Implications of the complex singlet field for noncommutative geometry model

Hosein Karimi

Supervisor: Prof. Ali Chamseddine

American University of Beirut

Corfu 2017

- There has been many attempts in mathematics to go beyond Riemannian geometry and establish more generalized notions.

- In 1980, Alain Connes was able to provide applicable set of axioms and defined a new notion of geometry based on Spectral Geometry and Operator Algebra

$$(A, H, D)$$

- In his original paper, he also provided an example of a noncommutative torus.

Alain Connes. C* algebras and dierential geometry. In: Compt. Rend. Hebd. Seances Acad. Sci. A290.13 (1980), pp. 599604. arXiv: hep-th/0101093 [hep-th]

- Later in 1996, Ali Chamseddine and Alain Connes found an application of this new geometry in physics.

- They assumed the space-time is a direct product of 4D Riemannian manifold with a noncommutative space.

- They also introduced the spectral action which is based on the spectrum of the Dirac operator and were able to show that the standard model arises naturally and almost uniquely from these assumptions.

Ali H. Chamseddine and Alain Connes. The Spectral action principle. In: Commun. Math. Phys. 186 (1997), pp. 731750. doi: 10.1007/s002200050126. arXiv: hep-th/9606001 [hep-th]

Generalized Space:

M×F

(A, H, D, J, γ)

$\gamma_5 \otimes \gamma_F$

$\mathcal{A} = C^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}))$

$J_M \otimes J_F$

16 fermions

$L^2(M, S) \otimes \mathcal{H}_F$

$D_M \otimes 1_{96\times 96} + \gamma_5 \otimes D_F$

Dirac Operator:

$$D_M \otimes 1_{96\times 96} + \gamma_5 \otimes D_F$$

$$\begin{pmatrix} D_A^B & D_A^{B'} \\ D_{A'}^B & D_{A'}^{B'} \end{pmatrix} \quad D_{A'B} = \overline{D}_{AB'}, \quad D_{A'B'} = \overline{D}_{AB}$$

$$D_{AB'} = \begin{pmatrix} \sigma & 0.. \\ 0.. & 0.. \end{pmatrix}$$

Parts:

$$D_M = \gamma^\mu \left(\partial_\mu + \omega_\mu\right)$$

$$J_F^2 = 1, \qquad J_F D_F = D_F J_F, \qquad J_F \gamma_F = -\gamma_F J_F$$

Invariant inner product

$$D_A = D + A + JAJ^{-1} \qquad A = \sum a\,[D, b]$$

$$\langle J\psi, D_A \psi \rangle$$

Ali H. Chamseddine and Alain Connes. Noncommutative Geometry as a Framework for Unification of all Fundamental Interactions including Gravity. Part I. In: Fortsch. Phys. 58 (2010), pp. 553600. doi: 10.1002/prop.201000069. arXiv: 1004.0464[hep-th]

Dirac Operator:

$$D_{AB} = \gamma^{\mu} \otimes \begin{pmatrix} D_\mu & & & & & \\ & D_\mu + i g_1 B_\mu & & & 0 & \\ & & (D_\mu + \frac{i}{2} g_1 B_\mu) I_{2\times 2} - \frac{i}{2} g_2 W^i_\mu \sigma_i & & & \\ & & & (D_\mu - \frac{2i}{3} g_1 B_\mu) I_{3\times 3} - \frac{i}{2} g_3 V^a_\mu \lambda_a & & \\ & 0 & & & (D_\mu + \frac{i}{3} g_1 B_\mu) I_{3\times 3} - \frac{i}{2} g_3 V^a_\mu \lambda_a & \\ & & & & & (D_\mu - \frac{i}{6} g_1 B_\mu) I_{6\times 6} - \frac{i}{2} g_3 V^a_\mu \lambda_a I_{2\times 2} - \frac{i}{2} g_2 W^i_\mu \sigma_i I_{3\times 3} \end{pmatrix} \otimes 1_3$$

$$+ \gamma^5 \otimes \begin{pmatrix} 0_3 & 0 & (\epsilon^{ab} H_b \otimes k^{*\nu})_{6\times 3} & 0 & 0 & 0 \\ 0 & 0_3 & (\bar{H}^a \otimes k^{*e})_{6\times 3} & 0 & 0 & 0 \\ (\epsilon_{ab} \bar{H}^b \otimes k^{\nu})_{3\times 6} & (H_a \otimes k^e)_{3\times 6} & 0_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0_9 & 0 & (\epsilon^{ab} H_b \delta^j_i \otimes k^{*u})_{18\times 9} \\ 0 & 0 & 0 & 0 & 0_9 & (\bar{H}^a \delta^j_i \otimes k^{*d})_{18\times 9} \\ 0 & 0 & 0 & (\epsilon_{ab} \bar{H}^b \delta^i_j \otimes k^u)_{9\times 18} & (H_a \delta^i_j \otimes k^d)_{9\times 18} & 0_{18} \end{pmatrix}$$

- The Spectral Action:

$$S = \mathrm{Tr}\left(f(D/\Lambda)\right) + \langle \psi, D\psi \rangle$$

$$\sum_{n=0}^{\infty} \Lambda^{4-n} F_{4-n} a_n$$

$$c\overline{\nu}_R \nu_R + C.C. + \text{fermionic and Yukawa interactions}$$

Seeley deWitt coefficients

- Ali H. Chamseddine and Alain Connes. The Spectral action principle. In: Commun. Math. Phys. 186 (1997), pp. 731750. doi: 10.1007/s002200050126. arXiv: hep-th/9606001 [hep-th]
- Peter B. Gilkey. The Spectral geometry of a Riemannian manifold. In: J. Di. Geom. 10.4 (1975), pp. 601618

