# N(N)LO calculations: an overview

Gionata Luisoni

gionata.luisoni@cern.ch

CERN

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TOOLS 2017

**Corfu**

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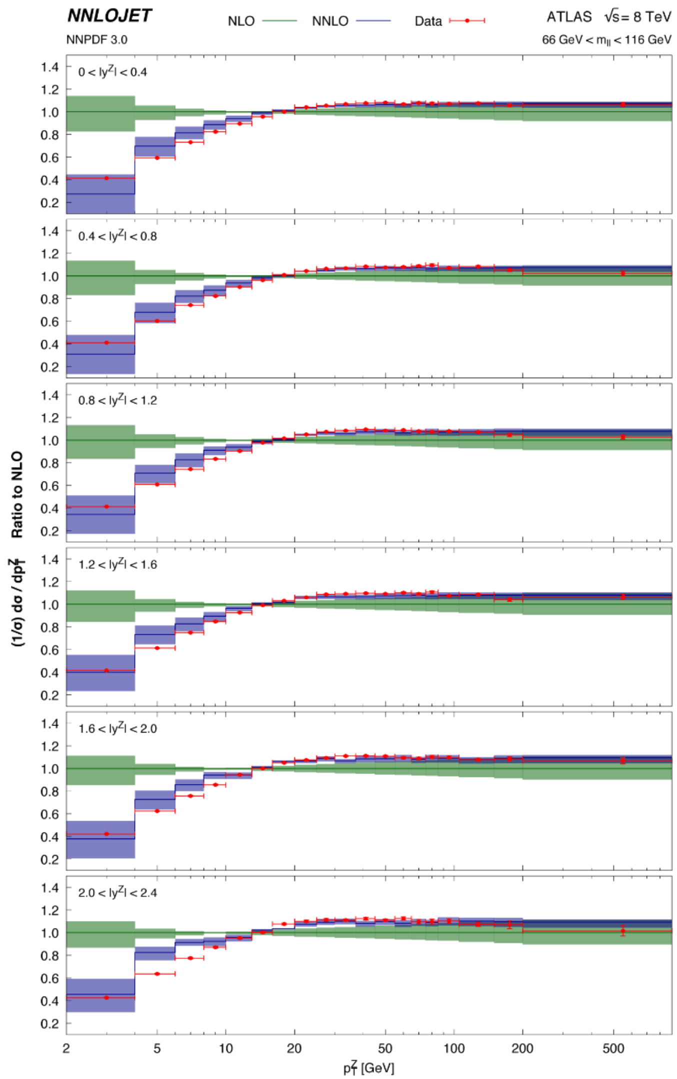
# Outline

- Motivation: why NLO, why automation?
- NLO automation: the key ingredients and the past challenges
- Latest developments
- Tools: state-of-the art
- Towards NNLO
- Conclusions and outlook

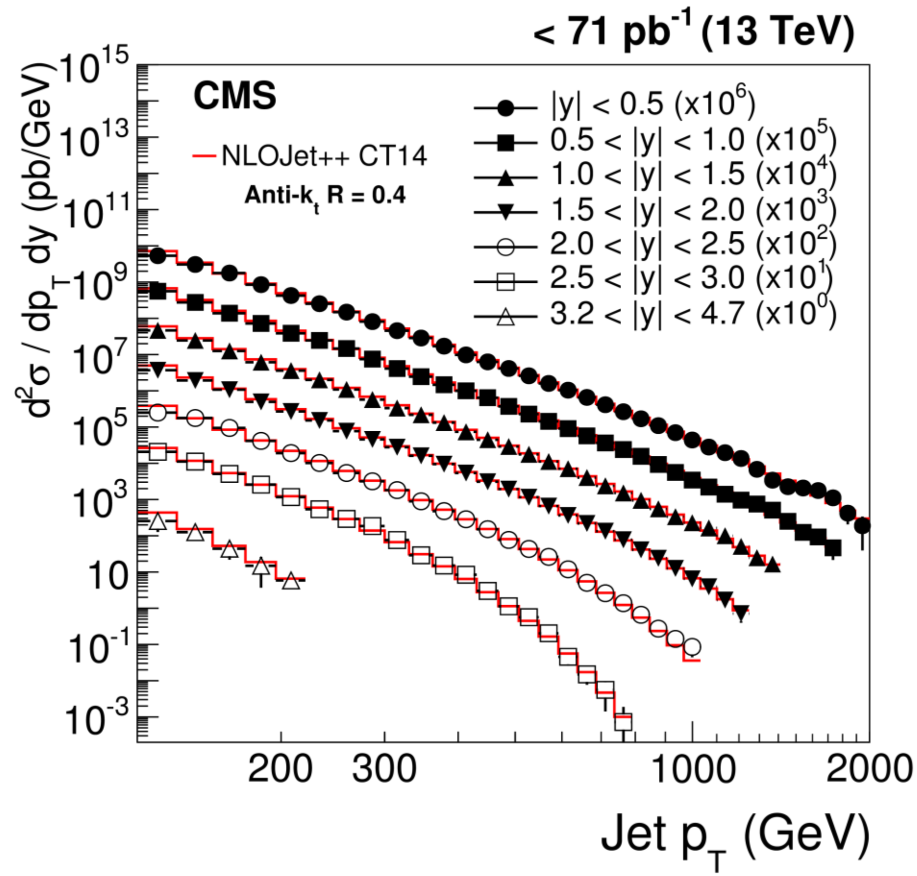
Disclaimer: despite trying to be comprehensive, this is a very biased selection of tools and I may have forgotten your favorite one. I apologize for potential omissions and if you point them out to me I would be glad to include them!

# Motivation: why NLO, why automation?

# Precision at the LHC



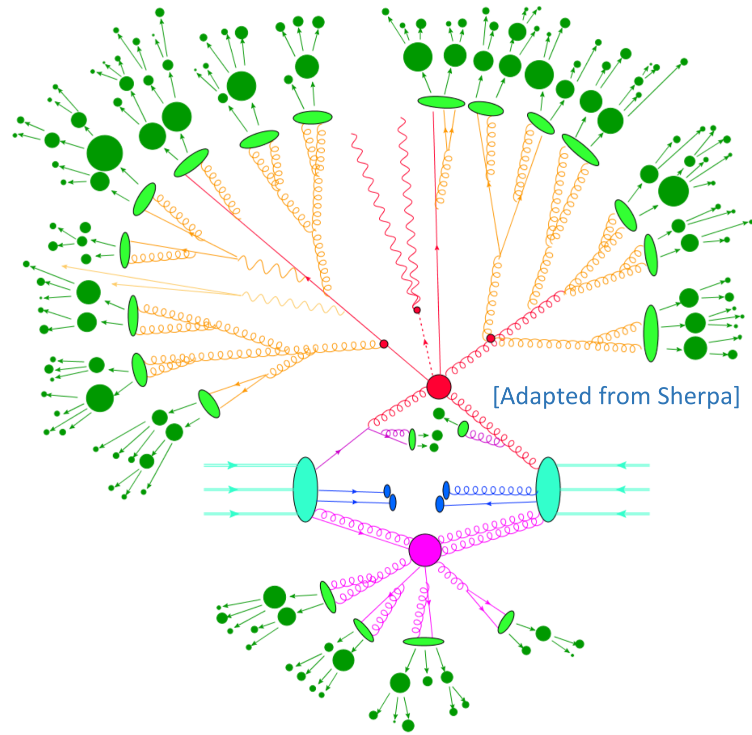

[1610.01843]




[1605.04436]

# LHC is a tough environment for precision..

- QCD is omnipresent at LHC:
  - PDF
  - Hard scattering and loop corrections
  - Parton Shower
  - Hadronization
  - Further non perturbative effects




- Master formula:

$$\sigma_{h_1 h_2 \to X} = \sum_{a,b} \int_0^1 dx_1 dx_2\, \underbrace{f_{h_1/a}(x_1, \mu_F^2)\, f_{h_2/b}(x_2, \mu_F^2)}_{\text{PDFs}} \times \underbrace{\hat{\sigma}_{a,b\to X}\left(x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right)}_{\text{partonic cross section}} \underbrace{\left[+\mathcal{O}\left(\frac{1}{Q^2}\right)\right]}_{\text{power corrections}}$$

- Would like to know all components with high precision!

