A double copy for N=2 supergravity in four dimensions

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with Silvia Nagy, Suresh Nampuri (arXiv:1609.05022 + 1611.04409)
and Gianluca Inverso, work in progress

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Gravity as a double copy of Yang-Mills?

Hints (in momentum space):

- construction of gravity amplitudes in terms of gluon amplitudes

  Bianchi, Elvang, Freedman, Bern, Carrasco, Johansson, Chiodarolli, Gunaydin, Roiban, ...

- tensoring of on-shell SYM/YM multiplets can accommodate for dof of on-shell supergravity multiplets

  Anastasiou, Borsten, Duff, Hughes, Nagy, ...

Here: space-time, on-shell double copy dictionary for linearized gravity

Related work: Monteiro, O'Connell, White, Luna, Goldberger, Ridgway, Nicholson, Ochirov, Westerberg, ...

Double copy construction for $D = 4$, $N = 2$ ungauged supergravity theories:

- linearized level
- on-shell (equations of motions), source free
- $$(N = 2)_{sugra} + n_V \; (N = 2)_{vector}$$

  in terms of two YM copies:

  $$(N = 2)_{SYM} \; \star \; \left[(N = 0)_{YM} + (n_V - 1) \, \mathrm{real} \;\; \mathrm{scalars} \right]$$
- double copy dictionary

$$\varphi_G = \varphi \star \tilde{\varphi}$$

Introduction

- Yang-Mills fields in adjoint of global non-Abelian group $\Longrightarrow$

  spectator field $\phi_{\alpha\tilde{\alpha}}$ in bi-adjoint of global non-Abelian group $G \times \tilde{G}$

$$\varphi_G = \varphi^{\alpha} \star \phi_{\alpha\tilde{\alpha}} \star \tilde{\varphi}^{\tilde{\alpha}}$$

- choice of $G \times \tilde{G} \Longrightarrow$ to ensure that on-shell independent dof of supergravity are captured by field theory dofs:

  sugra complex scalar fields $\quad z^A = (z^1, z^a), \quad a = 2, \dots, n_V$

  SYM: complex scalar σ, YM: real scalars $\tilde{\sigma}^a$

$$z^a = \sigma^{\alpha} \star \phi_{\alpha\tilde{\alpha}} \star \tilde{\sigma}^{\tilde{\alpha} a}$$

- in the following, omit spectator field for notational simplicity

Convolution

- $\star$ denotes a convolution: in Cartesian coordinates,

$$[f \star g](x) = \int d^4y \; f(y) \; g(x-y)$$

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

- Double copy (DC) dictionary: dictionary for fluctuations around a fixed background.

Gravity: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \; ,$ $\qquad X^I \rightarrow \langle X^I \rangle + X^I$

Field theory: $\eta_{\mu\nu}$

- Fluctuations: Sugra $(h_{\mu\nu}, \psi^i_\mu, W^I_\mu, \Omega^{Ii}, X^I) \; ,$ $\qquad I = 0, \dots, n_V$

$\partial_\mu X^I = \partial_\mu z^A \, \langle D_A X^I \rangle \quad , \quad z^A = X^A / X^0 \quad , \quad A = 1, \dots n_V$

YM: $(A_\mu, \lambda_i, \sigma \quad | \quad \tilde{A}_\mu, \tilde{\sigma}^a) \; ,$ $\qquad a = 2, \cdots, n_V$

Double copy dictionary

Derive DC dictionary at linearized level: $\varphi_G = \varphi \star \tilde{\varphi}$

Strategy:

- use field strengths to keep symmetries manifest: $\psi^i_{\mu\nu} = 2\,\partial_{[\mu}\psi^i_{\nu]}$
- general DC ansatz for linear combinations of sugra fermions:

$$\psi^i_{\mu\nu} + 2b_I\,\gamma_{[\nu}\partial_{\mu]}\Omega^{Ii} \equiv \varepsilon^{ij}\lambda_j \star \tilde{F}_{\mu\nu} + 2c_a\,\gamma_{[\nu}\partial_{\mu]}\lambda^i \star \tilde{\sigma}^a$$

  where $b_I, c_a \in \mathbb{C}$

- use convolution property and EOM's to obtain

$$\psi^i_{\mu\nu} = \varepsilon^{ij}\lambda_j \star \tilde{F}^-_{\mu\nu}$$

$$2b_I\,\partial_\mu\Omega^{Ii} = \varepsilon^{ij}\gamma^\rho\lambda_j \star \tilde{F}^+_{\mu\rho} + 2c_a\,\partial_\mu\lambda^i \star \tilde{\sigma}^a$$

