Scalar potential in models with spontaneous breaking of quantum scale invariance

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• New physics beyond SM

? - supersymmetry at TeV – "unwell".... at 10 TeV? large χ^2 cost.

? - scale symmetry (SS) of action:

$$x' = \rho\, x, \qquad \phi'(x') = \rho^{-d_\phi}\,\phi(x), \quad [\phi] = d_\phi. \quad \text{forbids} \qquad \int d^4x\; m^2\phi^2 + \cdots$$

- no dimensionful couplings; no scale $\rightarrow$ no hierarchy!
- SM with tree-level $m_h = 0$ has SS. "Higgs portal" models; etc. [Bardeen 1995]
- Quantum level: Regularization breaks it explicitly. In real world SS broken (M_{Planck}? etc).

• WANTED! Do a scale-invariant quantum calculation & Break SS only spontaneously.

- Toy model; flat space; S$\!\!\!/$S$\!\!\!/$ (vev$\gg \langle\phi\rangle$)? loop corrections to V, m_ϕ? renorm? effective ops?
  implications for hierarchy of scales? applications to SM?

• UV regularization breaks SS:

- Tree level: start with a scale-invariant $\mathcal{L}$: e.g. SM with $m_h = 0$, toy-model....
- Loop level? $\rightarrow$ divergences $\rightarrow$ subtraction scale (μ)

DR, d=4-2ϵ: $\lambda_\phi^0 = \mu^{2\epsilon}\left[\lambda_\phi + \sum_n a_n/\epsilon^n\right]$, $\mathcal{L} = (1/2)(\partial_\mu\phi)^2 - \lambda_\phi\,\mu^{2\epsilon}\,\phi^4$, (SS broken)

- UV regularizations break explicitly SS.

$\Rightarrow$ Solution: replace $\mu \rightarrow$ function $\mu(\sigma)$, [recall M_{string} moduli dep] [Deser 1970, Englert 1976, IZ book]

- powers $\mu^{2\epsilon} \Rightarrow$ need $\langle\sigma\rangle \neq 0$ i.e. spontaneous S$\!\!\!/$S (σ=Goldstone mode/dilaton).

$\Rightarrow$ spectrum extended by σ field!

• An example (tree level):

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma), \qquad \text{e.g.} \qquad V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\,\sigma^4$$

Extremum: $\langle\phi\rangle\big[\,\lambda_\phi\langle\phi\rangle^2 + \lambda_m\langle\sigma\rangle^2\,\big] = 0, \quad \langle\sigma\rangle\big[\,\lambda_m\langle\phi\rangle^2 + \lambda_\sigma\langle\sigma\rangle^2\,\big] = 0,$

$\langle\sigma\rangle \neq 0$ is a solution; then $\langle\phi\rangle \neq 0$; a non-trivial ground state exists if

(i) $\lambda_m^2 = \lambda_\phi\lambda_\sigma + \text{loops}, \;\; \lambda_m < 0;$ and **(ii)** $\dfrac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\dfrac{\lambda_m}{\lambda_\phi}(1+\text{loops}), \;\Rightarrow\; V = \dfrac{\lambda_\phi}{4}\Big(\,\phi^2 + \dfrac{\lambda_m}{\lambda_\phi}\,\sigma^2\,\Big)^2$

$\Rightarrow$ EWSB from Spontaneous S$\not{S}$.

$\Rightarrow$ S$\not{S}$ $\Leftrightarrow$ demand flat direction exist in (ϕ,σ): $\frac{\phi}{\sigma} = \pm\sqrt{\frac{-\lambda_m}{\lambda_\phi}}$ $\Leftrightarrow$ assume **(i)** $\Leftrightarrow$ tuning $V=0$.

masses:

$$m_{\tilde{\phi}}^2 = -2\,(1-\lambda_m/\lambda_\phi)\,\lambda_m\,\langle\sigma\rangle^2$$
$$m_{\tilde{\sigma}} = 0.$$

$\Rightarrow$ dof: 1 massive + 1 massless (Goldstone/dilaton). [Kobakhidze et al 2007, 2014]

