Finite-size scaling and intermittency studies in crucial measurements for the QCD critical point

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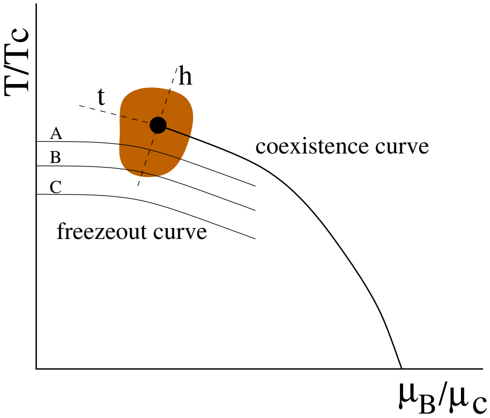
Workshop on the Standard Model and Beyond

Corfu, September 2-10

1. Critical region ; scaling properties
2. Measuring fractal dimensions in relativistic ion collisions
3. Locating the QCD critical point
4. Summary, conclusions and outlook

Phase diagram of QCD

A sketch for finite size system(s)



Objective:
Detection/existence of the
QCD Critical Point (CP)

from R. V. Gavai, Contemporary Physics 57, 350 (2016)

Critical region

Scaling laws	$\Leftrightarrow$	Fractal geometry
Order parameter fluctuations (baryon-number density $n_b(\mathbf{r})$)		Random fractal clusters formed by baryon excess
Density-density correlation $C(\mathbf{r},\mathbf{r_0}) = \langle n_b(\mathbf{r}) n_b(\mathbf{r_0}) \rangle$	**local**	Correlation dimension $C(\mathbf{r},\mathbf{r_0}) \propto \lvert \mathbf{r} - \mathbf{r_0} \rvert^{-(d-d_F)}$
Finite-size scaling (FSS) $\langle N_b \rangle = \langle \int_V n_b(\mathbf{r}) \rangle$	**global**	Mass fractal dimension $\langle N_b \rangle \propto V^{D_F/d}$

critical exponents $\Leftrightarrow$ fractal dimension(s)

(homogeneity: $D_F = d_F$)

Critical region (continued)

Scaling behaviour in the critical region

$\Downarrow$

Correlation length ξ versus linear size $L = V^{1/d}$

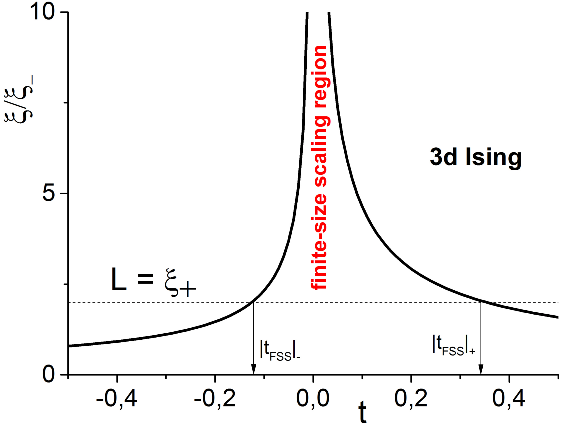
$L > \xi$: fractality seen in $C(\mathbf{r}, \mathbf{r_0})$
(up to scale ξ)
but **not** in $\langle N_b \rangle$

$\xi > L$: fractality seen in **both**
$C(\mathbf{r}, \mathbf{r_0})$ and $\langle N_b \rangle$
(all scales up to L)

ξ controlled by temperature T: $\xi = \begin{cases} \xi_+ \left| \frac{T - T_c}{T_c} \right|^{-\nu} & T \gtrsim T_c \\ \xi_- \left| \frac{T - T_c}{T_c} \right|^{-\nu} & T \lesssim T_c \end{cases}$

with $\frac{\xi_+}{\xi_-} = 2$ (universal ratio) and $\nu \approx 2/3$ (3d-Ising)

Finite-size scaling region



With $t = \frac{T - T_c}{T_c}$

Finite-size scaling region (continued)

Condition for applicability of FSS: $|t| < |t_{FSS}|_{\pm}$

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$|t_{FSS}|_{\pm}$ **depends on the system's size**

For a nucleus of size $R = R_0 A^{1/3}$ with $R_0 = 1.25\ fm$, A = mass number the limiting value $|t_{FSS}|_{\pm}$ is given by:

$$\xi(|t_{FSS}|_{\pm}) = R$$

Since the correlation length is:

$$\xi(|t|) = s_{\pm}\beta_c |t|^{-\nu} \ \text{with} \ s_{+} = 1 \ (T \gtrsim T_c), \ s_{-} = \frac{1}{2} \ (T \lesssim T_c)$$

we find: $|t_{FSS}|_{\pm} = \left(\frac{s_{\pm}\beta_c}{R}\right)^{1/\nu}$ ($\nu = \frac{2}{3}$ for 3d Ising)

