Addressing the missing matter problem in galaxies through a new fundamental gravitational radius

Salvatore Capozziello
Università di Napoli "Federico II"

Outline

- Extended Theories of Gravity
- The Noether Symmetry Approach
- The Baryonic Tully-Fisher relation
- The Fundamental Plane of Elliptical Galaxies
- Conclusions and discussion

Extended Theories of Gravity

- Extended Theories of Gravity work very well in cosmology at early and late epochs to address Inflation and Dark Energy issues
  - -A.A. Starobinsky, Phys. Lett.B 991, 99 (1980)
  - -S. Capozziello, M. De Laurentis, Phys. Rep. 509, 167 (2011),
  - -S. Nojiri, S.D. Odintsov, Phys. Rep. 505, 59 (2011).

- They have been proposed to explain galactic and extragalactic dynamics without introducing dark matter.

- As simple choice, one assumes a generic function *f(R)* of the Ricci scalar *R* (in particular, analytic functions) and searches for a theory of gravity having suitable behavior at small and large scale lengths.

- These theories need to be confirmed at different scales: for short distances, Solar system, spiral galaxies and galaxy clusters, besides cosmology

  S. Capozziello, M. De Laurentis, *Annalen der Physik* 524, 545 (2012).

Motivations

- Explaining the observed galactic and extragalactic dynamics using gravitational potentials derived from Extended Gravity without DM .

- Possible new fundamental gravitational radii which play analogue role in the case of weak gravitational field at galactic scales, as the Schwarzschild radius for strong gravitational field in the vicinity of some massive object (we have IR and UV gravitational radii).

- New gravitational radii come from the further degrees of freedom of Extended Gravity.

- Explaining extragalactic phenomena, such as the baryonic Tully-Fisher relation (BFT) of gas-rich galaxies and the fundamental plane (FP) of elliptical galaxies without the DM hypothesis.

f(R) gravity

Let us start from the action

$$A = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{L}_m]$$

The field equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R =$$

$$= \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\mu\nu}\,[f(R) - Rf'(R)] + f'(R)_{;\mu\nu} \; - g_{\mu\nu}\Box f'(R)\right\} + \frac{T^{(m)}_{\mu\nu}}{f'(R)}$$

Let us consider the power - law case $\quad f(R) = f_0 R^n$

with f_0 a dimensional constant.

f(R) gravity

- An important point is related to the choice of the power-law action for *f(R)* that could appear non-natural in order to discuss deviations with respect to GR. Being *n* any real number, it is always possible to recast the *f(R)* power-law function as

$$f(R) \propto R^{1+\epsilon}$$

- If we assume small deviation with respect to GR, that is $|\varepsilon| << 1$, it is possible to re-write a first-order Taylor expansion as

$$R^{1+\epsilon} \simeq R + \epsilon R \log R + O(\epsilon^2)$$

- one can control the magnitude of the corrections with respect to the Einstein gravity. This Lagrangian has been investigated from Solar System up to cosmological scales. In particular, applications to gravitational waves (Capozziello et al. 2008, Astropart. Phys.), binary star systems (De Laurentis et al. 2012, MNRAS), and neutron stars have been investigated (Astashenok, Capozziello, Odintsov 2014, PRD, 2015 JCAP).

f(R) gravity

Taking into account the gravitational field generated by a pointlike source and solving the field equations in the vacuum case, we write the metric as:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2$$

Combining the 00 – vacuum component and the trace of the field equations in absence of matter, we get the equation:

$$f'(R)\left(3\frac{R_{00}}{g_{00}} - R\right) + \frac{1}{2}f(R) - 3\frac{f'(R)_{;00}}{g_{00}} = 0$$

it reduces to:

$$R_{00}(r) = \frac{2n-1}{6n}A(r)R(r) - \frac{n-1}{2B(r)}\frac{dA(r)}{dr}\frac{d\ln R(r)}{dr}$$

and the trace equation reads:

$$\Box R^{n-1}(r) = \frac{2-n}{3n}R^n(r)$$

f(R) gravity

Expressing R_{00} and R in terms of the above metric, field equations become a system of differential equations for *A(r)* and *B(r)*.

A physically motivated hypothesis is assuming

$$A(r) = \frac{1}{B(r)} = 1 + \frac{2\Phi(r)}{c^2}$$

A general solution is

$$\Phi(r) = -\frac{Gm}{2r}\left[1 + \left(\frac{r}{r_c}\right)^{\beta}\right]$$

The parameter is:

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}$$

Let us search now for a fundamental motivation for power-law *f(R)* gravity

The Noether Symmetry Approach

Let us assume a static spherically symmetric metric of the form

$$ds^2 = A(r)c^2dt^2 - B(r)dr^2 - C(r)d\Omega^2$$

We recast the action considering the dimensionless curvature $\chi = R/R_0$

$$\mathcal{A} = -\frac{c^3}{16\pi G L_M^2}\int \left[f(\chi) - \lambda(\chi - \bar{\chi})\right]\sqrt{-g}\,\mathrm{d}^4x$$

The Ricci scalar can be expressed as

$$\bar{R} = \frac{A''}{A} + \frac{2C''}{C} + \frac{A'C'}{AC} - \frac{{A'}^2}{2A^2} - \frac{{C'}^2}{2C^2} - \frac{2}{C}$$

where the prime is the derivative with respect to *r*. Varying with respect to χ gives the Lagrange multiplier

$$\lambda = \frac{\mathrm{d}f(\chi)}{\mathrm{d}\chi} := f_\chi$$

The Noether Symmetry Approach

The point-like Lagrangian reduces to

$$\mathrm{L} = -\frac{L_M^2}{\sqrt{A}}\left[\frac{Af_\chi}{2C}C'^2 + f_\chi A'C' + Cf_{\chi\chi}A'\chi' + 2Af_{\chi\chi}C'\chi'\right] +$$

$$-\sqrt{A}\left[(2L_M^2 + C\chi)f_\chi - Cf\right]$$

Assuming the regime $R_c >> r$ and the related weak field approximation, the last two terms are both much smaller than $L_M^2 f_\chi$. This allows to rewrite the Lagrangian as

$$\mathrm{L} =$$

$$-\frac{L_M^2}{\sqrt{A}}\left[\frac{Af_\chi}{2C}C'^2 + f_\chi A'C' + Cf_{\chi\chi}A'\chi' + 2Af_{\chi\chi}C'\chi' + 2A\right]$$

The Noether Symmetry Approach

Solving the Noether vector equation means to find out the functions α_i which constitute the components of the Noether vector

$$\mathcal{L}_{\mathbf{X}}\mathrm{L} = \alpha_i \nabla_{q_i}\mathrm{L} + \alpha_i' \nabla_{q_i'}\mathrm{L} = 0$$

$$\mathbf{X} = \alpha_i \frac{\partial}{\partial q_i} + \alpha_i' \frac{\partial}{\partial q_i'}$$

A general form of the Noether vector, related to the Killing equations of the model, is:

$$\alpha_1 = k_1 A + p_1,$$

$$\alpha_2 = k_2 C + p_2,$$

$$\alpha_3 = k_3 \chi + p_3.$$

where k_i, p_i are constants

The Noether Symmetry Approach

The Lie condition is satisfied for

$$\boldsymbol{\alpha} = \left\{ 2(1-n)kA,\ 0,\ k\chi \right\}, \quad f(\chi) = \chi^n$$

That is for any *f(R)*= R^n a Noether Symmetry exists !

