the Lagrangian gives:

$$\mathcal{L} \supset \int d^4\theta \quad c_1\overline{\Phi}_+\Phi_+ - c_2|\overline{\Phi}_+\Phi_+|^2 + \dots \tag{4}$$

Therefore Φ_+ represents the goldstino and it is nilpotent

Third: supercurrent

The goldstino X controls the non-conservation of the Ferrara-Zumino supercurrent $\mathcal{J}_{\alpha\dot{\alpha}}$:

$$\overline{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X$$

Where

$$X = 4W - \frac{1}{3}\overline{DD}\left[K + 4g\xi(V + i\Lambda - i\Lambda^\dagger)\right] \tag{5}$$

Using equations of motion:

$$X \propto \Phi_+ \tag{6}$$

A constraint

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- project fermions of a gauge W_α and a matter multiplet $\mathcal{H}$

$$XW_\alpha = 0;$$
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- project the fermion, one real scalar and the auxiliary field of a chiral multiplet $\mathcal{A}$

$$X(\mathcal{A} + \overline{\mathcal{A}}) = 0.$$

Which microscopic theory gives rise to these constraints?

Komargodski - Seiberg '09 propose the constraint

$$\boxed{X(\mathcal{A}+\overline{\mathcal{A}})=0}. \tag{7}$$

Dall'Agata - Dudas - Farakos '16 prove that it is equivalent to

$$\begin{aligned} |X|^2(\mathcal{A}+\overline{\mathcal{A}}) &= 0, & (8)\\ |X|^2\overline{D}_{\dot\alpha}\overline{\mathcal{A}} &= 0, & (9)\\ |X|^2\overline{D}^2\overline{\mathcal{A}} &= 0. & (10) \end{aligned}$$

which remove the real scalar, the fermion and the auxiliary field

As a microscopic theory Dall'Agata - Dudas - Farakos '16 propose:

$m_0 \to \infty$, $c_F \to \infty$ and $m_{1/2} \to \infty$ in $\int d^4\theta$ of:

$$\frac{m_0}{2f^2}|X|^2(\mathcal{A}+\overline{\mathcal{A}})^2 - \frac{c_F}{f^2}|X|^2 D^2\mathcal{A}\overline{D}^2\overline{\mathcal{A}} - \frac{m_{1/2}}{2f^2}(|X|^2 D^\alpha \mathcal{A} D_\alpha \mathcal{A} + h.c.)$$

This contains higher derivatives.

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If $\mathcal{A}^a$ is in an adjoint representation:

$$\boxed{-\frac{m_D}{4\sqrt{2}f^2}\int d^2\theta \overline{D}^2 D^\alpha(\overline{X}X)W^a_\alpha \mathcal{A}^a}$$

Equation of motion to the $\mathcal{A}^a$:

$$\overline{D}^2 D^\alpha(\overline{X}X)W^a_\alpha = 0.$$

→ acting by $\overline{X}XD_\beta$ to the left hand side gives:

$$\boxed{\overline{X}XW^a_\alpha = 0},$$

(uses the non-zero property of the $\overline{D}^2 D^2(\overline{X}X)$ and the nilpotency $XD^\alpha X = 0$).

Next, we do the equation of motion to the W_α^a and get:

$$D^\alpha(\overline{D}^2 D^\alpha(\overline{X}X)\mathcal{A}^a) + h.c. = 0.$$

This leads to the elimination of the auxiliary field:

$$\boxed{\overline{X}XD^2\mathcal{A}^a = 0},$$

We can plug $\overline{X}X$ to the left hand side and using the nilpotency of X:

$$\boxed{\overline{X}X(\mathcal{A}^a + \overline{\mathcal{A}^a}) = 0},$$

which eliminates the real part of the scalar.

Minimal Gravitino Dark Matter

Gravitino as Dark Matter

Assumptions

- $T_{RH} < M_{SUSY} \longrightarrow$ Particles not produced after inflation ,
- $m_{3/2} \ll M_{SUSY} \longrightarrow$ Not gravity mediation SUSY breaking

Light states= SM + gravitino + $\cdots$
where $\cdots$ not super-particles and not dark matter

Gravitinos produced by the SM scattering in the thermal bath.

Gravitinos never in thermal equilibrium!

(K.B.-Chen-Dudas-Mambrini)

Gravitino as Dark Matter

Scattering described by the longitudinal modes (Goldstinos) through dim-8 operators, as

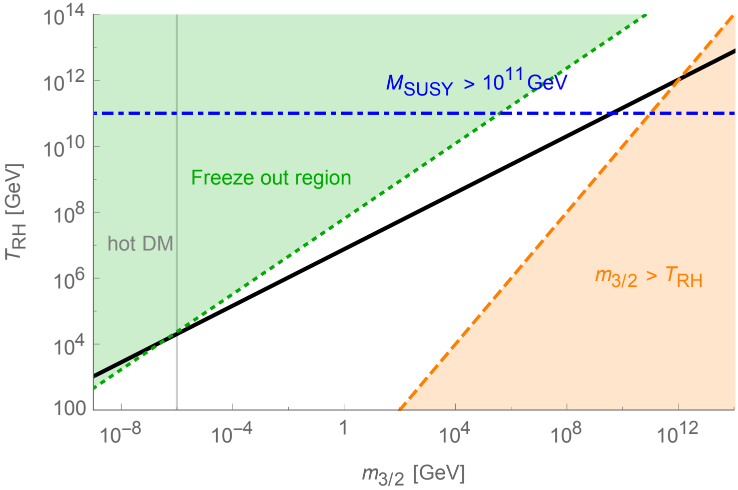
$$\frac{i}{2M_{Pl}^2 m_{3/2}^2}(G\sigma^\mu\partial^\nu\bar{G}-\partial^\nu G\sigma^\mu\bar{G})(\partial_\mu H\partial_\nu H^\dagger+\partial_\nu H\partial_\mu H^\dagger),$$

Leads to

$$\Omega_{3/2}h^2\simeq 0.11\left(\frac{100\ GeV}{m_{3/2}}\right)^3\left(\frac{T_{RH}}{5.4\times 10^7\ GeV}\right)^7$$

Therefore T_{RH} not very sensitive to the number of d.o.f's in the Universe.
SUSY provides the gravitino as the DM candidate

T_{RH} vs gravitino mass



Lowest allowed SUSY scale is of order 10 TeV

Summary

Summary

- Some non-linear SQFT can be written with the powerful tool of superfields
- We showed how this can be done in the case of FI model to use it.
- May be SUSY is there just to provide the gravitino as DM. How can we detect it?