Finite-size scaling region (continued)

To determine $|t_{FSS}|_{\pm}$ we need $T_c \Rightarrow$ **Lattice QCD**

We use $T_c = 150\ MeV$ compatible with Lattice results obtained from **chiral susceptibility** (S. Datta et al, Phys. Rev. D 95, 054512 (2017))

System (A)	$\lvert t_{FSS}\rvert_+$	$\lvert t_{FSS}\rvert_-$	$M = V/\beta_c^3$
Be (9)	0.36	0.13	33
C (12)	0.31	0.11	43
Si (28)	0.20	0.07	101
Ar (40)	0.17	0.06	144
Sc (45)	0.16	0.06	163
Xe (131)	0.09	0.03	473
La (139)	0.09	0.03	502
Au (197)	0.08	0.03	711
Pb (207)	0.07	0.03	748

Table: Finite-size scaling limits

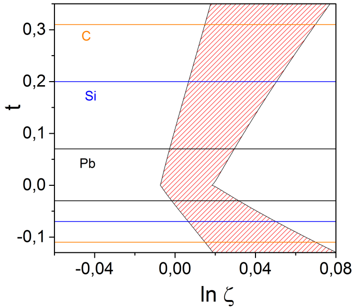
Finite-size scaling region (continued)

Using Ising-QCD partition function $\mathcal{Z}_{QCD}^{Ising}$ we find that the **mean baryon-number** $\langle N_b \rangle$ in the FSS region scales as:

$$\langle N_b \rangle \propto V^{\tilde{q}}$$

with $\tilde{q}$ depending on $\ln \zeta = \frac{\mu_B - \mu_c}{T_c}$ and $t = \frac{T - T_c}{T_c}$

For the critical region holds: $\frac{3}{4} < \tilde{q} < 1$ (definition)



Measuring $\tilde{q}$ - Transverse momentum space

Important: $\tilde{q}$ is **experimentally accessible!**

In the FSS region the **local** scaling: $\langle n_b(\mathbf{r}) n_b(\mathbf{r_0}) \rangle \propto |\mathbf{r} - \mathbf{r_0}|^{-(d-d_F)}$
holds also for large $|\mathbf{r} - \mathbf{r_0}|$

$\Downarrow$

Scaling is transferred to **(transverse) momentum space** for
small momentum differences

$\Downarrow$

Detectable through **intermittency analysis** $\Rightarrow$ **intermittency index** ϕ_2

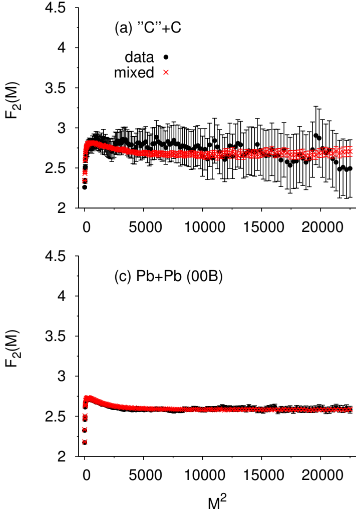
It holds: $\phi_2 = \tilde{q}$

Measurement of $\phi_2 \equiv$ Measurement of $\tilde{q}$!

Measurement of ϕ_2 in NA49 data

- Measurement of ϕ_2 performed in A+A data from NA49 experiment (CERN) at highest SPS energy $\sqrt{s} = 17.2\ GeV$
- $F_2(M)$ calculated for proton transverse momenta from **central** C+C, Si+Si and Pb+Pb collisions
- Evidence for intermittent behaviour in Si+Si
- High level of noise present in the data $\Rightarrow$ background has to be subtracted!

Measurement of ϕ_2 in NA49 data



Measurement of ϕ_2 in NA49 data

After subtraction of background we find

$$\phi_2^{(Si)} = 0.96^{+0.38}_{-0.25}$$

T. Anticic et al, NA49 Collaboration, Eur. J. Phys. C 75, 587 (2015)

$$\Downarrow$$

Very large errors $\Rightarrow$ Significantly more statistics are needed (and better control of systematic errors)!!!

An accurate measurement of ϕ_2, combined with a good estimation of the freeze-out parameters, is crucial for determining the location of QCD CP!