The related constant of motion Σ_0 is

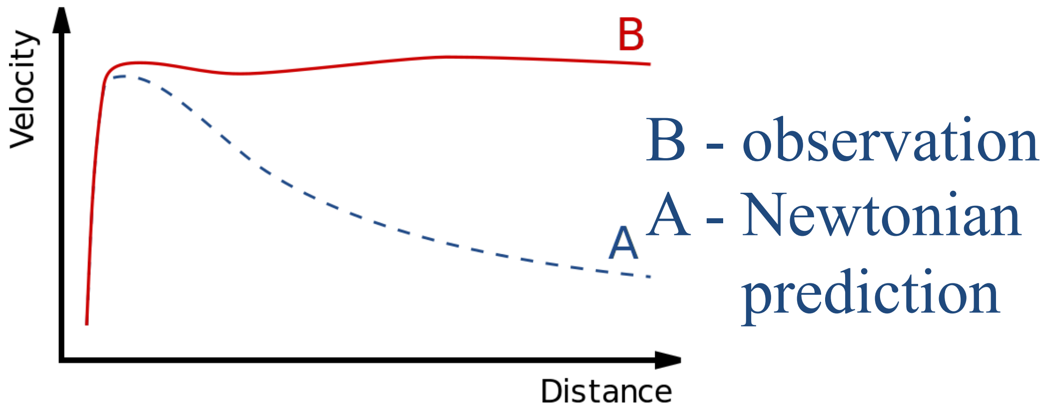
$$\Sigma_0 = \alpha_i \nabla_{q_i'} \mathrm{L}$$

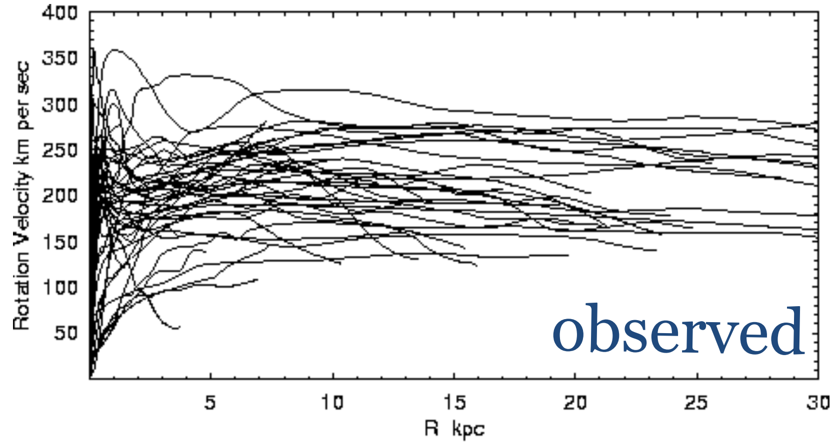
$$= L_M^2 n(n-1)kA^{-1/2} C\chi^{n-2}\left[2(n-1)A\chi' - A'\chi\right]$$

In the case of MOND, for $n = 3/2$, $C(r) = r^2$ and, at the lowest order of perturbation, $A(r) = 1 + 2\Phi/c^2$, the constant of motion is given by

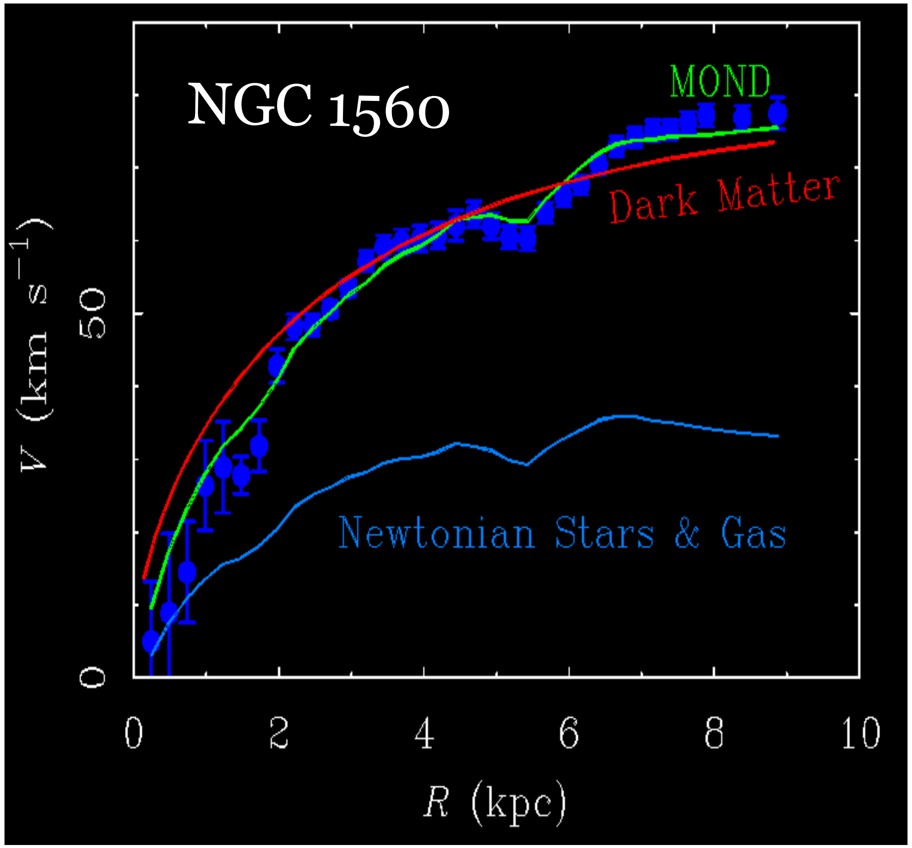
$$\Sigma_0 = \frac{3}{2} k r_{\mathrm{g}}^2 l_M$$