# Fixed order calculations

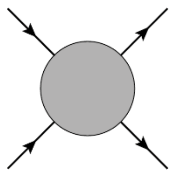
- Where the partonic cross section can be written as:

LO power of strong coupling

$$\hat{\sigma}_{a,b\to X} = \alpha_s^{n}\left[\sigma_0 + \alpha_s\sigma_1 + \alpha_s^2\sigma_2 + \alpha_s^3\sigma_3 + \mathcal{O}(\alpha_s^4)\right]$$

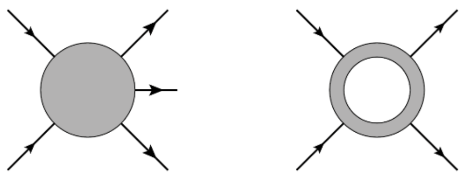
LO | NLO | NNLO | N$^3$LO

- LO:



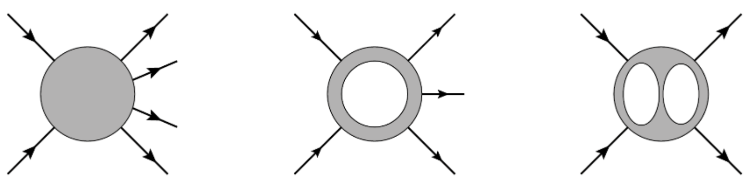
Predicts only the order of magnitude:
- ➢ scale in coupling is not defined
- ➢ 1 parton ⟷ 1 jet

- NLO:



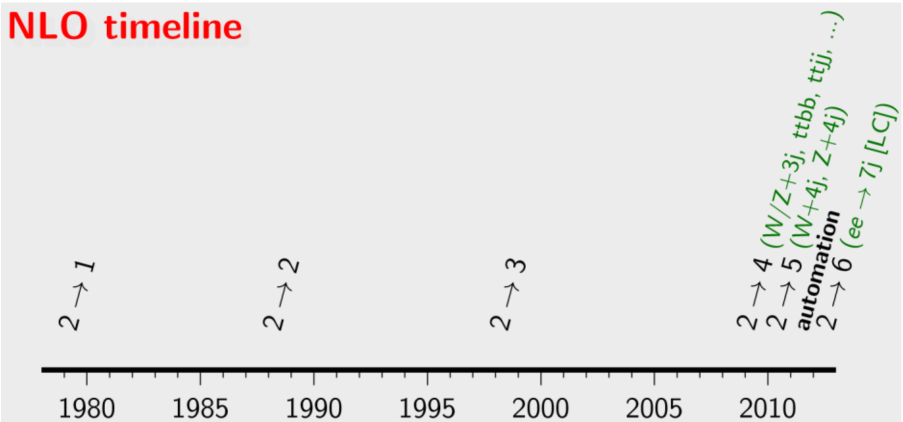
First reliable predictions:
- ➢ scale choices can be made
- ➢ first description of jet substructure

- NNLO:



Possible to quantify uncertainties:
- ➢ convergence can be checked
- ➢ richer jet substructure

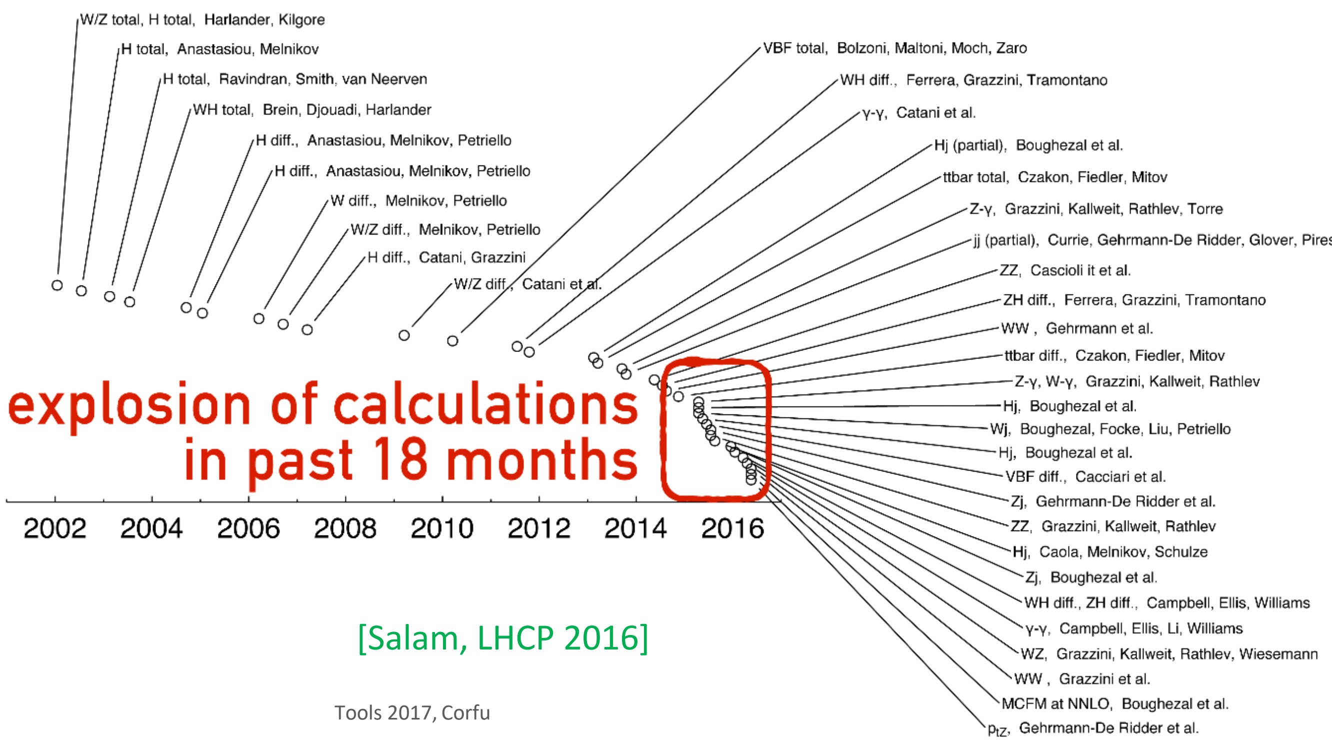
# Why automation?




[Salam, La Thuile 2012]

- flexibility
- reliability
- speed
- …

➢ more focus on phenomenology




[Salam, LHCP 2016]

# NLO automation: the key ingredients and the past challenges

# NLO calculation in a nutshell

- For a full NLO calculation the following ingredients are needed:

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to X} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{Born}} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}}\right) + \int_{\mathrm{d}\Phi_m} \left[\int_{\mathrm{d}\Phi_1} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \mathrm{d}\sigma^{V}_{\mathrm{NLO}}\right]$$

- Tree amplitude:
  - Born level matrix element
  - Real radiation matrix element
- Subtraction scheme
- Phase space integral

**Monte Carlo (MC)**

Interface: more later

- Virtual corrections

**One Loop Program (OLP)**

Note: for loop-induced processes this picture changes slightly

# NLO calculation: tree-level amplitudes

- For a full NLO calculation the following ingredients are needed:

$$\hat{\sigma}^{\text{NLO}}_{a,b\to X} = \int_{\text{d}\Phi_m} \text{d}\sigma_{\text{Born}} + \int_{\text{d}\Phi_{m+1}} \left(\text{d}\sigma^{\text{R}}_{\text{NLO}} - \text{d}\sigma^{\text{S}}_{\text{NLO}}\right) + \int_{\text{d}\Phi_m} \left[\int_{\text{d}\Phi_1} \text{d}\sigma^{\text{S}}_{\text{NLO}} + \text{d}\sigma^{V}_{\text{NLO}}\right]$$

- Tree amplitude:
  - Born level matrix element
  - Real radiation matrix element
- Subtraction scheme
- Phase space integral

- Virtual corrections

# Tree-level amplitude generators:

- Automated generation of tree-level matrix elements available since long time now
  - Many codes appeared for the first time in the '90s ⟶ continuously updated
  - Based on helicity amplitudes, off-shell currents, Dyson-Schwinger recursive equations or Berends-Giele recursion relations

Talk by J. Reuter

FeynArts
[Eck, Küblbeck; Hahn]

FeynCalc
[Mertig, Böhm, Denner]

O'Mega / WHIZARD
[Moretti, Ohl, Reuter; Kilian Ohl, Reuter]

Talk by T. Hahn

Helas
[Murayama, Watanabe, Hagiwara]

Grace
[Fujimoto et al.]

CompHEP
[Pukhov et al.]

Helac
[Kanaki, Papadopoulos]

MadGraph
[Stelzer, Long]

Comix
[Gleisberg, Höche]

Amegic++
[Krauss, Kuhn, Soff]

...

Disclaimer 1: most of the codes were further developed and refined by several other authors to become more flexible and automated. Here I list only the beginnings in a sort of historical perspective. More later…
Disclaimer 2: many of the automated 1-loop amplitude generators have also tree-level capabilities. Here only genuine tree-level codes are mentioned.