(5.49)

$$
\begin{aligned}
S_{\mathrm{b}} = {} & \frac{24}{\pi^2} F_4 \Lambda^4 \int d^4x \sqrt{g} \\
& - \frac{2}{\pi^2} F_2 \Lambda^2 \int d^4x \sqrt{g} \left(R + \frac{1}{2} a \overline{H} H + \frac{1}{4} c \sigma^2 \right) \\
& + \frac{1}{2\pi^2} F_0 \int d^4x \sqrt{g} \Bigg[\frac{1}{30} \left(-18 C_{\mu\nu\rho\sigma}^2 + 11 R^* R^* \right) + \frac{5}{3} g_1^2 B_{\mu\nu}^2 + g_2^2 \left(W_{\mu\nu}^{\alpha} \right)^2 + g_3^2 \left(V_{\mu\nu}^{m} \right)^2 \\
& \qquad + \frac{1}{6} a R \overline{H} H + b \left(\overline{H} H \right)^2 + a \left| \nabla_\mu H_a \right|^2 + 2e \overline{H} H \, \sigma^2 + \frac{1}{2} d \, \sigma^4 + \frac{1}{12} c R \sigma^2 + \frac{1}{2} c \left(\partial_\mu \sigma \right)^2 \Bigg] \\
& + F_{-2} \Lambda^{-2} a_6 + \cdots
\end{aligned}
$$

Ali H. Chamseddine and Alain Connes. Noncommutative Geometry as a Framework for Unification of all Fundamental Interactions including Gravity. Part I. In: Fortsch. Phys. 58 (2010), pp. 553600. doi: 10.1002/prop.201000069. arXiv: 1004.0464[hep-th]

Scalar sector:

$$V = \frac{1}{2} m_h^2 H^2 + \frac{1}{2} m_\sigma^2 |\sigma|^2 + \frac{1}{4} \lambda_\sigma |\sigma|^4 + \frac{1}{4} \lambda_h H^4 + \frac{1}{2} \lambda_{h\sigma} |\sigma|^2 H^2$$

$$\lambda_h = \frac{n^2+3}{(n+3)^2} \left(4g^2\right)$$

$$\lambda_{h\sigma} = \frac{2n}{n+3} \left(4g^2\right)$$

$$\lambda_\sigma = 2\left(4g^2\right)$$

$$\frac{5}{3} g_1^2 = g_2^2 = g_3^2$$

$$k^\nu = \sqrt{n}\, k^u$$

The scalar field can be complex or real. This adds vertice and changes the renormalization group equations (since there is the kinetic term)

Ali H. Chamseddine and Alain Connes. Resilience of the Spectral Standard Model. In: JHEP 09 (2012), p. 104. doi: 10.1007/JHEP09(2012)104. arXiv: 1208.1030 [hep-ph]

- The potential has local minimum which occurs when

$$\lambda_\sigma|\sigma|^2+\lambda_{h\sigma}H^2+m_\sigma^2=0, \quad \lambda_h H^2+\lambda_{h\sigma}|\sigma|^2+m_H^2=0$$

- We choose the following form of expectation values to formulate the symmetry breaking:

$$H=\begin{pmatrix}0\\ h+v\end{pmatrix}, \quad v=\langle h\rangle_0$$

$$\sigma=w+\sigma_1+i\sigma_2, \quad w=\langle\sigma_1\rangle_0$$

- Diagonalizing the mass matrix yields:

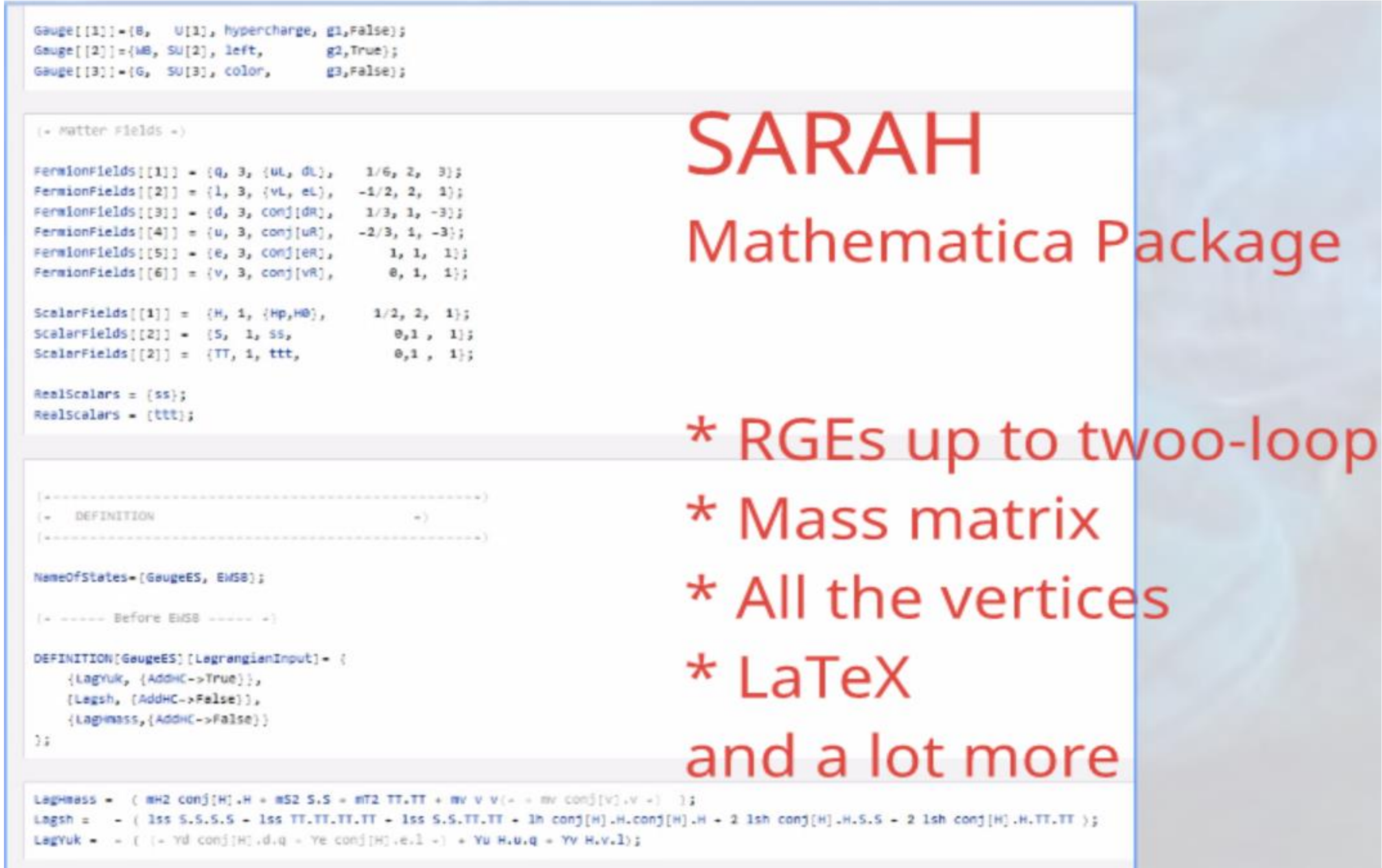
$$m_{\pm}^2 = \left(v^2\lambda_h + \frac{w^2}{4}\lambda_\sigma\right)\left(1 \pm \left(1 - \frac{v^2w^2\lambda_h\lambda_\sigma - v^2w^2(\lambda_{h\sigma})^2}{(v^2\lambda_h + \frac{w^2}{4}\lambda_\sigma)^2}\right)^{\frac{1}{2}}\right)$$