$\langle\sigma\rangle$: unknown (e.g. $\langle\sigma\rangle \sim M_{\text{Planck}}$, etc). Very weak coupling λ_m of visible to hidden sectors:

if $\lambda_\sigma \ll |\lambda_m| \ll \lambda_\phi$, then $m_{\tilde{\phi}} \sim \langle\phi\rangle \ll \langle\sigma\rangle$ $\quad\lambda_m^2 = \lambda_\phi\lambda_\sigma +$ loops

$\Rightarrow$ Loop level: is such hierarchy stable? Are there "dangerous" corrections like $\sim \lambda_\phi^2\,\langle\sigma\rangle^2$?

$\Rightarrow$ Ultraweak couplings? $\lambda_m \sim 1/\langle\sigma\rangle^2$, $\lambda_\sigma \sim 1/\langle\sigma\rangle^4$. All scales related to $\langle\sigma\rangle$. [Ross, Hill, Allison 2014]

More general V:

$V(\phi,\sigma) = \sigma^4\,W(\phi/\sigma)$, $\langle\sigma\rangle \neq 0$ min: $W(x) = W'(x) = 0$, $x = \phi/\sigma \Rightarrow W \propto \Big(\phi^2/\sigma^2 - \langle\phi\rangle^2/\langle\sigma\rangle^2\Big)^2$

$\Rightarrow$ A scale-invariant loop calculation would preserve these features.

• One-loop scale invariant V and "evanescent" interactions:

$$\mathcal{L} = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - V(\phi,\sigma), \qquad V = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\,\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\,\sigma^4$$

DR: $d = 4 - 2\,\epsilon$, $\mu \to \mu(\sigma)$. Then: $V(\phi,\sigma) \to \tilde{V} \equiv \mu(\sigma)^{2\epsilon}\,V(\phi,\sigma)$, and $\mu(\sigma) = z\,\sigma^{2/(d-2)}$.

$$\begin{aligned} V_1 &= \tilde{V} - \frac{i}{2}\int \frac{d^d p}{(2\pi)^d}\,\mathrm{Tr}\ln\left[p^2 - \tilde{M}^2(\phi,\sigma) + i\varepsilon\right], \\ &= \tilde{V} - \frac{1}{64\pi^2}\sum_{s=\phi,\sigma}\tilde{M}_s^4\left[\frac{1}{\epsilon} - \ln\tilde{M}_s^2/\kappa\right], \end{aligned} \qquad \begin{aligned} (\tilde{M}^2)_{\alpha\beta} &= \tilde{V}_{\alpha\beta} \equiv \frac{\partial^2\tilde{V}}{\partial\,\alpha\;\partial\,\beta} \\ (M^2)_{\alpha\beta} &= V_{\alpha\beta}. \end{aligned}$$

$$\begin{aligned} (\tilde{M}^2)_{\alpha\beta} &= \mu^{2\epsilon}\left[(M^2)_{\alpha\beta} + \epsilon\;N_{\alpha\beta}\right] + \mathcal{O}(\epsilon^2), \\ N_{\alpha\beta} &\equiv \frac{2}{\mu^2}\left[\mu\,(\mu_\alpha\,V_\beta + \mu_\beta\,V_\alpha) + (\mu\,\mu_{\alpha\beta} - \mu_\alpha\,\mu_\beta)\,V\right] \end{aligned} \qquad \alpha,\beta = \phi,\sigma. \quad \mu_\alpha = \frac{\partial\mu}{\partial\alpha},$$

$\Rightarrow$ New, finite one-loop corrections $\epsilon \times \frac{1}{\epsilon}$ = finite, from derivatives of μ.