# NLO calculation: phase space

- For a full NLO calculation the following ingredients are needed:

$$\hat{\sigma}^{\text{NLO}}_{a,b\to X} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\text{Born}} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\text{R}}_{\text{NLO}} - \mathrm{d}\sigma^{\text{S}}_{\text{NLO}}\right) + \int_{\mathrm{d}\Phi_m} \left[\int_{\mathrm{d}\Phi_1} \mathrm{d}\sigma^{\text{S}}_{\text{NLO}} + \mathrm{d}\sigma^{V}_{\text{NLO}}\right]$$

- Tree amplitude:
  - Born level matrix element
  - Real radiation matrix element
- Subtraction scheme
- Phase space integral

- Virtual corrections

# Phase space generators

- Often developed together with tree-level amplitude generators:
  - need the knowledge of the amplitude structure to optimize phase space sampling

BASES/SPRING
[Kawabata]

Mint
(in POWHEG-BOX)
[Nason]

Helac-Phegas
[Cafarella, Papadopoulos, Worek]

Kaleu
[Van Hameren]

MadEvent
(with MadGraph)
[Maltoni, Stelzer]

CompHEP
[Ilyin, Kovalenko, Pukhov ]

Sherpa
(with Amegic++ Comix)
[Gleisberg, Höche, Krauss, Schaelicke, Schumann, Winter]

Herwig 7
[Bellm et al.]

...

WHIZARD
Kilian Ohl, Reuter]

# NLO calculation: subtraction

- For a full NLO calculation the following ingredients are needed:

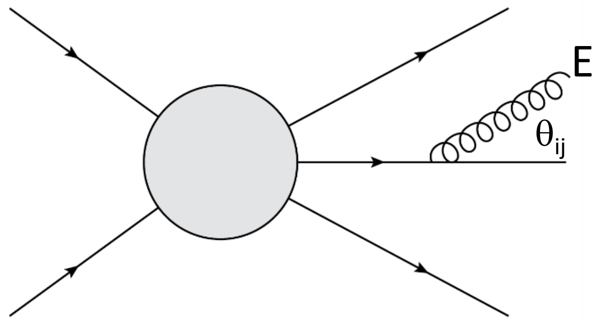
$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to X} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{Born}} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}}\right) + \int_{\mathrm{d}\Phi_m} \left[\int_{\mathrm{d}\Phi_1} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \mathrm{d}\sigma^{V}_{\mathrm{NLO}}\right]$$

- Tree amplitude:
  - Born level matrix element
  - Real radiation matrix element
- Subtraction scheme
- Phase space integral

- Virtual corrections

# Recap: Why do we need a subtraction scheme?

- When integrating over the inclusive (m+1)-particle phase space the real-radiation matrix element becomes singular in the soft ($E \longrightarrow 0$) and collinear ($\theta_{ij} \longrightarrow 0$) limit:




- Same divergent structure as virtual contribution, which becomes manifest only once the phase space integration is performed

- Introduce subtraction which reproduces the real-radiation singular behaviour, but can be integrated analytically (poles cancellation becomes manifest)

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to X} = \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma_{\mathrm{Born}} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}}\right) + \int_{\mathrm{d}\Phi_m} \left[\int_{\mathrm{d}\Phi_1} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \mathrm{d}\sigma^{V}_{\mathrm{NLO}}\right]$$

**separately finite**

# Subtraction schemes at NLO

- Most used subtraction schemes at NLO:
  - Catani-Seymour dipole method (CS) [Catani, Seymour; Catani, Dittmaier, Seymour, Trocsanyi]
  - Frixione-Kunszt-Signer (FKS) [Frixione, Kunszt, Signer]
  - Nagy-Soper [Nagy, Soper]

- Various tools have an implementation of these schemes along with the tree-level amplitude generators:

Sherpa
CS dipoles
[Gleisberg, Höche, Krauss, Schönherr, Schumann, Siegert, Winter]

MadGraph/MadEvent
MadDipole/MadFKS
[Frederix, Gehrmann, Greiner]
[Frederix, Frixione, Maltoni, Stelzer]

Helac-Dipoles
(+ Nagy-Soper)
[Bevilacqua, Czakon, Kuboz, Papadopoulos, Worek]

Autodipole
[Hasegawa, Moch, Uwer]

TevJet
[Seymour, Tevlin]

WHIZARD
FKS
[Reuter et al.]

Herwig7
[Bellm et al.]

POWHEG-BOX
FKS
[Alioli, Nason, Re, Oleari]

...

- Other schemes: (mainly developed for NNLO, but applicable also at NLO)

➢ Antenna [Kosower; Gehrmann et al.]
➢ CoLoRFul [Somogyi et al.]
➢ Residue-improved SD [Czakon et al.]
➢ $q_T$ - subtraction [Catani, Grazzini et al.]
➢ N-jettiness [Gaunt et al.; Boughezal et al.]
➢ Nested subtr. based on SD [Caola et al.]

# NLO calculation: virtual correction

- For a full NLO calculation the following ingredients are needed:

$$\hat{\sigma}^{\text{NLO}}_{a,b\to X} = \int_{\text{d}\Phi_m} \text{d}\sigma_{\text{Born}} + \int_{\text{d}\Phi_{m+1}} \left(\text{d}\sigma^{\text{R}}_{\text{NLO}} - \text{d}\sigma^{\text{S}}_{\text{NLO}}\right) + \int_{\text{d}\Phi_m} \left[\int_{\text{d}\Phi_1} \text{d}\sigma^{\text{S}}_{\text{NLO}} + \text{d}\sigma^{V}_{\text{NLO}}\right]$$

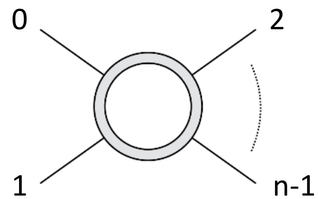
- Tree amplitude:
  - Born level matrix element
  - Real radiation matrix element
- Subtraction scheme
- Phase space integral

- Virtual corrections

For long time considered the bottleneck in the automation of NLO calculation!

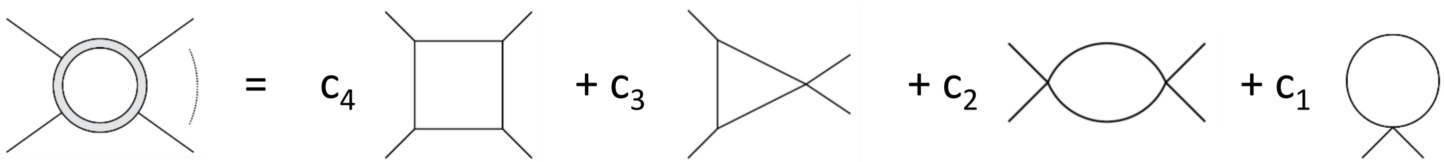
# 1-loop amplitudes computation

- Generic 1-loop amplitude:




$$= \int d^d\bar{q} \, \frac{\mathcal{N}(\bar{q}, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{n-1}}$$

- Can be decomposed in Master Integrals (MIs): [Passarino, Veltman]




- Reduce problem of computing 1-loop integral to the determination of the coefficients of the linear combination of MIs (**reduction**).

  Various way of doing this, mainly two techniques were automatized:

  ➔ Integrand reduction [Ellis, Giele, Kunszt, Melnikov, Mastrolia, Mirabella, Ossola, Papadopoulos, Peraro, Pittau, …]

  ➔ Tensor reduction [Binoth, Denner, Dittmaier, Fleischer, Guillet, Heinrich, v. Oldenborgh, Pilon, Reiter, Riemann, Vermaseren …]

- Tensor or scalar MIs coded into dedicated libraries

# 1-loop: several programs for several tasks

- From 1-loop amplitude generators to scalar 1-loop MIs libraries

FeynCalc [Mertig, Orellana, Shtabovenko]

FormCalc [Hahn]

GoSam [Chiesa, Greiner, Heinrich, Jahn, Jones, Kerner, GL, Mastrolia, Ossola, Peraro, Schlenk, Scyboz, Tramontano]

Helac-1loop [Bevilacqua, Czakon, Garzelli, v.Hameren, Kardos, Malamos, Papadopoulos, Pittau, Worek]

MadLoop [Hirschi, Frederix, Frixione, Garzelli, Maltoni, Mattelaer, Pittau]

OpenLoops [Cascioli, Lindert, Maierhöfer, Pozzorini]

Recola [Denner, Hofer, Lang, Uccirati]

Collier [Denner, Dittmaier, Hofer]

Cuttools [Ossola, Papadopoulos, Pittau]

Golem95 [Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers, v.Soden-Fraunhofen]

Ninja [Mastrolia, Mirabella, Peraro]

Samurai [Mastrolia, Ossola, Reiter, Tramontano]

PJFry [Yundin]

Package X [Patel]

FF [v.Oldeborgh]

LoopTools [Hahn, Perez-Victoria]

OneLOop [v.Hameren]

QCDLoop [Carrazza, Ellis, Zanderighi]

# 1-loop: several programs for several tasks

- From scalar 1-loop libraries to 1-loop amplitude generators

Amplitude & code generators:

FeynCalc | FormCalc | GoSam | MadLoop | OpenLoops | Recola | Helac-1loop

Reduction codes:

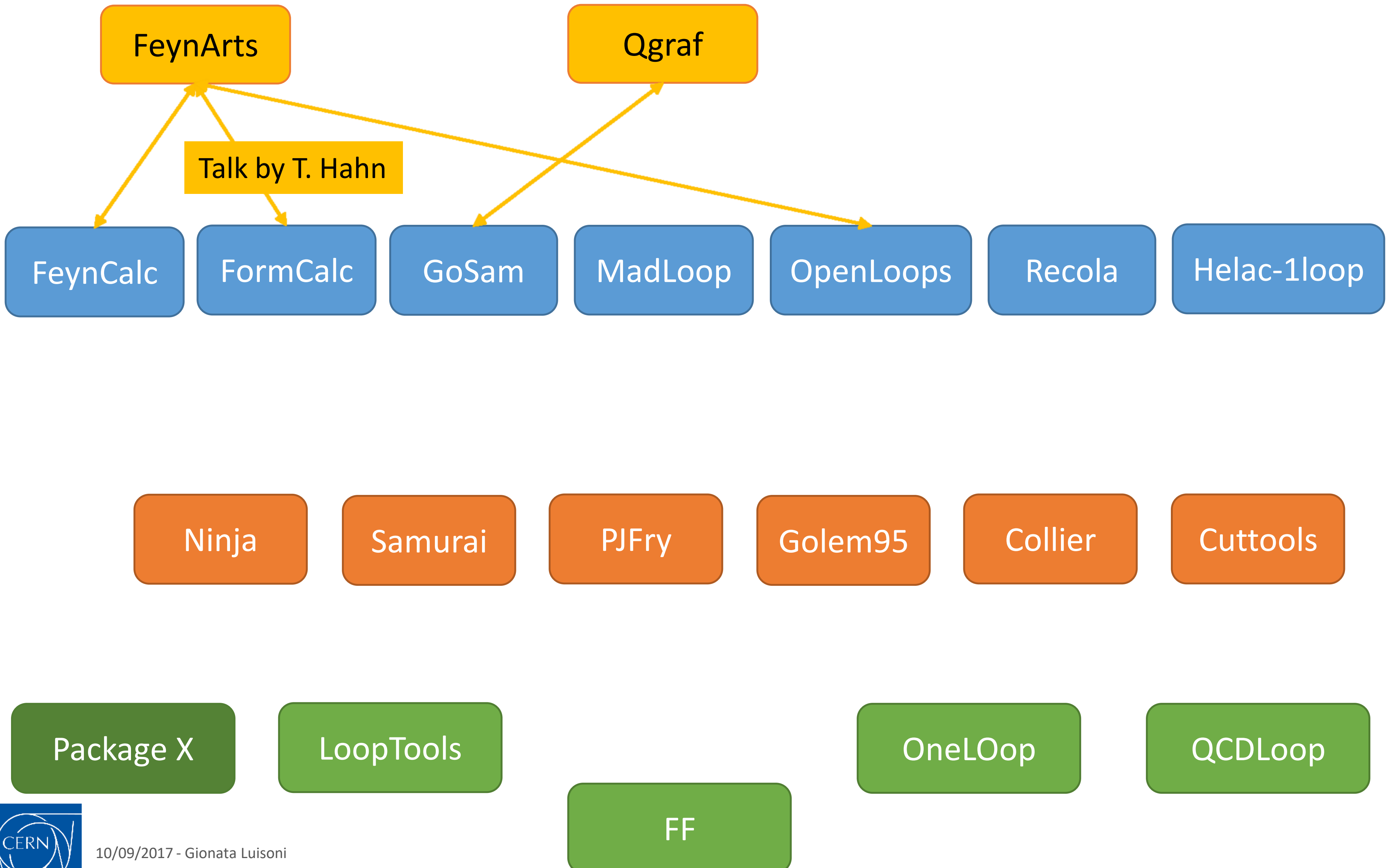
Ninja | Samurai | PJFry | Golem95 | Collier | Cuttools

Integral libraries:

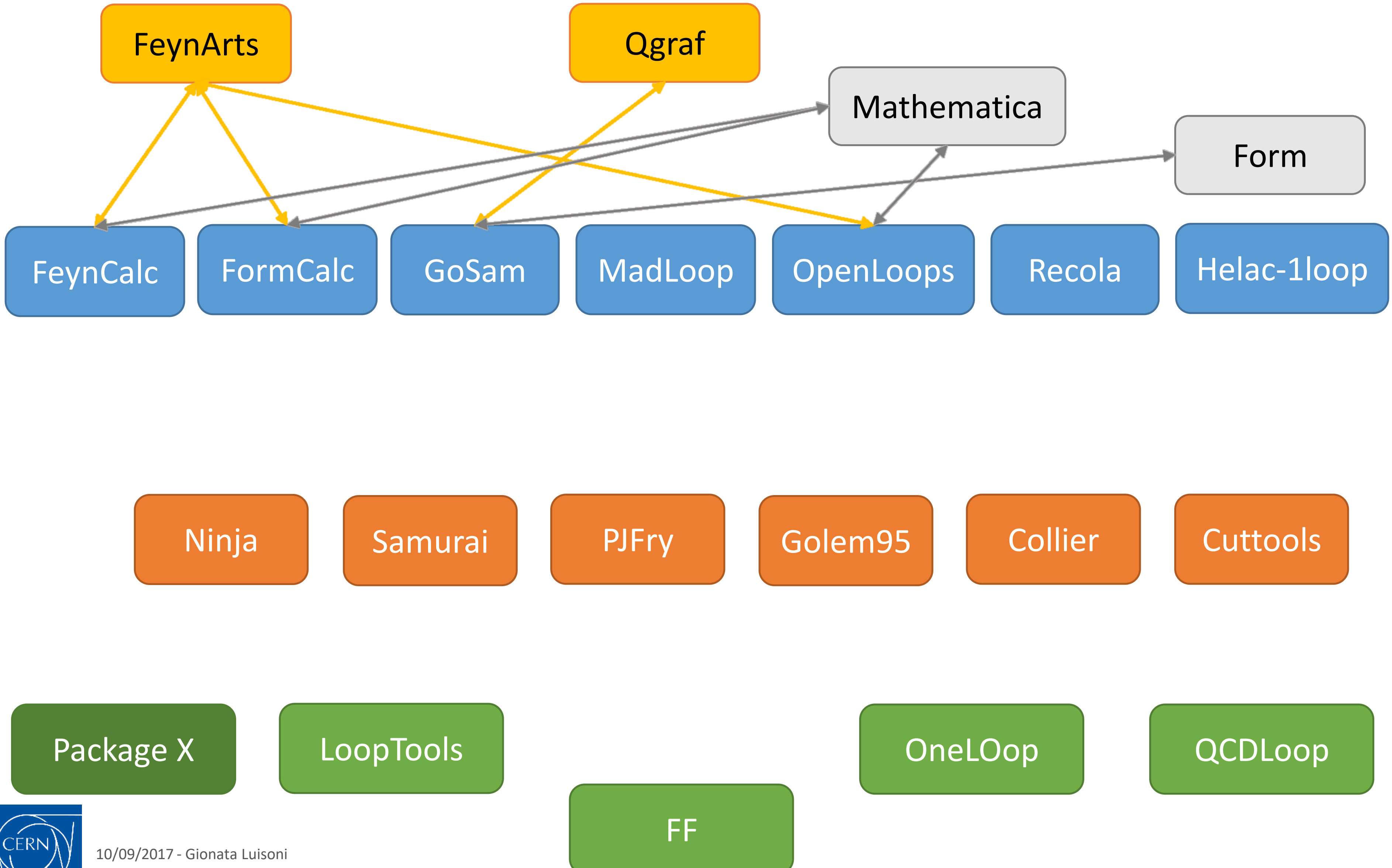
Package X | LoopTools | OneLOop | QCDLoop