$$M = w\sqrt{\frac{\lambda_\sigma}{2}}, \qquad m_h = v\sqrt{2\lambda_h}\sqrt{1 - \frac{\lambda_{h\sigma}^2}{\lambda_h\lambda_\sigma}}.$$

Ali H. Chamseddine and Alain Connes. Resilience of the Spectral Standard Model. In: JHEP 09 (2012), p. 104. doi: 10.1007/JHEP09(2012)104. arXiv: 1208.1030 [hep-ph]

- To check whether this is consistent with the experiments or not, we need to use renormalization group equations and run the couplings down toward low energies.
- The scalar couplings are related to Yukawa and gauge couplings. We use these relations as initial conditions at the unification scale.
- The value of unification scale is not predicted and is one of the free parameters of the model which can be explored by fitting the experimental masses of the particles.
- The Higgs mass is coming from a nonlinear relation and small corrections could in principle matter.

- SARAH is a very powerful package under Mathematica to find renormalization group equations.



Renormalization group equations with two loop corrections:

$$\frac{dg_1}{dt} = \frac{41\,{g_1}^3}{160\,\pi^2} + \frac{{g_1}^3}{12800\,\pi^4}\Big(-15\,{K_\nu}^2 - 85\,{K_t}^2 + 199\,{g_1}^2 + 135\,{g_2}^2 + 440\,{g_3}^2\Big),$$

$$\frac{dg_2}{dt} = -\frac{19\,{g_2}^3}{96\,\pi^2} + \frac{{g_2}^3}{7680\,\pi^4}\Big(-15\,{K_\nu}^2 - 45\,{K_t}^2 + 27\,{g_1}^2 + 175\,{g_2}^2 + 360\,{g_3}^2\Big),$$

$$\frac{dg_3}{dt} = -\frac{7\,{g_3}^3}{16\,\pi^2} + \frac{{g_3}^3}{2560\,\pi^4}\Big(-20\,{K_t}^2 + 11\,{g_1}^2 + 45\,{g_2}^2 - 260\,{g_3}^2\Big),$$

$$\frac{dK_\nu}{dt} = \frac{K_\nu}{16\pi^2}\Big(-9/20\,g_1^2 - 9/4\,{g_2}^2 + 3\,{K_t}^2 + 5/2\,{K_\nu}^2\Big) + \frac{1}{256\,\pi^4}\Bigg(1/40\,\Big(21\,{g_1}^4 - 54\,{g_1}^2{g_2}^2 -$$

$$230\,{g_2}^4 + 240\,{\lambda_h}^2 + 80\,{\lambda_{sh}}^2 + 5\,\left(17\,{g_1}^2 + 45\,{g_2}^2 + 160\,{g_3}^2\right){K_t}^2 + 15\,\left({g_1}^2 + 5\,{g_2}^2\right){K_\nu}^2$$

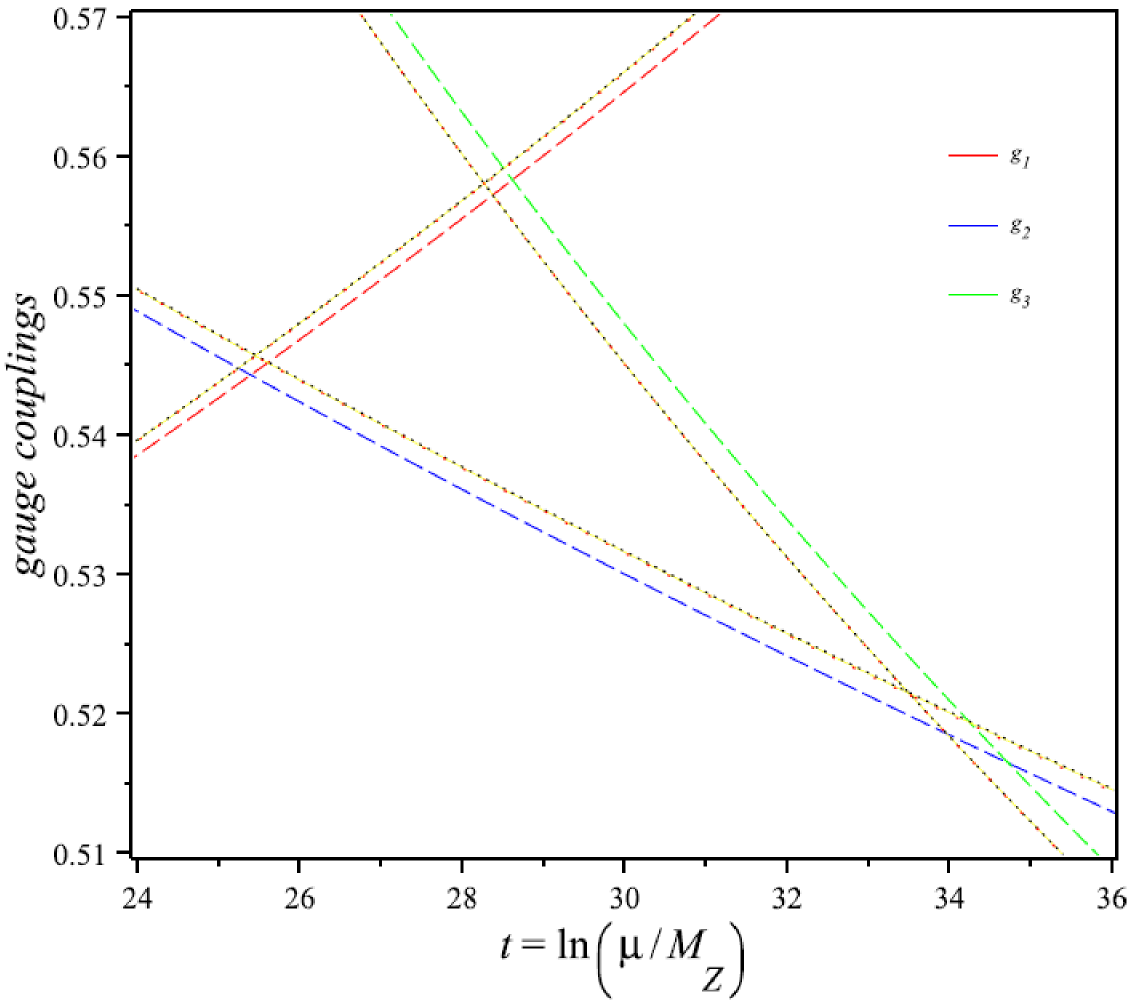
$$-270\,{K_t}^4 - 90\,{K_\nu}^4\Big)K_\nu + \frac{{K_\nu}^3}{80}\Big(-60\,{K_\nu}^2 - 540\,{K_t}^2 + 279\,{g_1}^2 + 675\,{g_2}^2 - 960\,\lambda_h\Big)\Bigg),$$