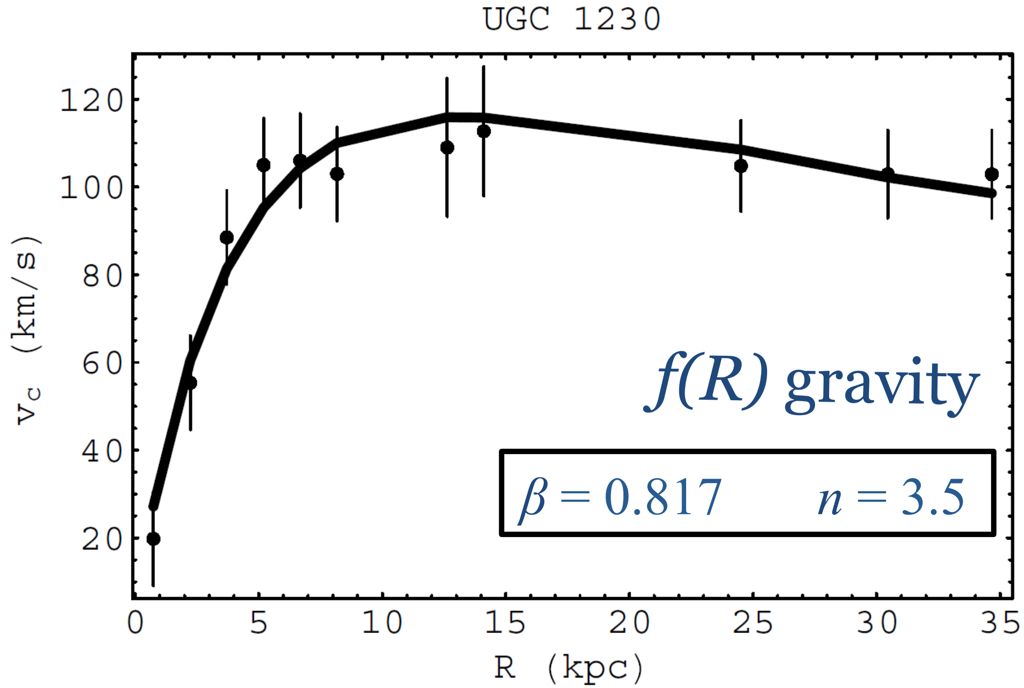
Modified gravity and flat rotation curves of spiral galaxies





Sofue & Rubin, 2001, ARA&A, 39, 137





Capozziello et al. 2007, MNRAS, 375, 1423

Observational constraints for r_c from BTF relation and circular velocity

- Starting from the above solution, an excellent agreement between theoretical and observed rotation curves of **low surface brightness galaxies** has been obtained for $\beta = 0.817$.

- This can be framed into the BTF relation with the aim to show that the new fundamental gravitational radius $\boldsymbol{r_c}$ can account for missing matter in galaxies.

- Specifically, the empirical BTF relation is a universal relationship between the baryonic mass of a galaxy and its rotational velocity of the form $M_b \propto v_c^4$. This follows from the fact that luminosity L traces baryonic mass M_b through the mass-to-light ratio γ. The BTF relation can be recovered from power-law $f(R)$ gravity.

The data from the Baryonic Tully-Fisher relation of gas rich galaxies as a test for ΛCDM and MOND considering

D - distance of the galaxy,

V_c - rotational velocity,

M_* - mass of the stars,

M_g - mass of the gas

(we used observational data from McGaugh, PRL (2011), which are given at the internet address: http://www.astro.umd.edu/~ssm/data/gasrichdatatable.txt)

Galaxy	D (Mpc)	V_c (km/s)	$\log M*$ $(Msun)$	$\log Mg$ $(Msun)$
DDO210	0.94	17 ± 4	5.88 ± 0.15	6.64 ± 0.20
CamB	3.34	20 ± 12	6.99 ± 0.15	7.33 ± 0.20
UGC8215	4.5	20 ± 6	6.81 ± 0.15	7.45 ± 0.20
DDO183	3.24	25 ± 3	7.24 ± 0.15	7.54 ± 0.20
UGC8833	3.2	27 ± 4	6.94 ± 0.15	7.30 ± 0.20
D564-8	6.5	29 ± 5	6.76 ± 0.20	7.32 ± 0.13
DDO181	3.1	30 ± 6	7.26 ± 0.15	7.56 ± 0.20
P51659	3.6	31 ± 4	6.67 ± 0.15	7.85 ± 0.20
KK9824	7.83	35 ± 6	7.72 ± 0.15	7.93 ± 0.20
UGCA92	3.01	37 ± 4	7.78 ± 0.15	8.32 ± 0.20
D512-2	14.1	37 ± 7	7.58 ± 0.20	7.96 ± 0.06
UGCA444	0.95	38 ± 5	7.34 ± 0.15	7.75 ± 0.20
KK98251	5.6	38 ± 5	7.34 ± 0.15	8.02 ± 0.20
UGC7242	5.4	40 ± 4	7.57 ± 0.15	7.78 ± 0.20
UGC6145	7.4	41 ± 4	7.20 ± 0.15	7.56 ± 0.20
NGC3741	3.0	44 ± 3	7.24 ± 0.15	8.45 ± 0.20
D500-3	18.5	45 ± 6	6.97 ± 0.20	7.94 ± 0.05
D631-7	5.5	53 ± 5	6.88 ± 0.20	8.29 ± 0.15
DDO168	4.3	54 ± 3	8.07 ± 0.15	8.74 ± 0.20
KKH11	3.0	56 ± 5	7.28 ± 0.15	7.85 ± 0.20
UGC8550	5.1	58 ± 3	8.25 ± 0.37	8.46 ± 0.39
D575-2	12.2	59 ± 7	7.63 ± 0.20	8.62 ± 0.07
UGC4115	7.5	59 ± 6	7.77 ± 0.15	8.58 ± 0.20
UGC3851	3.2	60 ± 5	8.45 ± 0.15	9.09 ± 0.20
UGC9211	12.6	64 ± 5	8.12 ± 0.39	9.21 ± 0.41
NGC3109	1.3	66 ± 3	7.41 ± 0.15	8.79 ± 0.20
UGC8055	17.4	66 ± 7	8.09 ± 0.15	9.02 ± 0.20
D500-2	17.9	68 ± 7	7.41 ± 0.20	9.06 ± 0.05
IC2574	4.0	68 ± 5	8.94 ± 0.15	9.20 ± 0.20
UGC6818	18.6	72 ± 6	9.22 ± 0.16	9.28 ± 0.20
UGC4499	13.0	74 ± 3	8.75 ± 0.27	9.32 ± 0.29
NGC1560	3.45	77 ± 3	8.70 ± 0.15	9.23 ± 0.20
UGC8490	4.65	78 ± 3	8.36 ± 0.15	8.96 ± 0.20
UGC5721	6.5	79 ± 3	8.17 ± 0.37	9.05 ± 0.39
F565-V2	48.	83 ± 8	8.30 ± 0.21	9.04 ± 0.24
F571-V1	79.	83 ± 5	9.00 ± 0.19	9.33 ± 0.22
IC2233	10.4	84 ± 5	8.96 ± 0.15	9.32 ± 0.20
NGC2915	3.78	84 ± 10	7.99 ± 0.15	8.78 ± 0.20
NGC5585	5.7	90 ± 3	8.98 ± 0.38	9.27 ± 0.40
UGC3711	7.9	95 ± 3	8.92 ± 0.15	9.01 ± 0.17
UGC6983	18.6	108 ± 3	9.53 ± 0.16	9.74 ± 0.20
F563-V2	61.	111 ± 5	9.41 ± 0.17	9.63 ± 0.21
F568-1	85.	118 ± 4	9.50 ± 0.18	9.87 ± 0.22
F568-3	77.	120 ± 6	9.62 ± 0.18	9.71 ± 0.22
F568-V1	80.	124 ± 5	9.38 ± 0.18	9.65 ± 0.22
NGC2403	3.18	134 ± 3	9.61 ± 0.15	9.77 ± 0.20
NGC3198	14.5	149 ± 3	10.12 ± 0.15	10.29 ± 0.20