FF

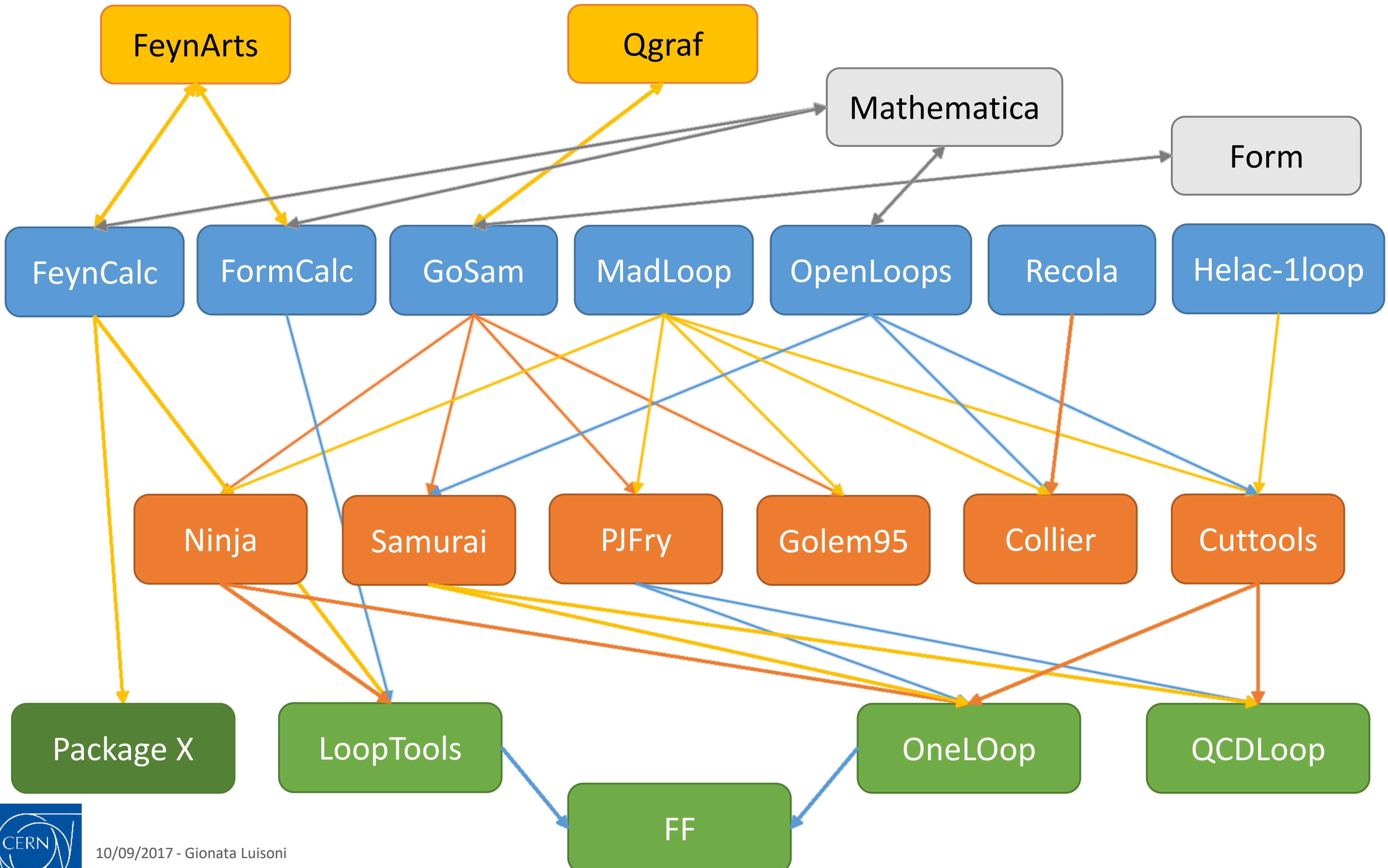
# 1-loop: several programs for several tasks



# 1-loop: several programs for several tasks

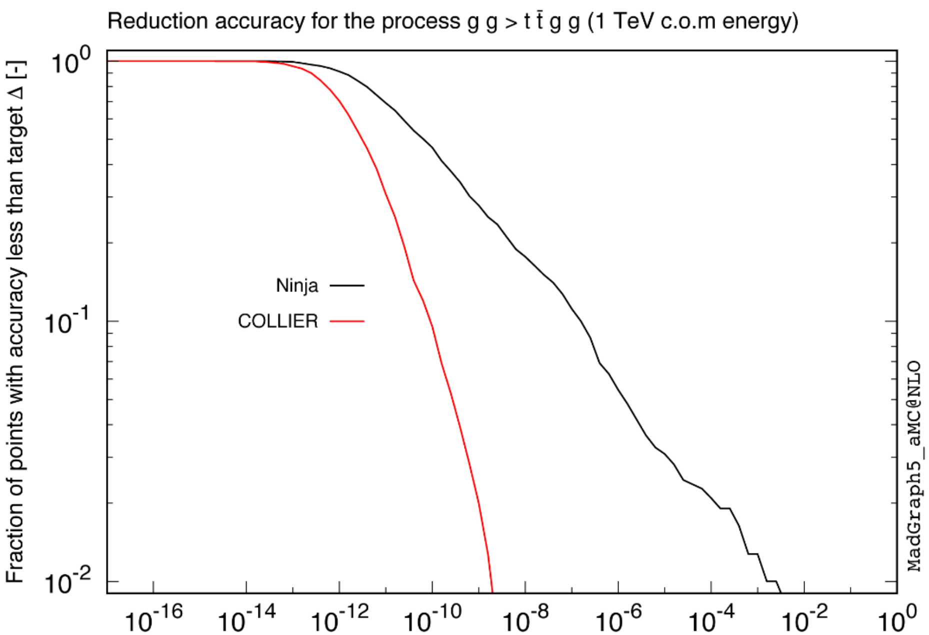


# 1-loop: several programs for several tasks

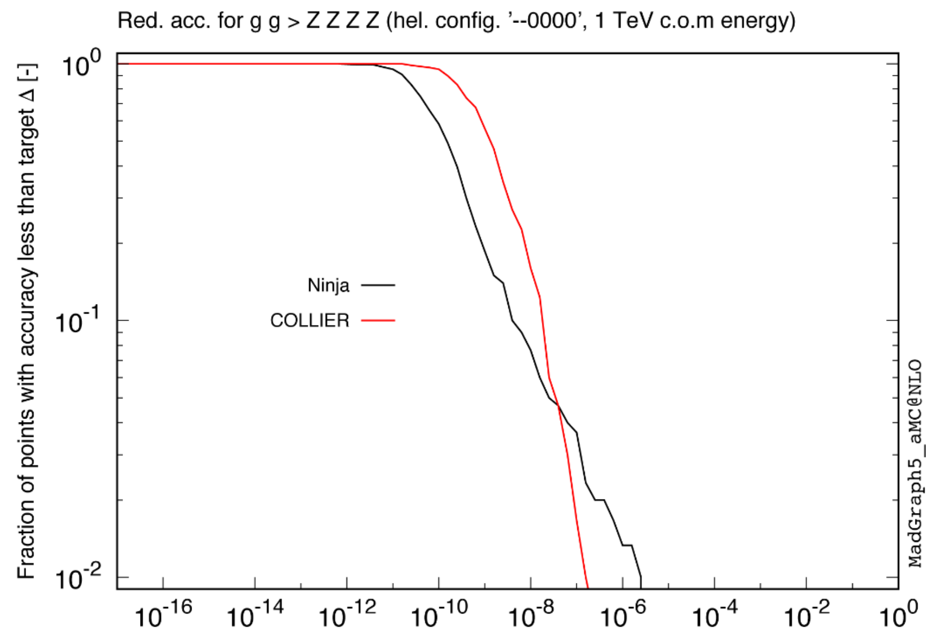


# 1-loop: several programs for several tasks

$$gg \to t\bar{t}gg$$




$$gg \to ZZZZ$$




[Hirschi MIAPP 2017]

# MC – OLP: the Binoth LH Accord interface

- In order to allow to easily interface the various MCs' with several OLPs', use a standard interface for communication

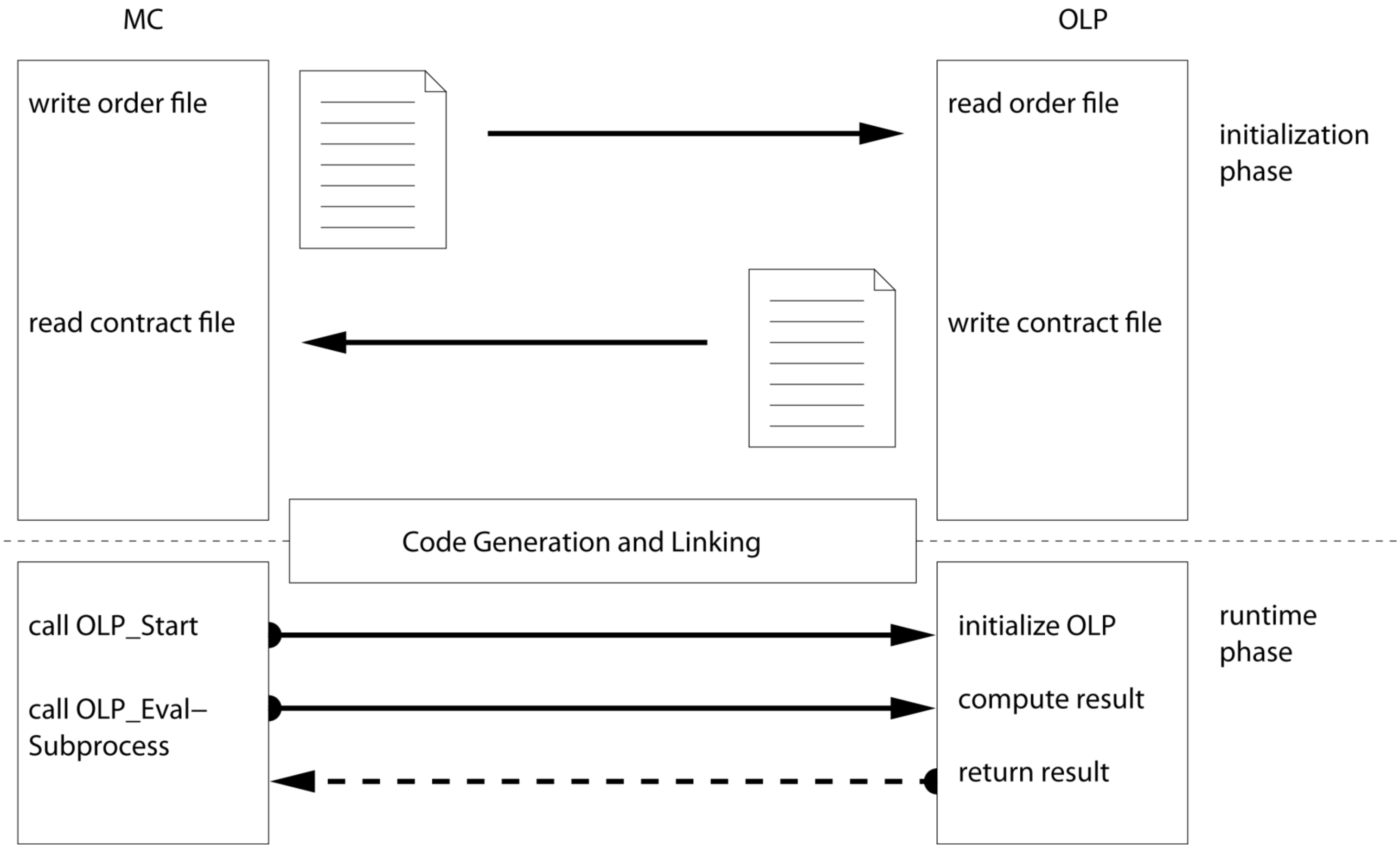
[Binoth et al.; Alioli et al.]