- One-loop scale invariant V

$$U_1 = V + V^{(1)} + V^{(1,n)}$$

$$V^{(1)} = \frac{1}{64\pi^2}\Big\{\sum_{s=\phi,\sigma} M_s^4(\phi,\sigma)\Big[\ln\frac{M_s^2(\phi,\sigma)}{\mu^2(\sigma)} - \frac{3}{2}\Big]\Big\}$$

$$V^{(1,n)} = \frac{-4}{\sigma^2}\Big\{2\sigma\left(V_\sigma V_{\sigma\sigma} + V_\phi V_{\phi\sigma}\right) - V V_{\sigma\sigma}\Big\}$$

where one can replace $\mu = z\,\sigma^{2/(d-2)} \to z\,\sigma$ after renormalization.

$\Rightarrow$ $V^{(1,n)}$: new one-loop finite correction, $z-$independent;

$\Rightarrow$ CW term itself modified to a SS version; Taylor expand it about $\sigma = \langle\sigma\rangle + \tilde{\sigma}$.

If μ=constant, then $V^{(1,n)} = 0$, i.e. usual DR result (explicit breaking of SS)

• One-loop scale invariant V.

- Using: $\lambda_m^2 = \lambda_\phi \, \lambda_\sigma + \text{loops}$, $\lambda_m < 0$, for EWSB, then:

$$V^{(1,n)} = \frac{\lambda_m}{\lambda_\phi} \Big(\frac{\phi^2}{\sigma^2} + \frac{\lambda_m}{\lambda_\phi} \Big) \big(\lambda_\phi^2 \, \phi^4 - 4 \, \lambda_\phi \, (4 \, \lambda_\phi + \lambda_m) \, \phi^2 \, \sigma^2 - 21 \lambda_m^2 \, \sigma^4 \big) \supset \frac{\phi^6}{\sigma^2} + \cdots$$

- Non-polynomial ops $\rightarrow$ polynomial, infinite series, Taylor expand U_1: $\sigma = \langle \sigma \rangle + \delta\sigma$, $\phi = \langle \phi \rangle + \delta\phi$.

$$\frac{\phi^6}{\sigma^2} = \frac{(\delta\phi + \langle \phi \rangle)^6}{\langle \sigma \rangle^2} \, \Big(1 - \frac{2\delta\sigma}{\langle \sigma \rangle} + \frac{3\delta\sigma^2}{\langle \sigma \rangle^2} + \cdots \Big)$$

$\Rightarrow$ Quantum effective non-polynomial operators, finite, independent of z (allowed by SS).

• Beta functions and Callan Symanzik. Counterterms:

$$\delta L_1 = -\mu(\sigma)^{2\epsilon}\Big\{\frac{1}{4!}(Z_{\lambda_\phi}-1)\lambda_\phi\phi^4 + \frac{1}{4}(Z_{\lambda_m}-1)\lambda_m\phi^2\sigma^2 + \frac{1}{4!}(Z_{\lambda_\sigma}-1)\lambda_\sigma\sigma^4\Big]$$

with

$$Z_{\lambda_\phi} = 1 + \frac{3}{2\kappa\,\epsilon}(\lambda_\phi + \lambda_m^2/\lambda_\phi), \qquad \text{similar for } Z_{\lambda_\sigma}$$
$$Z_{\lambda_m} = 1 + \frac{1}{2\kappa\,\epsilon}(\lambda_\phi + \lambda_\sigma + 4\lambda_m),$$

$\kappa = (4\pi)^2$. From: $d\,\underbrace{(\mu(\sigma)^{2\epsilon}\lambda_\phi Z_{\lambda_\phi})}_{\lambda_\phi^B}/d\ln z = 0$ one recovers usual beta functions:

$$\beta^{(1)}_{\lambda_\phi} \equiv \frac{d\lambda_\phi}{d\ln z} = \frac{3}{\kappa}(\lambda_\phi^2 + \lambda_m^2),$$
$$\beta^{(1)}_{\lambda_m} \equiv \frac{d\lambda_m}{d\ln z} = \frac{1}{\kappa}(\lambda_\phi + 4\lambda_m + \lambda_\sigma)\lambda_m$$

Callan Symanzik eq is verified: [see C. Tamarit 2014]

$$\frac{d\,U_1}{d\ln z} = \Big(\beta^{(1)}_{\lambda_j}\frac{\partial}{\partial\lambda_j} + z\frac{\partial}{\partial z}\Big)U_1 = \mathcal{O}(\lambda_j^3), \qquad (\text{sum over } j = \phi, m, \sigma).$$