Observational constraints for r_c from BTF relation and circular velocity

Circular velocity of a point mass, in the R^n gravity potential, can be found in the standard way, that is

$$v_c^2(r) = r\frac{d\Phi}{dr}$$

which gives

$$v_c^2(r) = \frac{GM}{2r}\left[1 + (1-\beta)\left(\frac{r}{r_c}\right)^{\beta}\right]$$

(For a detailed explanation see Capozziello et al., MNRAS (2007)

Observational constraints for r_c from BTF relation and circular velocity

Considering the Newtonian limit of *f(R)* gravity and discarding higher order terms than $O(2)$, the field equations for a perfect-fluid energy-momentum tensor of dust ($p = 0$) become:

$$\nabla^2 \Phi - \frac{R^{(2)}}{2} - f''(0)\nabla^2 R^{(2)} = \mathcal{X}\rho$$

$$-3f''(0)\nabla^2 R^{(2)} - R^{(2)} = \mathcal{X}\rho$$

ρ - the mass density

$X = 8\pi G/c^4$ - the gravitational coupling

$R^{(2)}$ - the Ricci scalar assumed up to the second order approximation

Observational constraints for r_c from BTF relation and circular velocity

Let us proceed step by step to demonstrate that BTF is given by the gravitational radius $\mathbf{r_c}$.

1. the Noether symmetries select a power-law for *f(R)* gravity. This is the only general form of *f(R)* function showing symmetries.

2. In particular, we assume $f(\chi) = \chi^n$, after introducing the dimensionless quantity $\chi := L_M^2 R$, where R is the Ricci scalar, L_M is the length fixed by the parameters of the theory, and n any real number.

Observational constraints for r_c from BTF relation and circular velocity

3. The trace of field Eqs. can be rewritten as

$$f'(\chi)\,\chi - 2f(\chi) + 3L_M^2\,\Delta f'(\chi) = \frac{8\pi G L_M^2}{c^4} T$$

By substituting the power-law, it becomes:

$$\chi^n\,(n-2) - 3nL_M^2\,\frac{\chi^{(n-1)}}{r^2} \approx \frac{8\pi G M L_M^2}{c^2 r^3}$$

Here, we are assuming the weak field approximation with $d/d\chi \sim 1/\chi$, $\Delta \sim -1/r^2$, and matter density $\rho \sim M/r^3$.

The second term in the l.h.s. of this Eq. is larger than the first if

$$Rr^2 \leq \frac{3n}{2-n}$$

In this approximation, the Ricci scalar corresponds to the Gaussian curvature and then $R \simeq R_c^{-2}$ where R_c is the Gauss curvature radius. Immediately we have $R_c >> r$, and then

$$R^{(n-1)} \approx -\frac{8\pi G M}{3nc^2 r L_M^{2(n-1)}}$$

Observational constraints for r_c from BTF relation and circular velocity

4. At the second order, the Ricci scalar is $R = -\frac{2}{c^2}\nabla^2\Phi = \frac{2}{c^2}\nabla \cdot \boldsymbol{a}$

that can be approximated as $R \approx -2\Phi/(c^2r^2) \approx 2a/(c^2r)$, with Φ the gravitational potential and a the acceleration. This gives:

$$a \approx -\frac{c^2 r}{2L_M^2}\left(\frac{8\pi GM}{3nc^2 r}\right)^{1/(n-1)}$$

$$\approx -c^{(2n-4)/(n-1)} r^{(n-2)/(n-1)} L_M^{-2} (GM)^{1/(n-1)}$$

which converges to a MOND-like acceleration $a \propto 1/r$ if $n - 2 = -(n - 1)$, that means $n = 3/2$.

5. With this value of n, we get the MOND relation $a \approx -\frac{(a_0 GM)^{1/2}}{r}$

In other words, the weak field limit of *f*(R) power-law gravity gives MOND as a particular case.

Observational constraints for r_c from BTF relation and circular velocity

According to this derivation, the above characteristic length $\boldsymbol{r_c}$ of R^n gravity can be related to the MOND acceleration constant a_0 using the following expression

$$r_c = \sqrt{\frac{GM}{a_0}}$$

Assuming that rotation curve is flat within the measurement uncertainties at some finite radius $\boldsymbol{r_f}$, i.e. $v_c(r_f) \approx v_f$, then $\boldsymbol{r_f}$ could be also related to a certain MOND acceleration $a_f > a_0$. This gives

$$r_f = \frac{\sqrt{a_0 GM}}{a_f} = \frac{a_0}{a_f} r_c$$

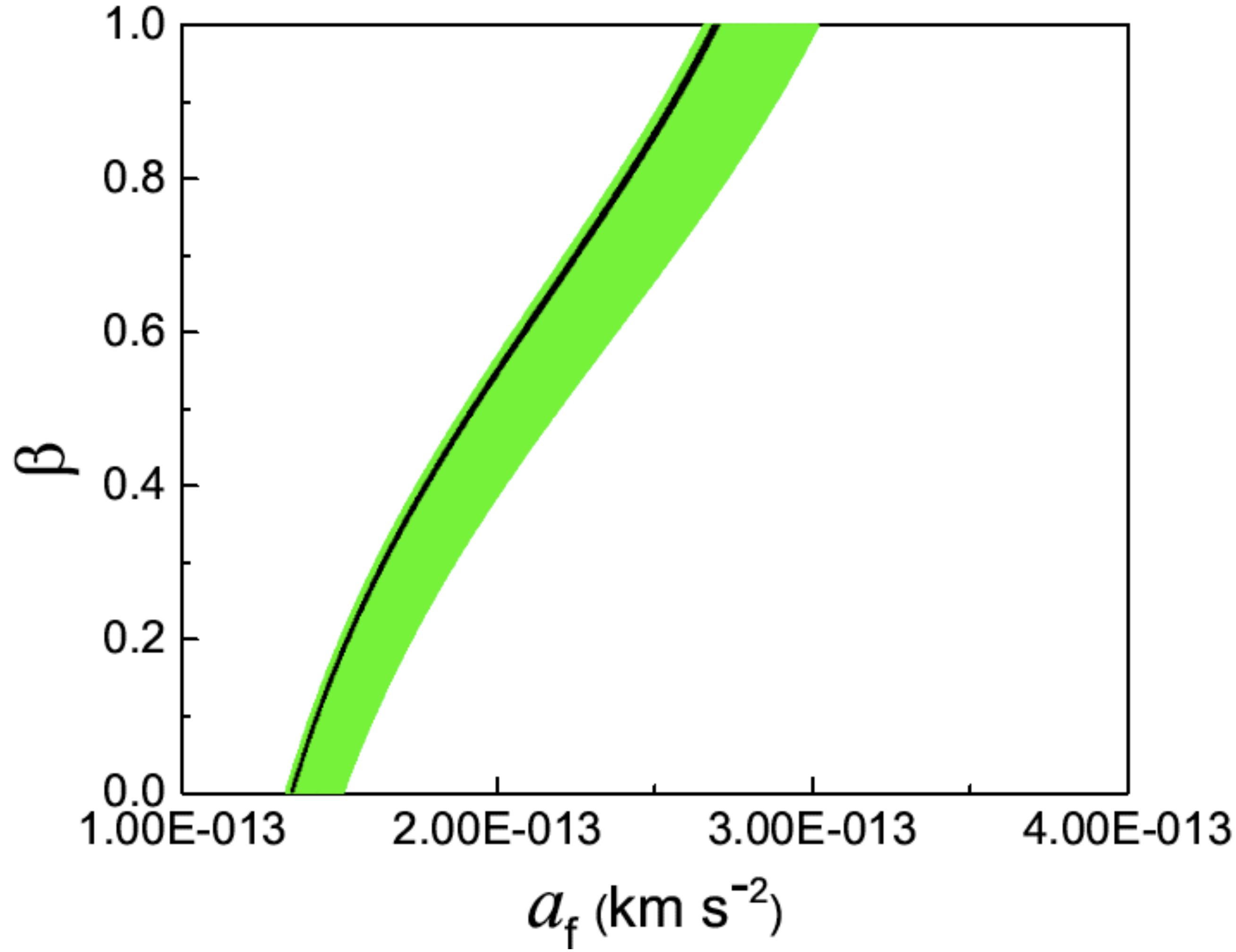
Hence, the BTF relation of R^n gravity expressed in terms of MOND accelerations is