- 2 step interface:
  - pre-runtime: fix conventions / tell OLP which processes are needed
  - runtime: call OLP for amplitude at a given phase space point

- Recently updated to increase automation and flexibility:
  - Support for dynamical parameters (coupling, masses, …)
  - Synchronization of EW schemes
  - Standards for treatment of unstable phase space points
  - Standards for merging different jet multiplicities
  - Extension to provide also colour correlated (CC) and helicity correlated (HC) tree amplitudes

# BLHA

- The Binoth Les Houches Accord Interface



# Order and contract files

- We can also compare order and contract files:

[OLE_order.lh]

```
# OLE_order.lh
# Created by Sherpa-2.2.2

MatrixElementSquareType  CHsummed
CorrectionType           QCD
IRregularisation         CDR
AlphasPower              2
AlphaPower               0
OperationMode            CouplingsStrippedOff
ResonanceTreatment       FixedWidthScheme
EWRenormalisationScheme  alphaMZ

# process list
1 -1 -> 6 -6
-1 1 -> 6 -6
21 21 -> 6 -6
```

[OLE_order.olc]

```
# vim: syntax=olp
#@OLP GoSam 2.0.4
#@IgnoreUnknown True
#@IgnoreCase False
#@SyntaxExtensions
MatrixElementSquareType CHsummed | OK
CorrectionType QCD | OK
IRregularisation CDR | OK
AlphasPower 2 | OK
AlphaPower 0 | OK
OperationMode CouplingsStrippedOff | OK
ResonanceTreatment FixedWidthScheme | OK # Ignored by OLP
EWRenormalisationScheme alphaMZ | OK # Ignored by OLP
1 -1 -> 6 -6 | 1 1
-1 1 -> 6 -6 | 1 2
21 21 -> 6 -6 | 1 0
```

Partonic process label used for communication between MC and OLP

# Latest developments

# EW corrections

- After automation of QCD, efforts started focusing on EW corrections
- Few additional aspects to be careful about:

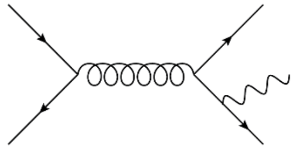
  - Bookkeeping
    - when tree levels at various orders in $\alpha_s$ and $\alpha$ lead to the same final state (example in the next slide)

  - Gauge invariant treatment of unstable particles via complex mass scheme

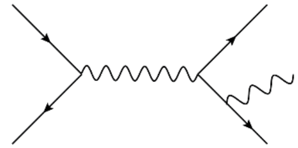
  - In 1-loop EW computation complexity grows faster than QCD
    - More possibilities for particles running in the loop, depending also on the chosen gauge

# EW corrections: bookkeeping in W+2 jets

qqW final state via QCD or EW:



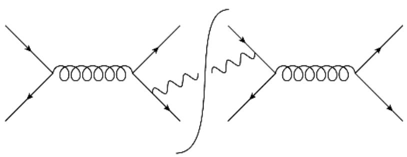
$\mathcal{O}(\alpha_s \alpha^{1/2})$



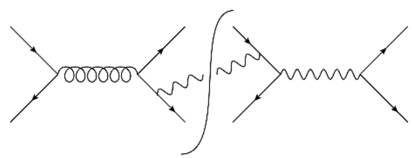
$\mathcal{O}(\alpha^{3/2})$

LO:

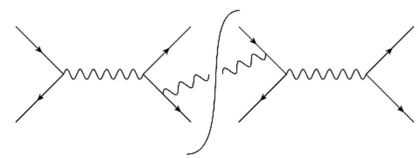
$\mathcal{O}(\alpha_s^2 \alpha)$



$\mathcal{O}(\alpha_s \alpha^2)$



$\mathcal{O}(\alpha^3)$



QCD EW QCD EW QCD EW

NLO:

$\mathcal{O}(\alpha_s^2 \alpha^2)$ $\mathcal{O}(\alpha_s^2 \alpha^3)$ $\mathcal{O}(\alpha_s \alpha^4)$ $\mathcal{O}(\alpha^5)$

# EW corrections

- Most recent NLO EW results:

  - Recola

| | |
|---|---|
| $pp \to lljj$ | [1411.0916] |
| $pp \to e^+e^-\mu^+\mu^- \,/\, \mu^+\mu^-\mu^+\mu^-$ | [1601.07787] [1611.05338] |
| $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$ | [1605.03419] |
| $pp \to t\bar{t} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ | [1607.06671] |
| $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu jj$ | [1611.02951] |
| $pp \to t\bar{t}H \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}\; H$ | [1612.07138] |
| $pp \to e^+\nu_e\mu^+\nu_\mu jj$ | [1708.00268] |

  - Sherpa/Munich + OpenLoops

| | |
|---|---|
| $pp \to W + 1,2,3\,\text{jets}$ | [1412.5157] |
| $pp \to ll/l\nu/\nu\nu + 0,1,2\,\text{jets}$ | [1511.08692] |
| $pp \to ll\nu\nu$ | [1705.00598] |

  - MadGraph5_aMC@NLO + MadLoop

| | |
|---|---|
| $pp \to t\bar{t}H/Z/W$ | [1504.03446] |
| $pp \to t\bar{t}$ | [1606.01915] [1705.04105] |
| $pp \to 2\,\text{jets}$ | [1612.06548] |

  - MadDipole/Sherpa + GoSam

| | |
|---|---|
| $pp \to W + 1,2,3\,\text{jets}$ | [1507.08579] |
| $pp \to \gamma\gamma + 0,1,2\,\text{jets}$ | [1706.09022] |

Many new results in the last 2-3 years!

# Tools: state-of-the-art

# Summary of (semi-) automated NLO tools

- Several existing frameworks for (semi-) automated NLO simulations and more:

Talk by M.Schönherr

**Helac-NLO**

[Bevilacqua, Czakon, Garzelli, v.Hameren, Kardos, Malamos, Papadopoulos, Pittau, Worek, Shao]

**Whizard**

[Brass, Chokoufe, Kilian, Ohl, Reuter, Rothe, Schmidt, Sekulla, Shim, Speckner, Stau, Steinemeier, Weiss, Zhao]

**Herwig-7 / Matchbox**

[Bellm, Gieseke, Grellscheid, Kirchgaeßer, Loshaj, Nail, Papaefstathiou, Plätzer, Podskubka, Rauch, Reuschle, Richardson, Schichtel, Seymour, Siódmok, Webber]

**Sherpa**

[Höche, Krauss, Kuttimalai, Schönherr, Schumann, Siegert, Thompson, Winter, Zapp]

**POWHEG-BOX**

[Alioli, Hamilton, Jezo, Nason, Oleari, Re, Zanderighi]

**MG5_aMC@NLO**

[Alwall, Artoisenet, Degrande, Frederix, Frixione, Fuks, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro]

# Summary of (semi-) automated NLO tools

- Several existing frameworks for (semi-) automated NLO simulations and more:

Talk by J. Reuter

Talk by M.Schönherr

**Helac-NLO**: Helac-1loop, Recola

**Whizard**: GoSam, OpenLoops, Recola

**Herwig-7 / Matchbox**: GoSam, NJet, OpenLoops

**POWHEG-BOX**: GoSam, MadLoop, OpenLoops

**Sherpa**: Blackhat, GoSam, Recola, NJet, OpenLoops

**MG5_aMC@NLO**: MadLoop, GoSam

Talks by B. Fucks, O.Mattelaer

- Many other more process specific tools: MCFM, VBFNLO,…
- Can be interfaced to further analysis tools: Fastjet, Rivet, … Talks by G. Soyez, A.Buckley
- Possible to perform LO/NLO computations in your favourite BSM model using interfaces to FeynRules, …