A systematic way to locate the critical point

To illustrate how an accurate ϕ_2-measurement may lead to the **determination of the QCD critical point** let us:

- Use the **NA49 measurement of ϕ_2** for proton transverse momenta in **central Si+Si collisions** ignoring experimental errors
- Use the estimated **freeze out parameters** measured in **NA49 experiment** at $\sqrt{s} = 17.2\ GeV$ ignoring experimental errors

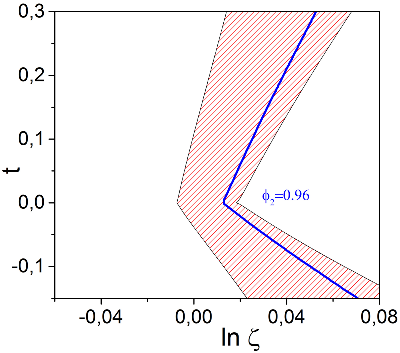
System (A)	T (MeV)	μ_B (MeV)	ϕ_2
C (12)	166	262.6	-
Si (28)	162.2	260	0.96
Pb (208)	157.5	248.9	-

Table: A+A freeze-out parameters and ϕ_2, centrality 0-12.5%

(freeze-out parameters from F. Becattini et al, Phys. Rev. C 73, 044905 (2006))

A systematic way to locate the critical point

- $\frac{3}{4} < \phi_2^{(Si)} < 1$, $\tilde{q}^{(Si)} = \phi_2^{(Si)} \Rightarrow$ Si + Si freezes-out in critical region
- Use Ising-QCD partition function to find all pairs $(\ln \zeta, t)$ leading to the scaling law: $\langle N_b \rangle \propto V^{0.96}$ $(\tilde{q} = 0.96)$ in the $(\ln \zeta, t)$ plane



Si+Si freezes-out on the blue line

Linear (piecewise) fit for the blue line:

$$\mu_c = \mu_{Si} - \frac{T_{Si} - T_c(1 + a_{\pm}^{(Si)})}{b_{\pm}^{(Si)}}$$

From (μ_{Si}, T_{Si}) (freeze-out) and $(a_{\pm}^{(Si)}, b_{\pm}^{(Si)})$ (fit) $\Rightarrow$ $\mu_c = f(T_c)$ (linear relation)

A systematic way to locate the critical point

Si+Si	a	b
+	-0.084	7.3
-	0.026	-2.48

Table: Fitting results for Si+Si

Using $(\mu_{Si},\ T_{Si}) = (260\ MeV,\ 162.2\ MeV)$ and the results of the table we obtain the two branches:

$$\mu_c = \begin{cases} 237.8\ MeV + 0.1225\,T_c & , \quad T_{Si} > T_c \\ 325.4\ MeV - 0.4137\,T_c & , \quad T_{Si} < T_c \end{cases}$$

These relations are entirely based on:

- ϕ_2 measurement for Si+Si (SPS-NA49)
- $\langle N_b \rangle$-scaling described by Ising-QCD partition function

A systematic way to locate the critical point

Use Lattice results for $T_c \Rightarrow$ Obtain μ_c!

Most recent: $T_c = 0.94\,\tilde{T}_c$ with $\tilde{T}_c$ pseudocritical temperature(s)

(S. Datta et al, Phys. Rev. D 95, 054512 (2017))

$\Downarrow$

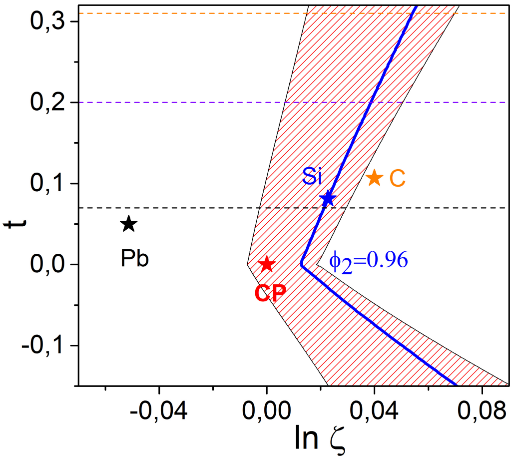
Peak of susceptibility(ies) vs. T at $\mu_B = 0$

Two main regimes:

- chiral suscept. $\Rightarrow T_c \approx 150\ MeV$
- strange quark or Polyakov loop suscept. $\Rightarrow T_c \approx 167\ MeV$