$$M = \frac{4 a_0 v_f^4}{G a_f^2 \left[1 + (1-\beta)\left(\frac{a_0}{a_f}\right)^{\beta}\right]^2}$$

Observational constraints for r_c from BTF relation and circular velocity

We have to point out that, in the case of BTFR for spiral galaxies, McGaugh (2011) has shown tha,t instead of standard MOND acceleration constant a_0, one should use a slightly different, empirically calibrated constant a (where $a_0 = 0.8a$), while the formula is unchanged. Therefore, for our calculations, we use the following expression:

$$M = \frac{4av_f^4}{Ga_f^2\left[1+(1-\beta)\left(\frac{a}{a_f}\right)^{\beta}\right]^2}$$



Region in (a_f,β) parameter space (shaded green area) where $41 \leq A \leq 53$ $M_\odot$ km^{-4} s^{4} according to the relation for M (our BTF relation). Black solid line represents the case when A = 51.4 $M_\odot$ km^{-4} s^{4} ($M_b = A\, v_f^4$).

Observational constraints for r_c from BTF relation and circular velocity

- we draw these lines at $M_b(v_f)$ graph:

(i) MOND $M_b = v_f^4/(a\,g)$

(ii) R^n $M_b = 4\,a\,v_f^4/(g\,a_1^2\,(1 + (1\text{-}\beta)\,(a/a_1)^\beta)^2)$

three R^n cases: $n = 3/2, 2, 7/2$ (correspond to $\beta = 0.518, 0.667, 0.817$)

a_0 - constant for point source in infinity

a - constant for spiral systems

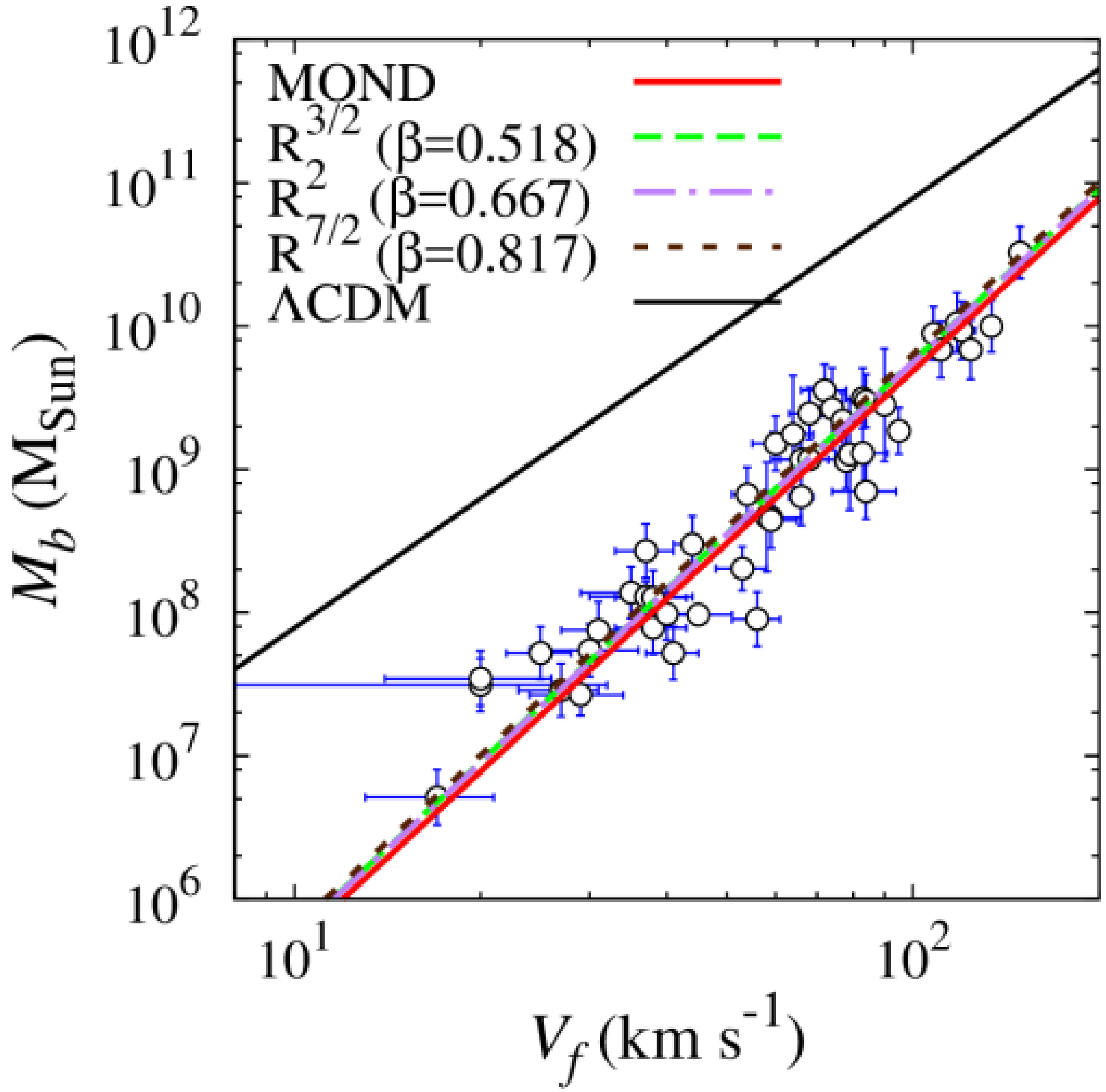
In the case of spiral galaxies, we have a instead of a_0