• Minimizing the one-loop U_1: $\lambda_\phi \gg |\lambda_m| \gg \lambda_\sigma$, and $\mu = z\,\sigma$. (*)

$$U_1 = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_m}{2}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4}\sigma^4 + \frac{1}{64\pi^2}\Big\{\sum_{s=1,2} M_s^4\Big[\ln\frac{M_s^2}{z^2\sigma^2} - \frac{3}{2}\Big]$$

$$+\ \lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2} - \big(16\,\lambda_\phi\lambda_m + 6\lambda_m^2 - 3\lambda_\phi\lambda_\sigma\big)\phi^4 - 16\,\lambda_m^2\,\phi^2\sigma^2\Big\} + \mathcal{O}(\lambda_m^3)$$

$$\text{min:}\quad \rho \equiv \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{\lambda_m}{\lambda_\phi}\Big[1 - \underbrace{\frac{6\lambda_\phi}{64\pi^2}\big(4\ln 3\lambda_\phi - 17/3\big)}_{\text{one-loop}}\Big] + \mathcal{O}(\lambda_m^2)$$

- fix subtraction "parameter": $z = \langle\phi\rangle/\langle\sigma\rangle \Rightarrow \mu = \langle\phi\rangle$ (usual). $m_{\tilde\phi}^2 = \text{tr}(U_1)_{ij}\Big|_{min}$.

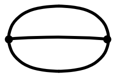
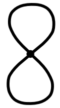
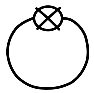
$$\delta m_{\tilde\phi}^2 = \frac{-\langle\sigma\rangle^2}{32\pi^2}\Big[4\lambda_m^2(4+13\rho) + 18\lambda_\sigma\,(7\lambda_\sigma - \lambda_\phi\rho) + \lambda_m\big[25\lambda_\sigma(1+\rho) - 3\lambda_\phi\rho(-32+5\rho+\rho^2)\big]\Big] \sim \lambda_m^2\,\langle\sigma\rangle^2$$

- where tuning relation among $\lambda_{\phi,m,\sigma}$ for $V=0$ was used (min cond)

- no tuning beyond classical (*) to have $\delta m_{\tilde\phi}^2 \ll \langle\sigma\rangle^2$. no $\lambda_\phi^2\langle\sigma\rangle^2$; (spontaneous $\to$ all orders?)

- Two-loop scale-invariant V: new poles! (background field method)

Davydychev, Tausk 93

$$V_2 = \frac{i}{12}$$  $$+ \frac{i}{8}$$  $$+ \frac{i}{2}$$ 

$$= \frac{i}{12} \tilde{V}_{ijk} \tilde{V}_{lmn} \int \frac{d^d p}{(2\pi)^d} \int \frac{d^d q}{(2\pi)^d} (\tilde{D}_p^{-1})_{il} (\tilde{D}_q^{-1})_{jm} (\tilde{D}_{p+q}^{-1})_{kn} + \cdots$$

$$= \frac{1}{\epsilon^2} (....\mu = \text{const}.....) + \frac{1}{\epsilon} (...\mu = \text{const}....) + V_2^{1/\epsilon} + V^{(2)}(\mu = \text{const}) + V^{(2,n)}$$

$\tilde{V} = \mu^{2\epsilon} V = (z\sigma)^{2\epsilon} V \Rightarrow \tilde{V}_{ijk...} = V_{ijk..} + \epsilon \times (....)_{ijk...} \Rightarrow (1/\epsilon^2) \times \tilde{V}_{ijk} = V_2^{1/\epsilon} \sim 1/\epsilon$. New c-terms:

$$V_2^{1/\epsilon} = \frac{\mu(\sigma)^{2\epsilon}}{16\kappa^2\,\epsilon} \Big[\phi^4 \Big(\frac{20}{3} \lambda_\phi^2 \lambda_m + \frac{7}{6} \lambda_\phi \lambda_m^2 - 2\lambda_m^3 - \frac{1}{2} \lambda_\phi^2 \lambda_\sigma - \frac{4}{3} \lambda_\phi \lambda_m \lambda_\sigma + \frac{7}{12} \lambda_m^2 \lambda_\sigma + \frac{1}{4} \lambda_\phi \lambda_\sigma^2 \Big)$$

$$+ \phi^2 \sigma^2 \Big(8\lambda_\phi \lambda_m^2 + \frac{41}{2} \lambda_m^3 + \lambda_\phi \lambda_m \lambda_\sigma + \frac{43}{3} \lambda_m^2 \lambda_\sigma + \frac{1}{2} \lambda_m \lambda_\sigma^2 \Big) + \sigma^4 \Big(4\lambda_m^3 + \frac{1}{3} \lambda_m^2 \lambda_\sigma + \frac{7}{4} \lambda_\sigma^3 \Big)$$

$$+ \frac{\phi^6}{\sigma^2} \Big(-\frac{7}{6} \lambda_\phi^2 \lambda_m + \frac{7}{3} \lambda_\phi \lambda_m^2 - \frac{1}{6} \lambda_\phi \lambda_m \lambda_\sigma \Big) - \frac{1}{4} \lambda_\phi \lambda_m^2 \, \frac{\phi^8}{\sigma^4} \Big].$$

- also non-polynomial c-terms, respect SS $\Rightarrow$ ∞-many polynomial ones! $\Rightarrow$ non-renormalizability.

- also new finite terms, $V^{(2,n)}$.

- Beta functions at two-loop: new corrections

$$\delta L_2 = \frac{1}{2}(Z_\phi - 1)(\partial_\mu \phi)^2 + \frac{1}{2}(Z_\sigma - 1)(\partial_\mu \sigma)^2 - \mu(\sigma)^{2\epsilon}\Big\{(Z_{\lambda_\phi} - 1)\frac{\lambda_\phi}{4!}\phi^4 + \\ +(Z_{\lambda_m} - 1)\frac{\lambda_m}{4}\phi^2\sigma^2 + (Z_{\lambda_\sigma} - 1)\frac{\lambda_\sigma}{4!}\sigma^4 + (Z_{\lambda_6} - 1)\frac{\lambda_6}{6}\frac{\phi^6}{\sigma^2} + (Z_{\lambda_8} - 1)\frac{\lambda_8}{8}\frac{\phi^8}{\sigma^4}\Big\},$$

where

$$Z_{\lambda_\phi} = 1 + \frac{\delta_0^\phi}{\kappa\,\epsilon} + \frac{1}{\kappa^2}\Big(\frac{\delta_1^\phi + \nu_1^\phi}{\epsilon} + \frac{\delta_2^\phi}{\epsilon^2}\Big), \text{ etc, } \kappa = (4\pi)^2, \qquad \text{bare coupling} \quad \lambda_\phi^B = \mu(\sigma)^{2\epsilon}\lambda_\phi\, Z_{\lambda_\phi}\, Z_\phi^{-2},$$

$(d/d\ln z)\,\lambda_\phi^B = 0$,

$$\Rightarrow \quad \beta_{\lambda_\phi} = \frac{3}{\kappa}(\lambda_\phi^2 + \lambda_m^2) - \frac{1}{\kappa^2}\Big(\frac{17}{3}\lambda_\phi^3 + 5\lambda_\phi\lambda_m^2 + 12\lambda_m^3\Big) + \beta_{\lambda_\phi}^{(2,\mathrm{n})},$$

$$\beta_{\lambda_\phi}^{(2,\mathrm{n})} = \frac{1}{2\kappa^2}\left[\lambda_m^2(24\lambda_m - 7\lambda_\sigma) + \lambda_\phi(-14\lambda_m^2 + 16\lambda_m\lambda_\sigma - 3\lambda_\sigma^2) + \lambda_\phi^2(-80\lambda_m + 6\lambda_\sigma)\right],$$

$\Rightarrow$ new $\beta_\lambda^{(2,n)}$ at 2-loop, due to new $1/\epsilon$ poles in V, from field derivatives of $\mu(\sigma)^{2\epsilon}$

$\Rightarrow$ RGE: distinguishes spontaneous ($\mu \sim z\sigma$) from explicit S̸S (μ =const).