$$\frac{dK_t}{dt} = \frac{K_t}{16\pi^2}\Bigg(-\frac{17\,{g_1}^2}{20} - 9/4\,{g_2}^2 - 8\,{g_3}^2$$

$$+ 9/2\,{K_t}^2 + {K_\nu}^2\Bigg) + \frac{1}{256\,\pi^4}\Bigg(\frac{K_t}{600}\,(1187\,{g_1}^4 - 270\,{g_1}^2{g_2}^2 - 3450\,{g_2}^4 + 760\,{g_1}^2{g_3}^2 + 5400\,{g_2}^2{g_3}^2 -$$

$$64800\,{g_3}^4 + 3600\,{\lambda_h}^2 + 1200\,{\lambda_{sh}}^2 + 75\,\left(17\,{g_1}^2 + 45\,{g_2}^2 + 160\,{g_3}^2\right){K_t}^2 + 225\,\left({g_1}^2 + 5\,{g_2}^2\right){K_\nu}^2 - 4050$$

$${K_t}^4 - 1350\,{K_\nu}^4) + \left(\frac{223\,{g_1}^2}{80} + \frac{135\,{g_2}^2}{16} + 16\,{g_3}^2 - 12\,\lambda_h - \frac{27\,{K_t}^2}{4} - 9/4\,{K_\nu}^2\right){K_t}^3 + 3/2\,{K_t}^5\Bigg),$$

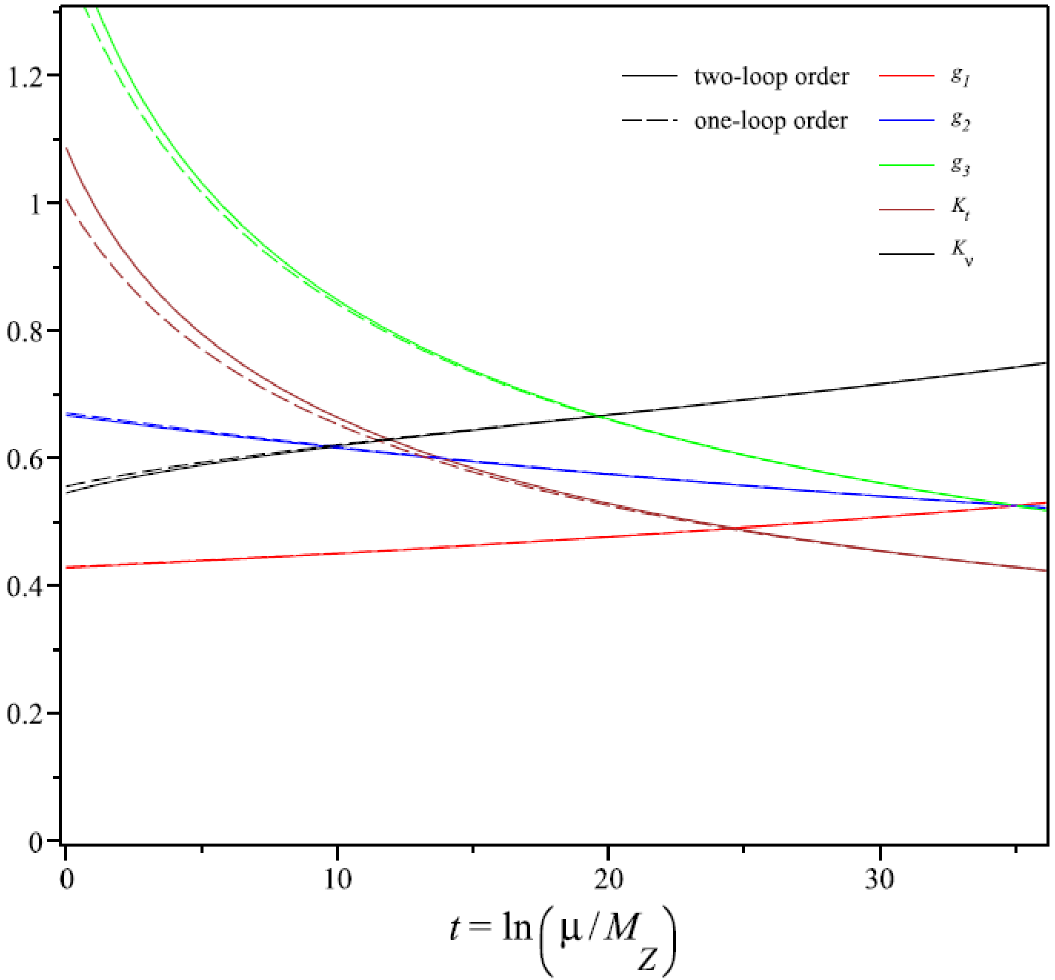


0.57
0.56
0.55
0.54
0.53
0.52
0.51
gauge couplings
24
26
28
30
32
34
36
$t = \ln\left(\mu / M_Z\right)$
g_1
g_2
g_3