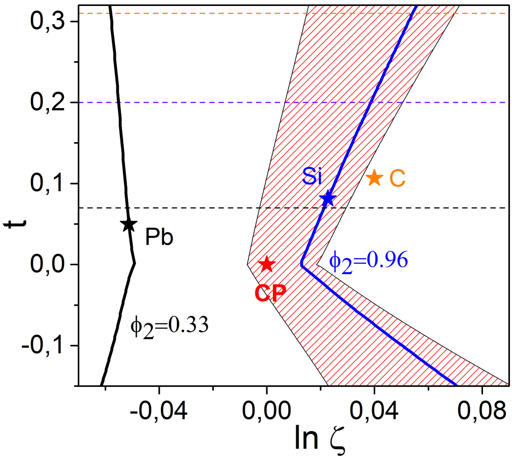
(Y. Aoki et al, Phys. Lett. B 643, 46 (2006))

Regime I: $T_c = 150\ MeV \Rightarrow \mu_c = 256.6\ MeV$



A systematic way to locate the critical point

Regime I: $T_c = 150\ MeV \Rightarrow \mu_c = 256.6\ MeV$



Predictions for NA61 and RHIC

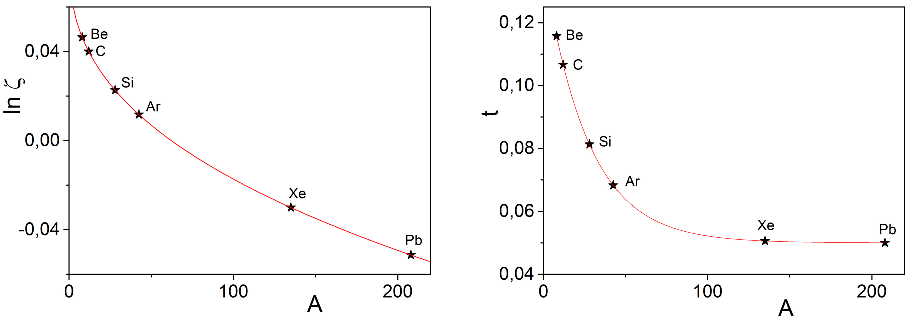
The value $\phi_2^{(Pb)} = 0.33$ is a prediction based on the proposed approach

It is compatible with intermittency analysis results for proton transverse momenta in central Pb+Pb collisions at $\sqrt{s} = 17.2\ GeV$ (SPS-NA49) in the **01I** run (N. Davis, PhD thesis, 2015)

$\Downarrow$

Using the scheme developed so far one can make **predictions** for **freeze-out parameters and ϕ_2-values** in **NA61 (CERN-SPS)** and **RHIC (BNL-BES)** experiments!

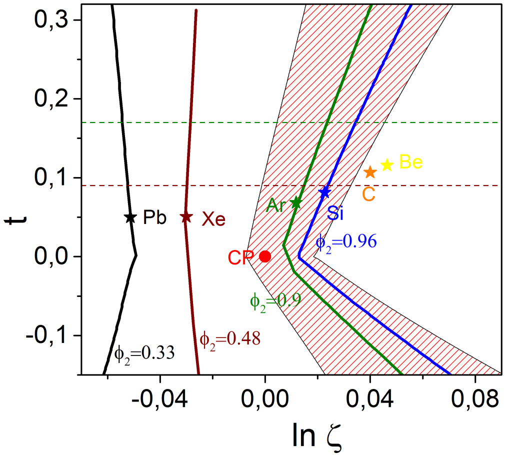
Predictions for NA61 ($\sqrt{s} = 17.2\ GeV$)



System	T (MeV)	μ_B (MeV)
Be+Be	167.4	263.6
Ar+Sc	160.2	258.4
Xe+La	157.6	252.1

Table: Prediction for NA61 freeze-out parameters ($\sqrt{s} = 17.2\ GeV$)

Predictions for NA61 ($\sqrt{s} = 17.2\ GeV$)



Measurement of $\phi_2^{(Ar)}$ in NA61 is crucial!!!

Predictions for RHIC ($11.5\ GeV < \sqrt{s} < 19.6\ GeV$)

In the energy region **11.5 GeV** $<$ $\sqrt{\mathbf{s}}$ $<$ **19.6 GeV** the freeze-out parameters for central Au+Au collisions at RHIC are well described by:

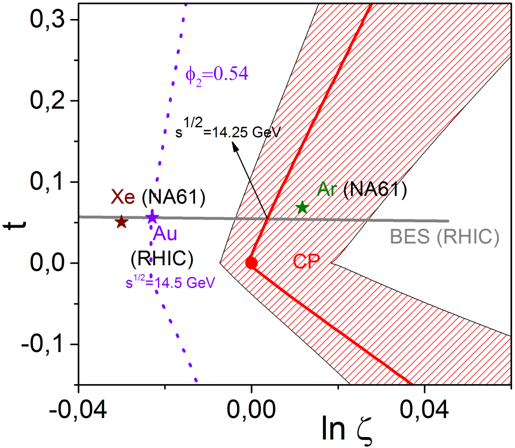
$$T = \frac{164\ MeV}{1 + \exp[2.60 - \ln(\sqrt{s_{NN}(GeV)})/0.45]}$$

$$\mu = \frac{1303\ MeV}{1 + 0.286\sqrt{s_{NN}(GeV)}}$$

(A. Adronic et al, Nucl. Phys. A 834, 237C (2010))

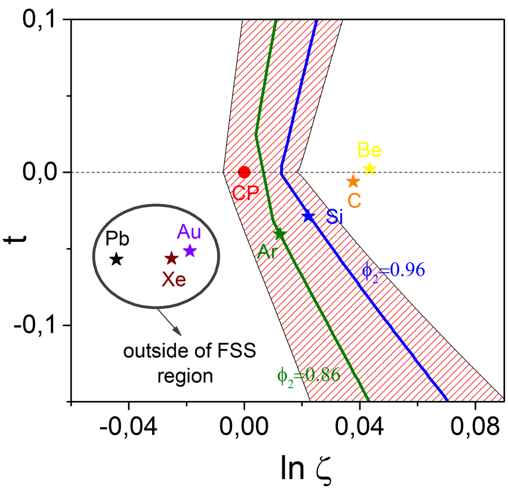
We use this parametrization for the freeze-out to predict ϕ_2-values at RHIC within the Ising-QCD finite-size scaling approach!

Predictions for RHIC ($11.5\ GeV < \sqrt{s} < 19.6\ GeV$)



- Energy region in BES relevant to QCD CP search: **14 GeV $< \sqrt{s_{NN}} <$ 15 GeV**
- **Extreme sensitivity to $\sqrt{s_{NN}}$!**
- Energy beam fine tuning **needed** to approach the critical region in RHIC-BES

Regime II: $T_c = 167\ MeV \Rightarrow \mu_c = 256.3\ MeV$



μ_c remains the same!

The large systems fall outside of the finite-size scaling region!

$\phi_2^{(Ar+Sc)}$ **is robust!**

Ar+Sc at $\sqrt{s_{NN}} = 17.2\ GeV$ $\rightarrow$ **the best candidate for critical fluctuations!**

RHIC needs $\sqrt{s_{NN}}$ **fine tuning** to see critical fluctuations: $\sqrt{s_{NN}} \in [14.1, 14.3]\ GeV$

Summary, conclusions and outlook

- It has been demonstrated that the finite-size scaling implied by the Ising-QCD effective dynamics of the baryon number density is a powerful tool for the search of the QCD critical point
- In particular it leads to local power-law fluctuations in transverse momentum space detectable through intermittency analysis
- The corresponding intermittency index ϕ_2 coincides with the finite-size scaling critical exponent q
- To illustrate the road towards the detection of the critical point within this framework we used a recent measurement of ϕ_2 in Si+Si central collisions at $\sqrt{s_{NN}} = 17.2\ GeV$ (NA49, CERN-SPS)

Summary, conclusions and outlook

- The ϕ_2 measurement **combined** with the measurement of the freeze-out parameters for the **same system** leads to a linear relation between μ_c and T_c
- This in turn leads to an accurate determination of the critical chemical potential $\mu_c \approx 256\ MeV$
- Using the obtained μ_c-value one can make a set of predictions concerning freeze-out parameters and ϕ_2-values of forthcoming measurements at **NA61 (CERN-SPS)** and **RHIC (BNL)**
- **Experimental errors** are **neglected** in our **illustrative** analysis $\Rightarrow$ **accurate** measurement(s) of ϕ_2 and **freeze-out parameters** are needed!

Summary, conclusions and outlook

- **Our analysis indicates that central Ar + Sc collisions at $\sqrt{s_{NN}} = 17.2\ GeV$ freeze-out very close to the critical point!**
- Thus an **accurate measurement** of ϕ_2 and the **freeze-out parameters** (T, μ_B) for **Ar + Sc** is of **high priority!**
- Furthermore, we find that for central **Au + Au** collisions at RHIC a **strong fine tuning** of the **beam energy** is needed for approaching the **critical point!**
- **The relevant window for BES at RHIC is:**

$$14.1\ GeV < \sqrt{s_{NN}} < 14.3\ GeV$$

Thank you!

Financial support from NATIONAL AND KAPODISTRIAN UNIVERSITY OF ATHENS, S.A.R.G, 13233 is appreciated