empirical calibration is $a_0 = 0.8a$

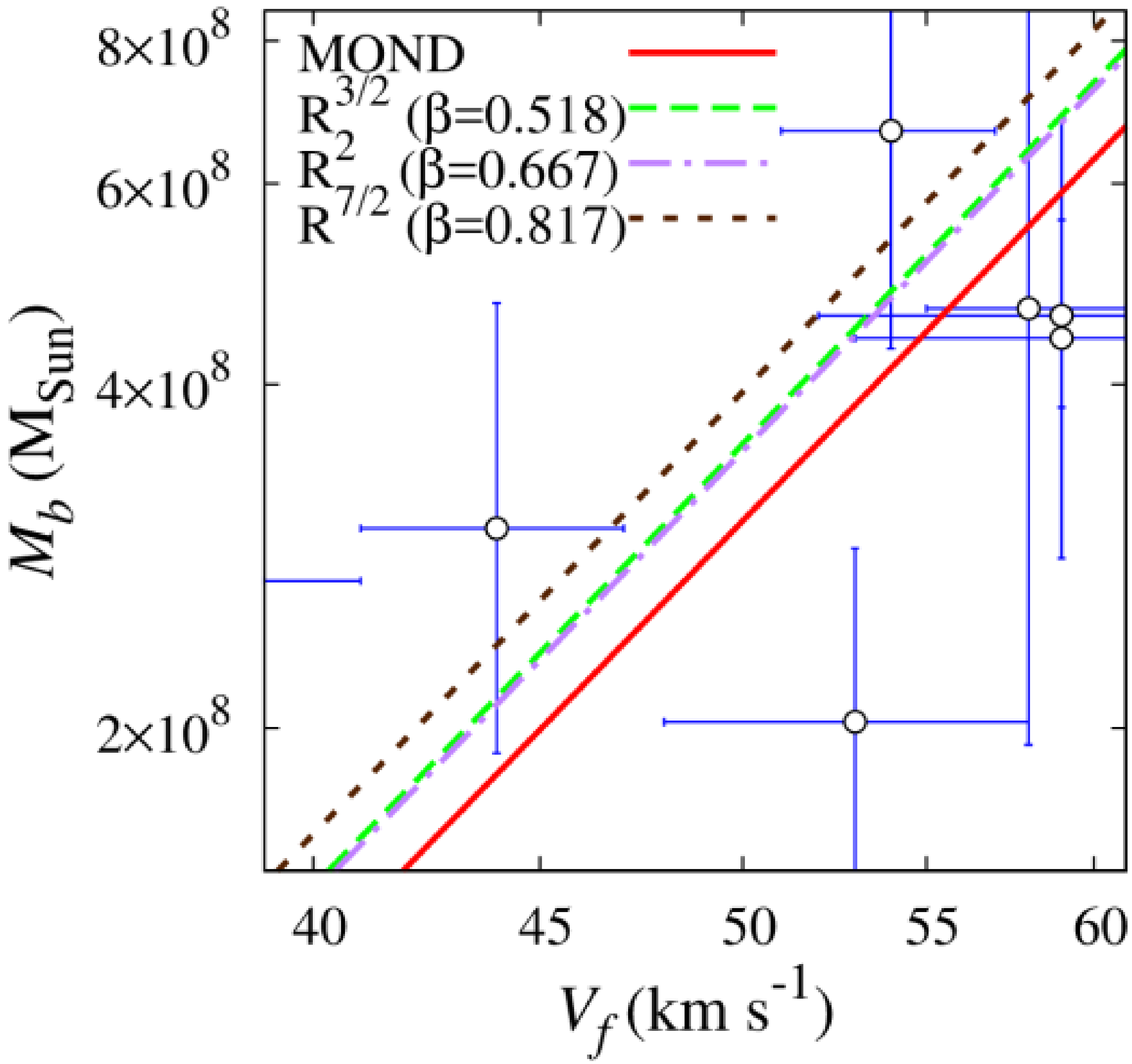
(iii) ΛCDM $M_b = 0.17\,M_{vir}$, $v_f = v_{vir}$

- formula for ΛCDM is taken from the paper McGaugh 2012, AJ:

$M_{vir} = (4.6 \cdot 10^5\,M_{sun}\,\mathrm{km}^{-3}\,\mathrm{s}^3)\,v_{vir}{}^3$



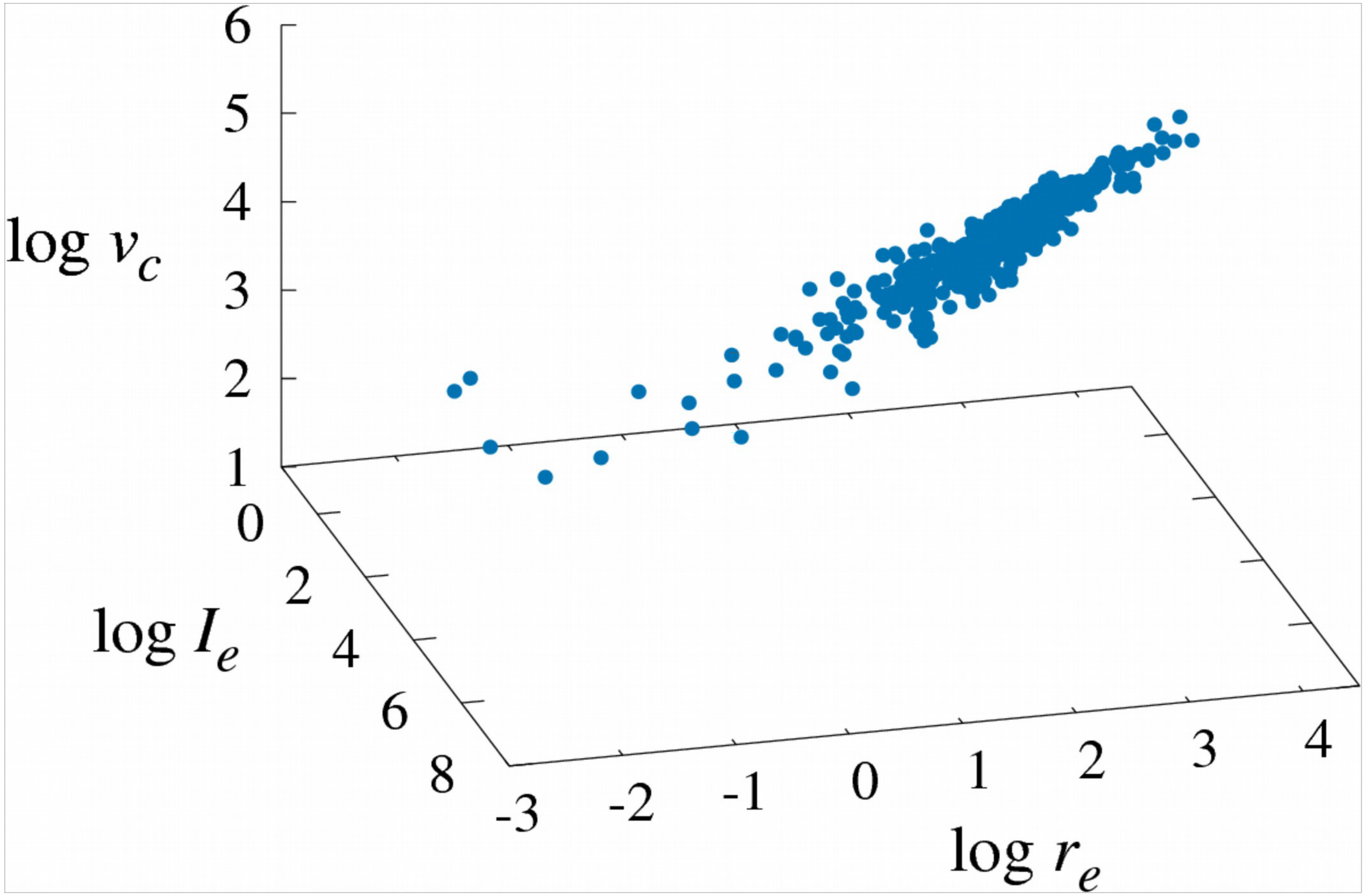
Comparison between best fit BTF relations of gas-rich galaxies (for a sample of galaxies), in MOND, R^n gravity for values of n = 1.5, 2 and 3.5 (corresponding are 0.518, 0.667 and 0.817, respectively) and ΛCDM.