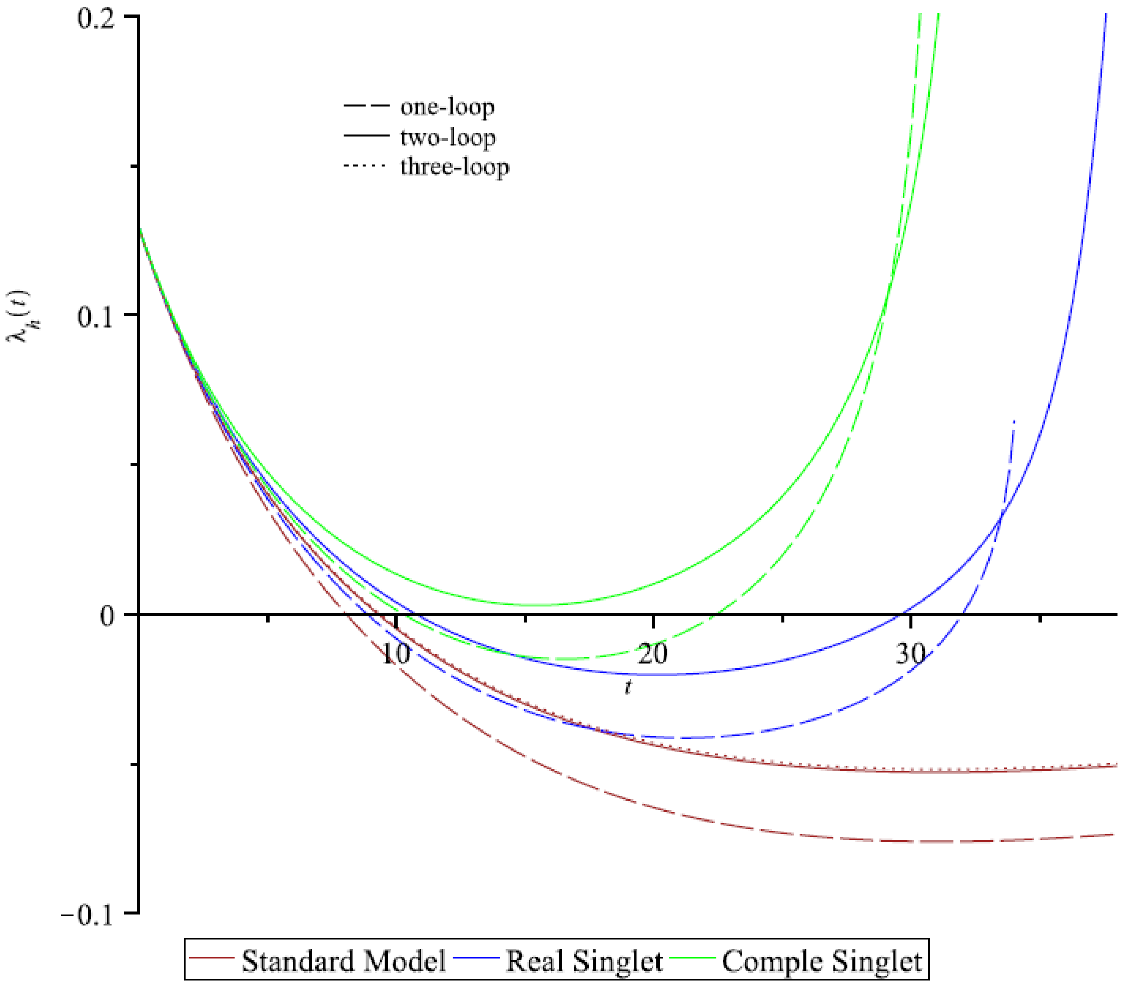
$$
\begin{aligned}
\frac{d\lambda_h}{dt} = \frac{1}{16\pi^2}\Bigg(&\frac{27\,{g_1}^4}{200} + \frac{9\,{g_1}^2{g_2}^2}{20} + \frac{9\,{g_2}^4}{8} \\
&- 9/5\,{g_1}^2\lambda_h - 9\,{g_2}^2\lambda_h + 24\,{\lambda_h}^2 + 4\,{\lambda_{sh}}^2 + 12\,\lambda_h\,{K_t}^2 + 4\,\lambda_h\,{K_\nu}^2 - 6\,{K_t}^4 - 2\,{K_\nu}^4\Bigg) \\
+ \frac{1}{256\,\pi^4}\Bigg(&-\frac{3411\,{g_1}^6}{2000} - \frac{1677\,{g_1}^4{g_2}^2}{400} - \frac{289\,{g_1}^2{g_2}^4}{80} + \frac{305\,{g_2}^6}{16} + \frac{1887\,{g_1}^4\lambda_h}{200} + \frac{117\,{g_1}^2{g_2}^2\lambda_h}{20} - \frac{73\,{g_2}^4\lambda_h}{8} \\
&+ \frac{108\,{g_1}^2{\lambda_h}^2}{5} + 108\,{g_2}^2{\lambda_h}^2 - 312\,{\lambda_h}^3 - 40\,\lambda_h\,{\lambda_{sh}}^2 - 32\,{\lambda_{sh}}^3 \\
&+ \left(-\frac{171\,{g_1}^4}{100} - 9/4\,{g_2}^4 + \frac{45\,{g_2}^2\lambda_h}{2} + 80\,{g_3}^2\lambda_h - 144\,{\lambda_h}^2 + 1/10\,{g_1}^2\left(63\,{g_2}^2 + 85\,\lambda_h\right)\right){K_t}^2 - \frac{{K_\nu}^2}{200}\left(18\,{g_1}^4 + 15\,{g_1}^2\left(4\,{g_2}^2 - 20\,\lambda_h\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\lambda_{sh}}{dt} = &\frac{\lambda_{sh}}{160\,\pi^2}\left(60\,{K_t}^2 + 20\,{K_\nu}^2 - 9\,{g_1}^2 - 45\,{g_2}^2 + 120\,\lambda_h + 80\,\lambda_{sh} + 80\,\lambda_s\right) \\
&- \frac{1}{102400\,\pi^4}\lambda_{sh}\,\Big(-1671\,{g_1}^4 - 450\,{g_1}^2{g_2}^2 + 3625\,{g_2}^4 - 5760\,{g_1}^2\lambda_h - 28800\,{g_2}^2\lambda_h + 24000\,{\lambda_h}^2 \\
&\quad - 480\,{g_1}^2\lambda_{sh} - 2400\,{g_2}^2\lambda_{sh} + 57600\,\lambda_h\,\lambda_{sh} + 17600\,{\lambda_{sh}}^2 + 38400\,\lambda_{sh}\,\lambda_s\,(t) + 16000\,\left(\lambda_s\,(t)\right)^2 \\
&- 100\,\left(17\,{g_1}^2 + 45\,{g_2}^2 + 160\,{g_3}^2 - 288\,\lambda_h - 96\,\lambda_{sh}\right){K_t}^2 - 100\,\left(3\,{g_1}^2 + 15\,{g_2}^2 - 96\,\lambda_h - 32\,\lambda_{sh}\right){K_\nu}^2 \\
&\quad + 5400\,{K_t}^4 + 1800\,{K_\nu}^4\Big)\,,
\end{aligned}
$$