Talks by B. Fucks, O.Mattelaer

CERN

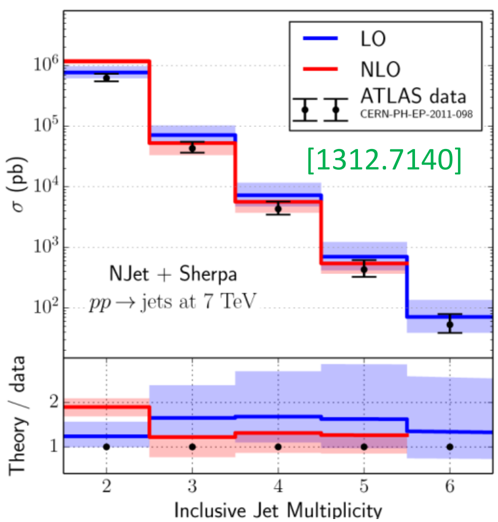
# Other 1-loop programs

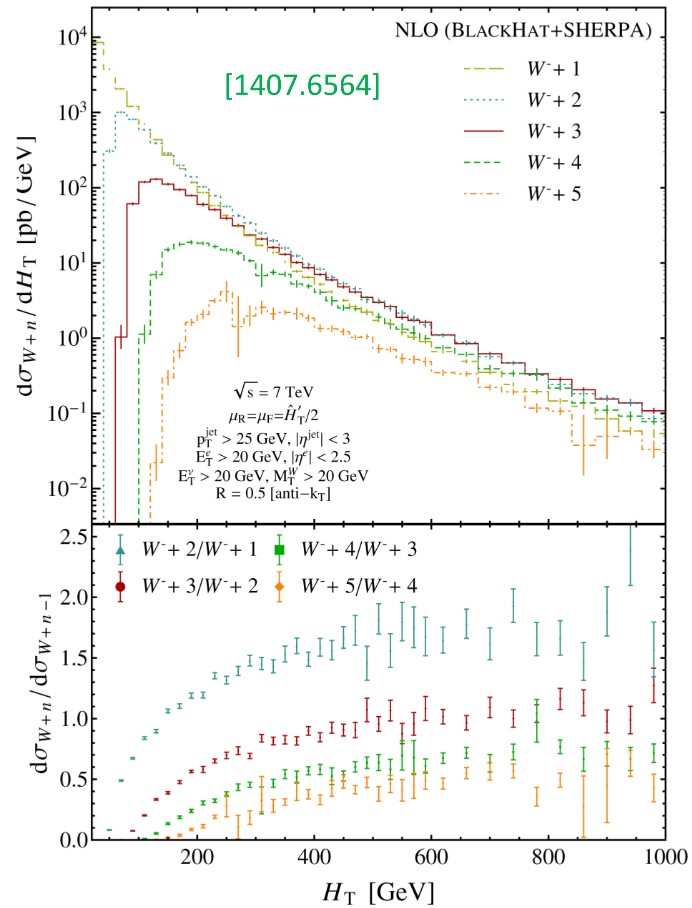
- Other codes for the computation of 1-loop amplitudes, which are specialized on massless processes with many legs:

- Record multiplicity in jet and vector boson + jets calculations at NLO in QCD
- Based on generalized unitarity

Blackhat — [Bern, Dixon, Febres Cordero, Höche, Ita, Kosower, Maître, Ozeren]

Njet — [Badger, Biedermann, Uwer, Yundin]



# Towards NNLO

# Towards NNLO automation

- NNLO starts to be the new automation frontier
- Several challenges ahead:

$$\hat{\sigma}^{\mathrm{NNLO}}_{a,b\to X} = \hat{\sigma}^{\mathrm{NLO}}_{a,b\to X} + \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}}\right) + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{\mathrm{S}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{\mathrm{V,1}}_{\mathrm{NNLO}} - \mathrm{d}\sigma^{\mathrm{VS,1}}_{\mathrm{NNLO}}\right) + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{\mathrm{VS,1}}_{\mathrm{NNLO}} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{\mathrm{V,2}}_{\mathrm{NNLO}}$$

$\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{NNLO}}$ - double real: tree-level radiation of 2 additional partons to tree-level

$\mathrm{d}\sigma^{\mathrm{V,1}}_{\mathrm{NNLO}}$ - real-virtual: interference between 1-loop + 1-emission and tree-level 1-emission amplitude

$\mathrm{d}\sigma^{\mathrm{V,2}}_{\mathrm{NNLO}}$ - double virtual: interference between 2-loops virtual and born tree-level, and 1-loop amplitude squared

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✓ Double real radiation

✗ Subtraction more IR limits:

- several methods
- how well can we automatize them? How efficient are they?

✗ 1-Loop calculation to higher epsilon and for unresolved particles:

- Can in principle be computed with OLPs, which need potentially to be extended

✗ 2-Loop amplitudes:

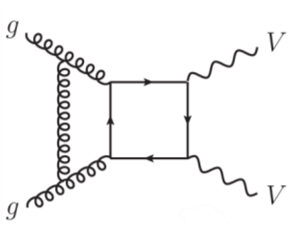
- hard to go beyond 2 to 2 for massless particles but work is in progress..

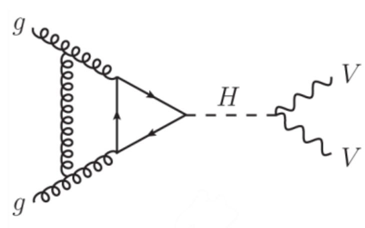
# Loop-induced processes

- A first step towards NNLO: presence of 2-loop matrix elements but same IR complexity as NLO

- Nevertheless some first additional complications:
  - real radiation amplitude is 1-loop: challenge for numerical stability
  - virtual amplitude is 2-loop: in general very hard! More later..
  - phenomenologically relevant:
    - E.g. Higgs and double Higgs production:

$gg \to VV$ and $gg \to H \to VV$



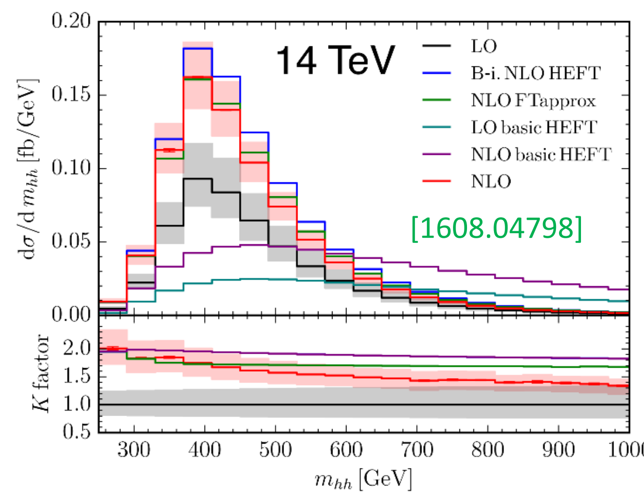




Background, signal and interference @ NLO

➔ Relevant for off-shell Higgs width measurements

[Caola et al.] [Campbell, Ellis, Czakon, Kirchner]

$gg \to HH$




[Borowka, Greiner, Heinrich, Jones, Kerner, Schlenck, Schubert, Zirke]
[Heinrich, Jones, Kerner, Luisoni, Vryonidou]

# NNLO Subtraction

- Double real radiation introduces several additional complications:
  - double soft / triple collinear configurations

- Several approaches:
  - ➢ Antenna [Kosower; Gehrmann et al.]
  - ➢ CoLoRFul [Somogyi et al.]
  - ➢ Residue-improved SD [Czakon et al.]
  - ➢ Projection-to-Born [Brucherseifer, Caola, Melnikov; Cacciari, Dreyer, Karlberg, Salam, Zanderighi]
  - ➢ $q_T$ - subtraction [Catani, Grazzini et al.]
  - ➢ N-jettiness [Gaunt et al.; Boughezal et al.]
  - ➢ Nested subtr. based on SD [Caola et al.]