- non-renormalizability:

$\Rightarrow$ non-polynomial operators $\Leftrightarrow$ ∞-many polynomial ones, at given loop order:

$$\lambda_6 \frac{\phi^6}{\sigma^2} + \lambda_8 \frac{\phi^8}{\sigma^4} + \cdots \qquad \Leftrightarrow \qquad \frac{\phi^6}{\sigma^2} = \frac{(\delta\phi + \langle\phi\rangle)^6}{\langle\sigma\rangle^2} \left(1 - \frac{2\delta\sigma}{\langle\sigma\rangle} + \frac{3\delta\sigma^2}{\langle\sigma\rangle^2} + \cdots\right) \qquad \langle\sigma\rangle \sim f$$

β_{λ_6}, β_{λ_8}, etc....

$$\beta_{\lambda_6}^{(2,n)} = \frac{1}{4\kappa^2} \lambda_\phi \lambda_m \left(7\lambda_\phi - 14\lambda_m + \lambda_\sigma\right) \to 0 \;\; (\lambda_m \to 0), \quad etc, \qquad \kappa = (4\pi)^2.$$

that would otherwise be difficult to compute.

- if $\lambda_{6,8,\ldots} \neq 0$ in tree-level V $\Rightarrow$ β_{λ_6}, β_{λ_8}, ... $\neq 0$ at one-loop.

• Callan-Symanzik at two-loop

$$\frac{d\,U(\lambda,z)}{d\ln z} = \Big(z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial \phi} - \sigma\gamma_\sigma\frac{\partial}{\partial \sigma}\Big)U(\lambda,z) = 0\ ,$$

$$\beta_{\lambda_j} \;=\; \beta_{\lambda_j}^{(1)} + \beta_{\lambda_j}^{(2)} + \beta_{\lambda_j}^{(2,\mathsf{n})}.$$

$$\frac{\partial\,V^{(2)}}{\partial\ln z} + \left(\beta_{\lambda_j}^{(2)}\frac{\partial}{\partial\lambda_j} - \gamma_\phi\phi\frac{\partial}{\partial\phi} - \gamma_\sigma\sigma\frac{\partial}{\partial\sigma}\right)V + \beta_{\lambda_j}^{(1)}\frac{\partial V^{(1)}}{\partial\lambda_j} = 0, \qquad \text{as for } \mu\text{=contant}$$

$$\frac{\partial\,V^{(2,\mathsf{n})}}{\partial\ln z} + \beta_{\lambda_j}^{(2,\mathsf{n})}\frac{\partial V}{\partial\lambda_j} + \beta_{\lambda_j}^{(1)}\frac{\partial V^{(1,\mathsf{n})}}{\partial\lambda_j} = 0\ .$$

⇒ second equation: non-trivial check, for new corrections $V^{(1,n)}$, $V^{(2,n)}$, depends on $\beta_{\lambda_{6,8}}$.

• Dilatation current D_μ: [Komargodski, Rattazzi et al]

$$D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} (x^\nu \partial_\nu \phi_j + d_\phi) - x^\mu \mathcal{L}, \qquad d_\phi = (d-2)/2 \quad \text{(scalars)},$$

$$\partial_\mu D^\mu = (d_\phi + 1)(\partial_\mu \phi_i) \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} + d_\phi \, \phi_j \frac{\partial \mathcal{L}}{\partial \phi_j} - d\mathcal{L}$$

canonical k.t., potential $\mathcal{V}$, onshell:

$$\partial_\mu D^\mu = d\,\mathcal{V} - \frac{d-2}{2} \phi_j \frac{\partial \mathcal{V}}{\partial \phi_j}$$

[Tamarit, arXiv:1309.0913]

- in SS theories: $\mathcal{V}$ homogeneous function: $\mathcal{V}(\rho\,\phi_j) = \rho^{2d/(d-2)}\,\mathcal{V}(\phi_j)$ from which (*) follows.
- $\partial_\mu D^\mu = 0$ if $\mathcal{V}$ is scale invariant in $d = 4 - 2\epsilon$ such as $\mathcal{V} = (z\,\sigma)^{2\epsilon}\, V(\phi, \sigma)$.
- Non-zero in loops. Couplings still run (with momentum), $\beta_\lambda \neq 0$.