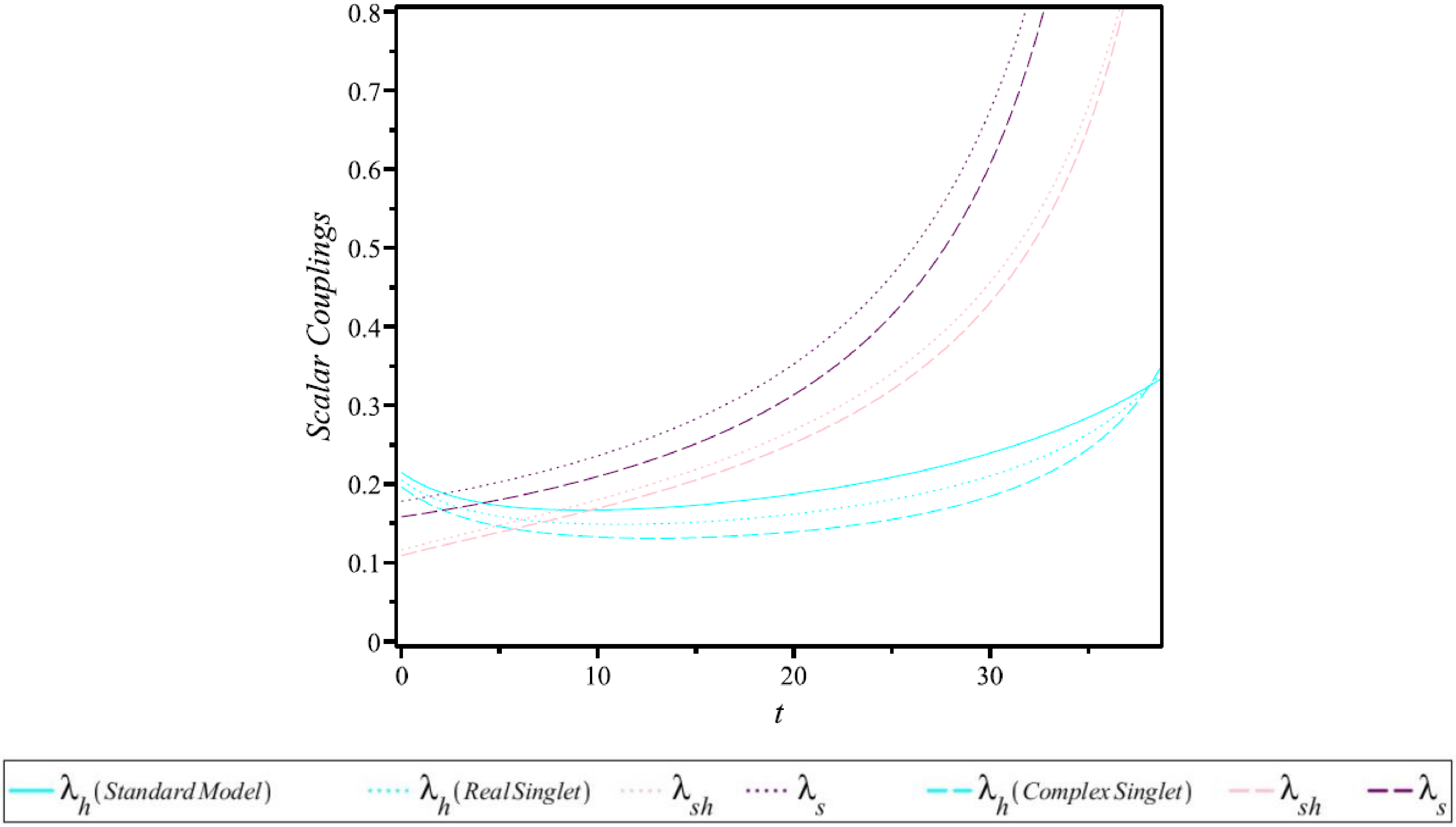
$$
\begin{aligned}
\frac{d\lambda_s}{dt} = \frac{1}{16\,\pi^2}\left(8\,{\lambda_{sh}}^2 + 20\,\left(\lambda_s\,(t)\right)^2\right) + \frac{1}{256\,\pi^4}\Bigg(&\frac{48\,{g_1}^2{\lambda_{sh}}^2}{5} + 48\,{g_2}^2{\lambda_{sh}}^2 - 64\,{\lambda_{sh}}^3 - 80\,{\lambda_{sh}}^2\lambda_s\,(t) \\
&- 60\,\left(\lambda_s\,(t)\right)^3 - 48\,{\lambda_{sh}}^2{K_t}^2 - 16\,{\lambda_{sh}}^2{K_\nu}^2\Bigg)\,.
\end{aligned}
$$