- Can be categorized into 2 big families:

  - Local subtraction (as used for NLO)
    - Cancel divergences locally with counter term
    - ✓ Better convergence
    - ✗ Integrated subtraction terms can be hard to compute

  - Phase space slicing
    - Split phase space according to singular configuration and use NLO local subtraction for NLO-like singularities
    - ✓ Simpler to implement (from resummation)
    - ✗ Large cancellation on cut-off – check of slicing parameter dependence

# 2-loop amplitudes

- As it was for 1-loop 15 years ago, the bottleneck seems to be again the loop part
  - 2-loops computations available for 2 $\rightarrow$ 2 processes (massless internal particles)
  - Tools for the reduction of the loop amplitudes to coefficient x MIs:
    - Highly nontrivial since no general MIs basis is known (contrary to 1 loop)

| Fire | Kira | LiteRed | Reduze |
|---|---|---|---|
| [Smirnov] | [Maierhöfer, Usovitsch, Uwer] | [Lee] | [Schabinger, Studerus, v.Manteuffel] |

    - Based on Integration-by-parts (IBPs) relations: $\int d^D k \frac{\partial}{\partial k^\mu} v^\mu f(k, p_i) = 0$

  - Many promising developments in the last years

[Abreu, Badger, Febres Cordero, Feng, Huang, Frellesvig, Henn, Kosower, Ita, Jaquier, Larsen, Mastrolia, Mirabella, Mogull, Ossola, Papadopoulos, Page, Peraro, Primo, Zeng, Zhang, …]

# 2-loop amplitudes

- Many techniques both analytical and numerical or semi-numerical

  - Direct integration [Feynman; t'Hooft, Veltman, … ; Brown; Panzer; Schnetz; v.Manteuffel, Panzer, Schabinger; … ]
  - Mellin-Barnes representation [Tausk; Smirnov; …]
  - Differential equations [Kotikov; Remiddi; Gehrmann, Remiddi; Henn; …]

    [Argeri, Caola, Caron-Huot, Di Vita, Gehrmann, Grozin, Korchemsky, Henn, Lee, v.Manteuffel, Marquard, Mastrolia, Melnikov, Meyer, Mirabella, Papadopoulos, Primo, Schabinger, Schlenk, Schubert, Smirnov, Tancredi, Tommasini, Weihs, Wever, Yundin, …]

  - Numerical solution of differential equations [Caffo, Czyz, Laporta, Remiddi; Czakon, Mitov; …]
  - Dispersion relation [Bauberger et al.; Bauberger, Freitag; …]
  - Via Bernstein-Sato-Tkachov theorem [Passarino; Uccirati et al.; …]
  - Numerical evaluation via Mellin-Barnes [Czakon; Dubovyk, Freitas, Gluza, Riemann, Usovitsch;…]
  - Numerical extrapolation [De Doncker, Yuasa, Kato, Fujimoto Kurihara, Ishikawa, Olagbemi, Shimizu]
  - Direct integration in momentum space [Soper; Gong, Soper, Nagy; Weinzierl, Reuschle et al.;…]
  - Loop-tree duality [Rodrigo, Buchta, Chachamis, Sborlini, Driencourt-Mangin et al.; …]
  - Sector decomposition [Hepp; Denner, Roth; Binoth, Heinrich; …]

sector_decomposition [Bogner, Weinzierl]

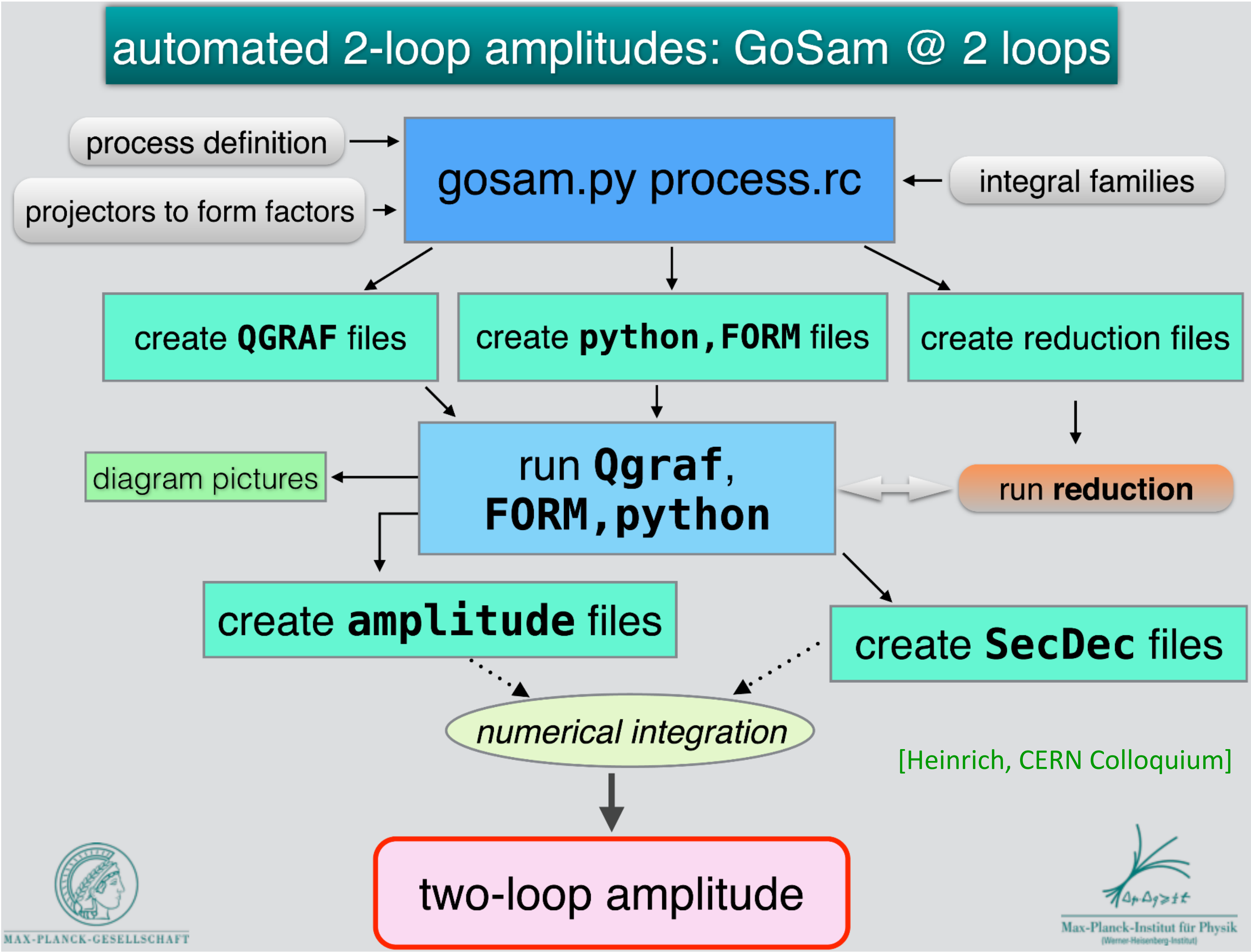
CSectors [Gluza, Kajda, Riemann, Yundin]

Fiesta [Smirnov, Smirnov, Tentyukov]

SecDec/pySecDec [Borowka, Carter, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]

Talk by S. Jahn

# Towards automation




[Heinrich, CERN Colloquium]

# Available tools for NNLO predictions

- Some tools for dedicated NNLO predictions:

**NNLOJET** Based on antenna subtraction

[Chen, Cruz-Martinez, Currie, Gehrmann, Gehrmann De-Ridder, Glover, Huss, Jaquier, Morgan, Pires]

$pp \to 2\,\text{jets}$
$pp \to H + 1\,\text{jet}$
$pp \to Z + 1\,\text{jet}$
$ep \to 2 + 1\,\text{jets}$

**MATRIX** Based on $q_T$-subtraction

[Grazzini, Kallweit, Rathlev, Wiesemann]

$pp \to Z/\gamma^* \ (\to l^+l^-)$
$pp \to W \ (\to l\nu)$
$pp \to H$
$pp \to \gamma\gamma$
$pp \to W\gamma \to l\nu\gamma$
$pp \to Z\gamma \to l^+l^-\gamma$
$pp \to ZZ \to 4\,l$
$pp \to WW \to l\nu l'\nu'$
$pp \to ZZ/WW \to ll\nu\nu$
$pp \to WZ \to l\nu l'^+l'^-$
$pp \to HH$

**MCFM-NNLO** Based on N-jettiness

[Boughezal, Campbell, Ellis, Focke, Giele, Liu, Neumann, Petriello, Williams]

$pp \to Z/\gamma^* \ (\to l^+l^-)$
$pp \to W \ (\to l\nu)$
$pp \to H$
$pp \to \gamma\gamma$
$pp \to Z\gamma \to l^+l^-\gamma)$
$pp \to HZ \to Hl^+l^-$
$pp \to HW \to Hl\nu$

# Conclusions & Outlook

- NLO automation is a reality: many tools for several tasks
  - **NOT** everything is possible "out of the box" but many pheno-relevant computations can be performed in little time compared to 10-15 years ago
  - "Conceptually" solved although large multiplicity / multiscale calculations are still computationally very tough
  - Allows to produce precise NLO predictions also for BSM scenarios
  - Possibility to produce exclusive final state NLO predictions interfacing to PS Monte Carlo generators

Talk by M.Schönherr

- Experimental accuracy reached at LHC calls for NNLO predictions for several processes
  - Very active field of research: collective effort towards automation
  - Many challenges still ahead, but very fast progresses...

    ... how long for NNLO automation?

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**Stay tuned!!!**