• Dilatation current D_μ:

- in "traditional reg" $\mathcal{V} = \mu^{2\epsilon} V(\phi, \sigma)$, with V scale invariant in $d = 4$ but not in $d = 4 - 2\epsilon$:

$$\partial_\mu D^\mu = d\,\mu^{2\epsilon}\,V - 2(d-2)\,\mu^{2\epsilon}\,V(\phi_j) = 2\,\epsilon\;\mu^{2\epsilon}\,V \sim 2\,\epsilon\,\mu^{2\epsilon}\left[\lambda_j + \frac{\beta_{\lambda_j}}{\epsilon} + \cdots\right]\frac{\partial V}{\partial \lambda_j} \propto \beta_{\lambda_j}\,\frac{\partial V}{\partial \lambda_j}$$

(trace anomaly).

• notice:

- μ=constant: explicit S$\not{S}$: $\mathcal{L}$: renormalizable (both fields massive).

- $\mu = \mu(\sigma)$, S$\not{S}$ spont: $\mathcal{L}$: non-renormalizable! (1 field massless).

• Scale invariant SM + dilaton: one-loop potential

$$V = \frac{\lambda_\phi}{3!}\,(H^\dagger H)^2 + \frac{\lambda_m}{2}\,(H^\dagger H)\,\sigma^2 + \frac{\lambda_\sigma}{4!}\,\sigma^4 + \frac{4\,\lambda_6}{3}\,\frac{(H^\dagger H)^3}{\sigma^2} + \cdots$$
$$= \frac{1}{4!}\,\lambda_\phi\,\phi^4 + \frac{1}{4}\,\lambda_m\,\phi^2\,\sigma^2 + \frac{1}{4!}\,\lambda_\sigma\,\sigma^4 + \frac{\lambda_6}{6}\,\frac{\phi^6}{\sigma^2}\cdots; \qquad (\lambda_\phi\lambda_\sigma = \lambda_m^2 + \text{loops}).$$

SM one-loop potential $U_1 = V + V^{(1)} + V^{(1,n)}$

$$V^{(1)} \equiv \frac{1}{4\kappa}\sum_{j=\phi,\sigma;G,t,W,Z} n_j\,m_j^4(\phi,\sigma)\,\ln\frac{m_j^2(\phi,\sigma)}{c_j\,(z\sigma)^2}$$

$$V^{(1,n)} \equiv \frac{1}{48\kappa}\Big[(-16\lambda_m\lambda_\phi - 18\lambda_m^2 + \lambda_\phi\lambda_\sigma)\,\phi^4 - \lambda_m(48\lambda_m + 25\lambda_\sigma)\phi^2\sigma^2 - 7\lambda_\sigma^2\sigma^4$$
$$+\ (\lambda_\phi\lambda_m + 6\lambda_6\lambda_\sigma)\frac{\phi^6}{\sigma^2} + 8\,(4\lambda_\phi - 2\lambda_m)\,\lambda_6\,\frac{\phi^8}{\sigma^4} + (192\lambda_6 + 2\lambda_\phi)\lambda_6\,\frac{\phi^{10}}{\sigma^6} + 40\lambda_6^2\,\frac{\phi^{12}}{\sigma^8}\Big]$$

- scale invariant. New effective operators. Taylor-expand them about $\langle\sigma\rangle$.
- as shown earlier, $\delta m_\phi^2 \sim \lambda_\sigma^2\langle\sigma\rangle^2$: under control, no extra tunning beyond classical one.