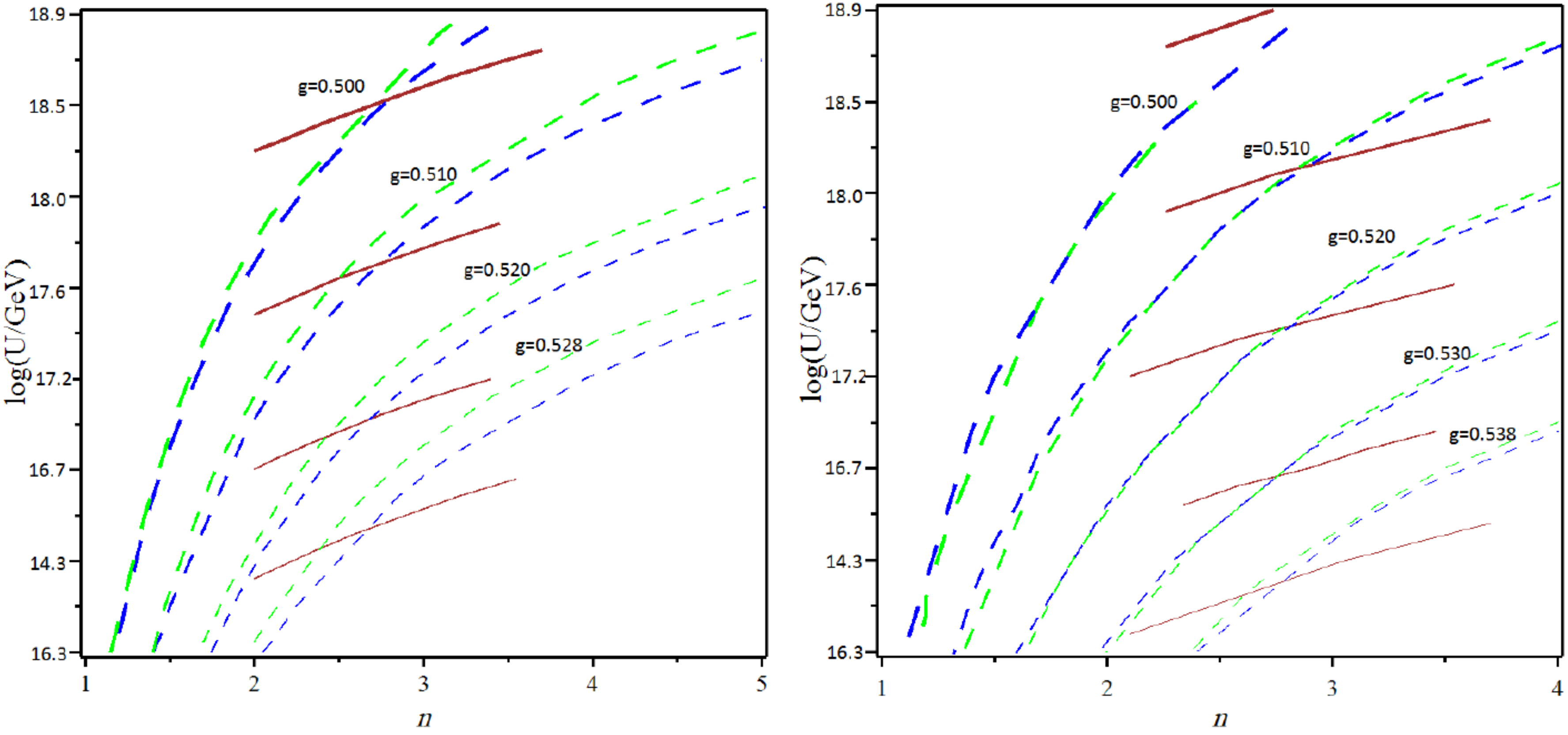
two-loop order
one-loop order
g_1
g_2
g_3
K_t
K_ν
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
$t = \ln\left(\mu / M_Z\right)$



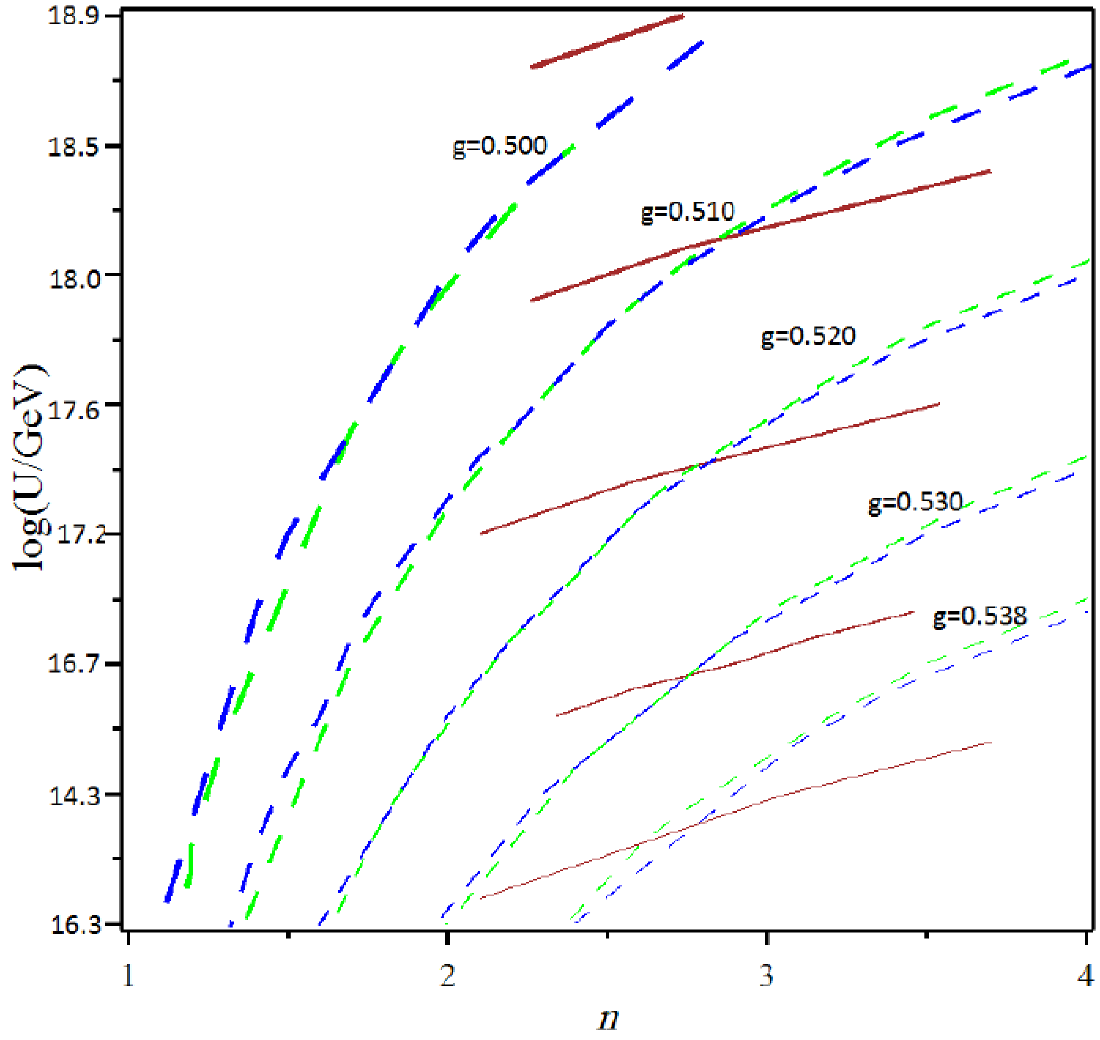
0.2
0.1
0
−0.1
10
20
30
t
λ_h(t)
one-loop
two-loop
three-loop
Standard Model
Real Singlet
Comple Singlet