- Verify consistency with EOM's for $\psi^i_\mu, \Omega^{Ii}, \tilde{A}_\mu, \tilde{\sigma}^a$

Double copy dictionary

Apply (linearized) supersymmetry transformations to the DC relations.

Use Lorentz gauge $\partial_\mu \tilde{A}^\mu = 0$, for simplicity. Omit spectator field $\phi_{\alpha\tilde{\alpha}}$.

$$
\begin{aligned}
R^-_{\mu\nu\alpha\beta} &= -\frac{1}{2}\Big[F_{\mu\nu}\star\tilde{F}^-_{\alpha\beta} + F^-_{\alpha\beta}\star\tilde{F}_{\mu\nu} \\
&\qquad +2\,\left(\eta_{[\alpha[\mu}F_{\nu]\lambda}\star\tilde{F}^\lambda{}_{\beta]} + \eta_{[\mu[\alpha}F_{\beta]\lambda}\star\tilde{F}^\lambda{}_{\nu]}\right)^-\Big] \\
\psi^i_{\mu\nu} &= \varepsilon^{ij}\lambda_j\star\tilde{F}^-_{\mu\nu} \\
F^{I-}_{\mu\nu} &= -2\,\langle\bar{X}^I\rangle\sigma\star\tilde{F}^-_{\mu\nu} + l^I\bar{\sigma}\star\tilde{F}^-_{\mu\nu} + r^I{}_a F^-_{\mu\nu}\star\tilde{\sigma}^a \\
\partial_\mu\Omega^I_i &= \frac{l^I}{2}\varepsilon_{ik}\gamma^\rho\lambda^k\star\tilde{F}^-_{\rho\mu} + r^I{}_a\partial_\mu\lambda_i\star\tilde{\sigma}^a \\
\partial_\mu\bar{X}^I &= -\frac{\bar{l}^I}{2}F^-_{\mu\rho}\star\tilde{A}^\rho + \bar{r}^I{}_a\,\partial_\mu\bar{\sigma}\star\tilde{\sigma}^a
\end{aligned}
$$

- RHS depends on $\langle\bar{X}^I\rangle$, $\langle N_{IJ}\bar{X}^I\rangle l^J = 0\,,\ \langle N_{IJ}\bar{X}^I\rangle r^J_a = 0$

Double copy dictionary

- DC dictionary has freedom $\phi_{\alpha\tilde{\alpha}}$, l^I, $r^I{}_a$.

  Additional input to fix these.

- For instance: l^I , $r^I{}_a \propto \langle D_A X^I \rangle$

  $D_A X^I = \partial_A X^I + \frac{1}{2}\,(\partial_A K)\; X^I \quad , \quad \partial_A = \frac{\partial}{\partial z^A} \quad , \quad z^A = \frac{X^A}{X^0}$

- DC relation for Riemann tensor satisfies first and second Bianchi identities.

  $\Rightarrow$ peel off two derivatives to obtain DC relation for $h_{\mu\nu}$.

  In Lorentz gauge $\partial \cdot \tilde{A} = 0$:

$$h_{\mu\nu} = 2A_{(\mu} \star \tilde{A}_{\nu)} - \eta_{\mu\nu}\, A_\rho \star \tilde{A}^\rho$$

  (up to linearized diffeomorphisms $2\partial_{(\mu}\xi_{\nu)}$)

Outlook

Work in progress:

- DC dictionary in general gauge, with sources?

  Dictionary invariant under $A \to A + d\alpha$, $\tilde{A} \to \tilde{A} + d\tilde{\alpha}$

  $\Rightarrow$ should hold in general (work in progress)

- Going beyond the linearized approximation?
  Luna, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White

Thanks!