A zoomed part of the previous figure, for a small range of parameters.

Open circles are observed data from McGaugh (2011).

The Fundamental Plane of Galaxies



The three parameters of FP: surface brightness $\boldsymbol{I_e}$, effective radius $\boldsymbol{r_e}$ and circular velocity $\boldsymbol{v_c}$, for a sample of elliptical galaxies from Burstein et al 1997.

Basic theory of Fundamental Plane

The **Fundamental Plane** of elliptical galaxies is an empirical relation between the global properties of these galaxies:

$$\log r_e = a \log \sigma_0 + b \log I_e + c$$

$\boldsymbol{r_e}$ - effective (half-light) radius (the radius within which half of the galaxy's luminosity is contained)

$\boldsymbol{\sigma_0}$ - central velocity dispersion

$\boldsymbol{I_e}$ - mean surface brightness within the effective radius

- there is the so-called "**tilt**" of the fundamental plane, with respect to the virial plane expectation, meaning that the coefficients of its equation (a,b,c) differ from those predicted by virial theorem (VT): when written in logarithmic form, the two planes appear to be tilted by an angle of ~ 15°.

- VT prediction: $a = 2$, $b = -1$
- Estimates from data (Bender et al. 1992): $a = 1.4$, $b = -0.85$

(see e.g.: G. Busarello, M. Capaccioli, S. Capozziello, G. Longo, E. Puddu, *The relation between the virial theorem and the fundamental plane of elliptical galaxies*, Astron. Astrophys. 320, 415 (1997))

Recovering the fundamental plane from f(R)

-To recover the FP using R^n gravity, we have to find relations between FP parameters and values of *f(R)* potential. In this sense, the three addends of FP have to be connected to *f(R)* parameters:

1. addend with $\mathbf{r_e}$: correlation between $\mathbf{r_e}$ and $\mathbf{r_c}$ ($\mathbf{r_c}$ – from R^n potential)

2. addend with $\boldsymbol{\sigma_0}$: correlation between $\boldsymbol{\sigma_0}$ and $\mathbf{v_{vir}}$ ($\mathbf{v_{vir}}$ - virial velocity in R^n)

3. addend with $\mathbf{I_e}$: correlation between $\mathbf{I_e}$ and $\mathbf{r_e}$ (through the $\mathbf{r_c}/\mathbf{r_e}$ ratio)

- for the mass distribution, we take into account the Hernquist profile:

$$\rho(r) = a\, M / (2\, \pi\, r\, (r + a)^3), \text{ where } a = r_e/(1 + \sqrt{2})$$

see L. Hernquist, ApJ 356, 359 (1990)

The Data

- We use the data given in Table I by Burstein, Bender, Faber, Nolthenius, *Global relationships among the physical properties of stellar systems*, Astron. J. 114, 1365 (1997).
These data are the result of the collected efforts over the years
- data in ASCI format are given in table *'metaplanetab1'* see arXiv:astro-ph/9707037

column (5): log $\mathbf{v_c}$ (km/s)

column (6): log $\boldsymbol{\sigma_0}$ (km/s)

column (7): log $\mathbf{r_e}$ (kpc)

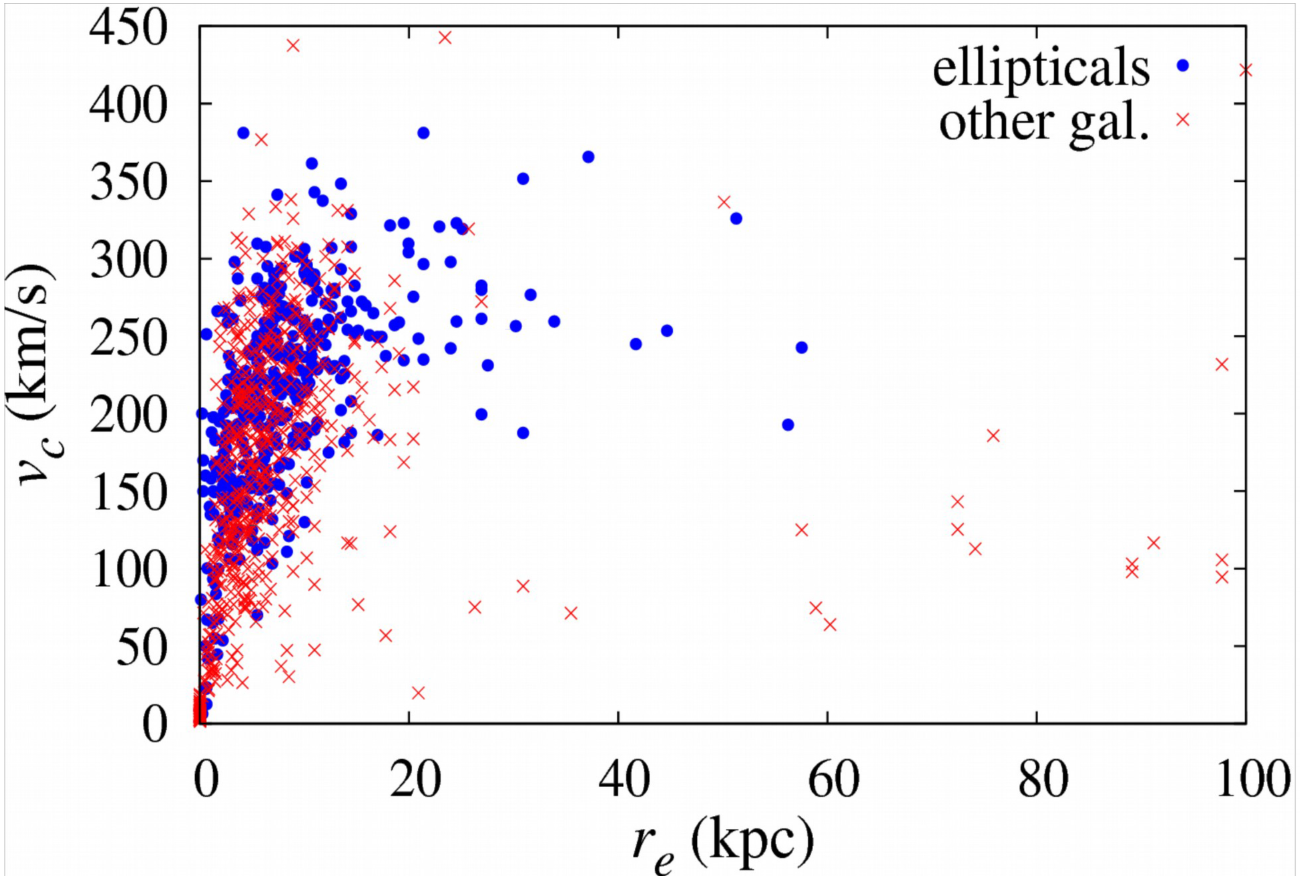
column (8): log $\mathbf{I_e}$ (Lsun / pc^2)