where beta functions, similar to usual case (μ = constant)

$$\beta_{\lambda_\phi} = \frac{1}{\kappa}\left[3\left(\frac{9}{4}g_2^4 + \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2 g_2^2 - 12h_t^4\right) - 4\lambda_\phi\left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right) + 4\lambda_\phi^2 + 3\lambda_m^2 + 96\lambda_m\lambda_6\right]$$

$$\beta_{\lambda_m} = \frac{2\lambda_m}{\kappa}\left[\lambda_\phi + 2\lambda_m + \frac{1}{2}\lambda_\sigma - \left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right)\right]$$

$$\beta_{\lambda_\sigma} = \frac{3\lambda_\sigma}{\kappa}\left[\lambda_\sigma + 4\frac{\lambda_m^2}{\lambda_\sigma}\right]$$

but corrected by λ_6 and

$$\beta_{\lambda_6} = \frac{3\lambda_6}{\kappa}\left[6\lambda_\phi - 8\lambda_m + \lambda_\sigma - 2\left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right)\right]$$

$$\beta_{\lambda_8} = \frac{2}{\kappa}\left[2\lambda_6\left(28\lambda_6 + \lambda_m\right) - 4\lambda_8\left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right)\right]$$

$$\beta_{\lambda_{10}} = 10\left[4\lambda_6^2 - \lambda_{10}\left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right)\right]$$

$$\beta_{\lambda_{12}} = 2\left[3\lambda_6^2 - 6\lambda_{12}\left(\frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 3h_t^2\right)\right]$$

$\Rightarrow$ if $\lambda_{6,8,10,..} = 0$ at tree level, then $\beta_{\lambda_{6,8,10,12}} = 0$ at one-loop, but emerge at 2-loops.

- Phenomenology?

• Conclusions

Application of a scale-invariant UV regularization, with spontaneous SS breaking.

⇒ 1. One- and two-loop V with S$\!\!\!/$S. Manifest scale invariant quantum potential. Flat direction.

New corrections ("evanescent" interaction).

⇒ 2. $\lambda_6\, \phi^6/\sigma^2 + \lambda_8\, \phi^8/\sigma^4$, etc: non-polynomial/c-terms ⇒ non-renormalizability.

⇒ 3. Quantum stable hierarchy of scales from hierarchy of dimensionless couplings.

Quantum correction to m_ϕ^2 under control. Higher orders?

⇒ 4. Two-loop beta's and RGE: differ in spontaneous ($\mu = z\sigma$) from explicit S$\!\!\!/$S (μ=const)

⇒ 5. Applications to scale inv SM + dilaton.

Other applications? non-minimal coupling, Brans-Dicke theory. [Ross,Hill,Ferreira; Shaposhnikov et al]

see talk by Graham Ross at this workshop.

• A decoupling limit

[Fubini 1976]

At two-loop level:

$$U = V + \frac{V_{\phi\phi}}{4(4\pi)^2}\ln\frac{V_{\phi\phi}}{(z\sigma)^2} + \frac{\lambda_\phi^3\phi^4}{32(4\pi)^4}\Big\{c_0 - 10\ln\frac{V_{\phi\phi}}{(z\,\sigma)^2} + 3\ln^2\frac{V_{\phi\phi}}{(z\,\sigma)^2}\Big\}$$

$$+ \frac{5\lambda_\phi^3}{24(4\pi)^4}\frac{\phi^6}{\sigma^2} + \frac{7\lambda_\phi^3}{576(4\pi)^4}\frac{\phi^8}{\sigma^4}, \qquad [\mu(\sigma)\to z\sigma, \;\; c_0 = \text{constant}]$$

- two loop: usual counterterms as if μ =constant, but $\mu \to z\sigma$ scale inv.
- new, finite two-loop operators ϕ^6/σ^2, ϕ^8/σ^4 (not counterterms).
- three loop: new poles $(1/\epsilon)\,\phi^6/\sigma^2$, $(1/\epsilon)\,\phi^8/\sigma^4$.
- higher loops more effective operators. Can Taylor-expand them about $\langle\sigma\rangle$.