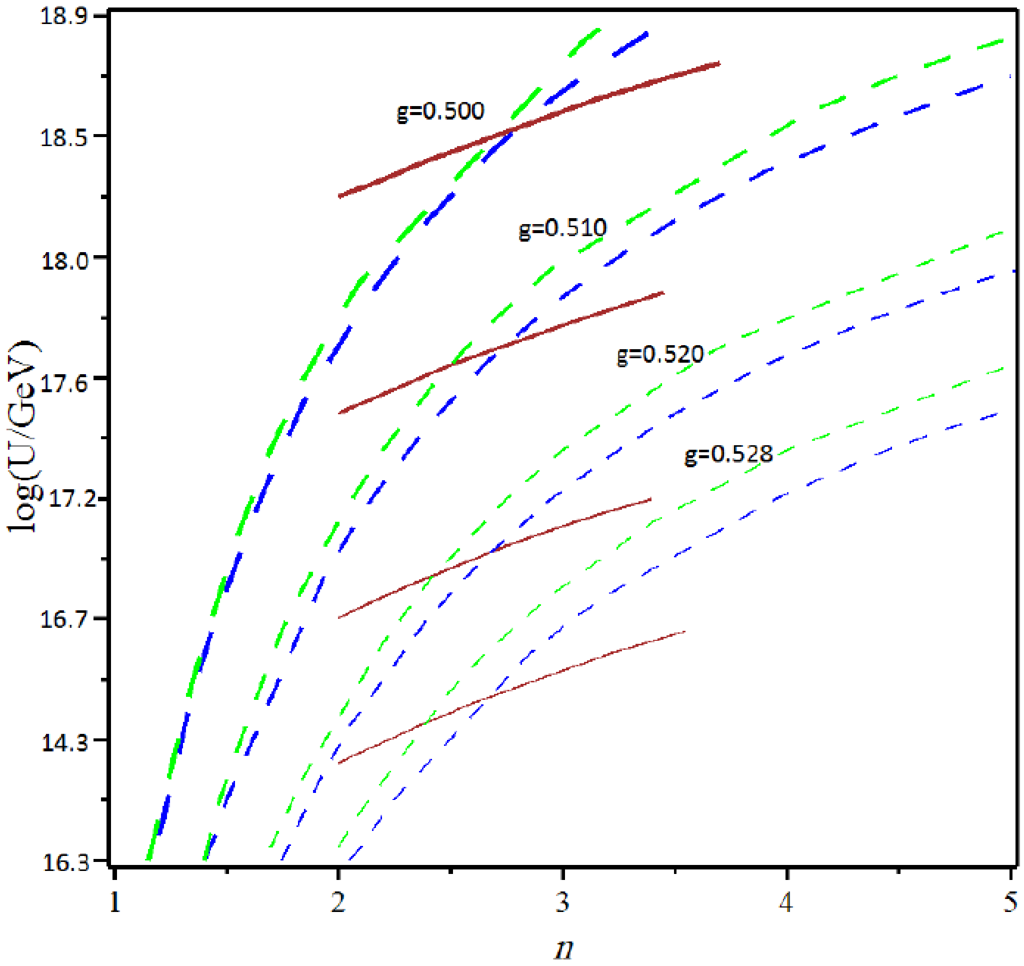
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Scalar Couplings
0
10
20
30
t
λh(Standard Model)
λh(Real Singlet)
λsh
λs
λh(Complex Singlet)
λsh
λs



H. Karimi, Implications of the Complex Singlet Field for Noncummutative Geometry Model, arXiv:1705.01605 **[hep-ph]**



log(U/GeV)
18.9
18.5
18.0
17.6
17.2
16.7
14.3
16.3
g=0.500
g=0.510
g=0.520
g=0.530
g=0.538
1
2
3
4
n



18.9
18.5
18.0
17.6
17.2
16.7
14.3
16.3
log(U/GeV)
1
2
3
4
5
n
g=0.500
g=0.510
g=0.520
g=0.528

conclusions:

- The simplest model derived from noncommutative geometry approach contains standard model along with some familiar features such as:
    - Einstein Action
    - Higher order gravitational terms
    - Right handed neutrino
    - A singlet which is coupled to the Higgs
    - Unification of gauge couplings at high energies
    - Constraints on the scalar and Yukawa couplings at unification scale which are consistent
      with the experimental values of the particle masses

- There is a small window for the values of unification scale, unified gauge coupling, and ratio of the Yukawa couplings of the top and neutrino, where the model is consistent with the masses of the top and Higgs particles.

- This model survives low Higgs mass and, with a complex singlet, makes the situation with instability better.

The fact that the relation between couplings are restrictive and there are only three degrees of freedom, yet the model is consistent with the experimental values at low energies, suggests to take the noncommutative geometry approach to the standard model series and try to consider its richer forms.

In addition, the fact that the gauge couplings cannot be recovered suggests more physics must be involved.

We think it is important to work on the more general model derived from noncommutative geometry which is Pati-Salam model. That model is richer in the scalar and Higgs sector and at the same time does not suffer from GUT theory disadvantages.

Thanks!