Obj Name (1)	ID # (2)	Obj Code (3)	Dist (Mpc) (4)	log V_{char} Obs (5)	log σ_c Used (6)	log r_e (kpc) (7)	log I_e $L_\odot$ pc^{-2} (8)
NGC 221	8	1	0.7	1.903	1.903	−0.95	3.47
NGC 315	14	1	107.2	2.546	2.546	1.49	1.86
NGC 720	56	1	35.8	2.392	2.392	0.84	2.34
NGC 777	64	1	99.4	2.542	2.542	1.13	2.16
NGC 821	67	1	37.7	2.298	2.298	0.92	2.06
NGC 1399	100	1	26.4	2.491	2.491	0.74	2.53

for elliptical galaxies, the circular velocity inside effective radius is $\mathbf{v_c(r_{\it e})} = \boldsymbol{\sigma_0}$, for other stellar systems $\mathbf{v_c} \neq \boldsymbol{\sigma_0}$

Results

- we plot the graph v_c (r_e) for ellipticals and for other galaxies



Newtonian contribution

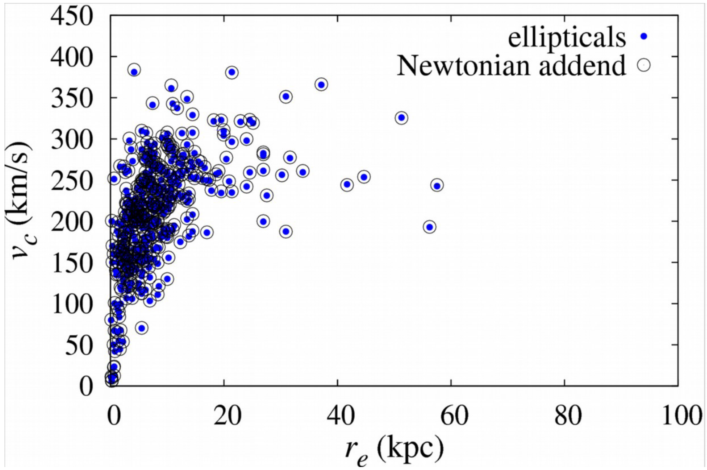
$$v_c^2(r) = \frac{v_{c,N}^2(r)}{2} + \frac{r}{2}\frac{\partial \Phi_c}{\partial r}$$

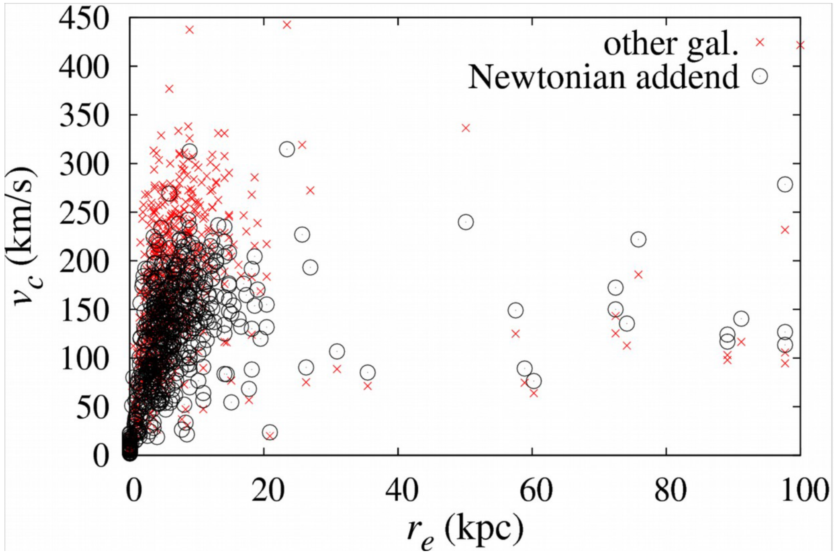
correction term from *f(R)*

Circular velocity v_c as a function of effective radius r_e for a sample of galaxies listed in Table 1 by Burstein et al 1997.

Results

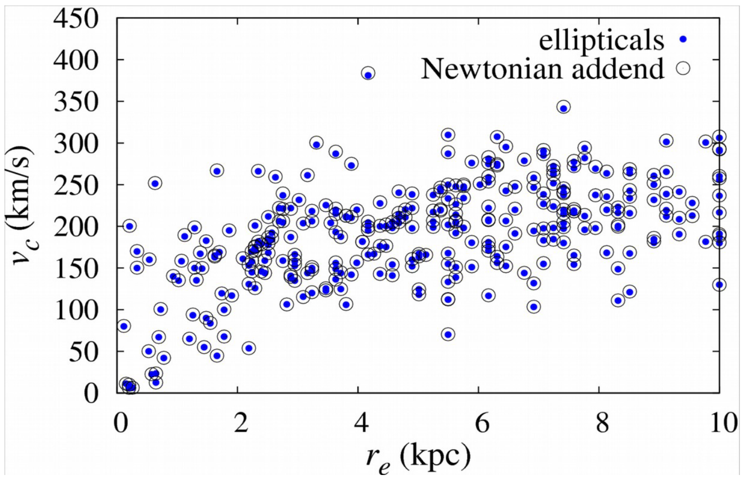
- circular velocity $\boldsymbol{v_c}$ as a function of effective radius $\mathbf{r_e}$ and their Newtonian circular velocity $\boldsymbol{v_{c,N}}$ **(*r*$_\mathbf{e}$)** (open circles)



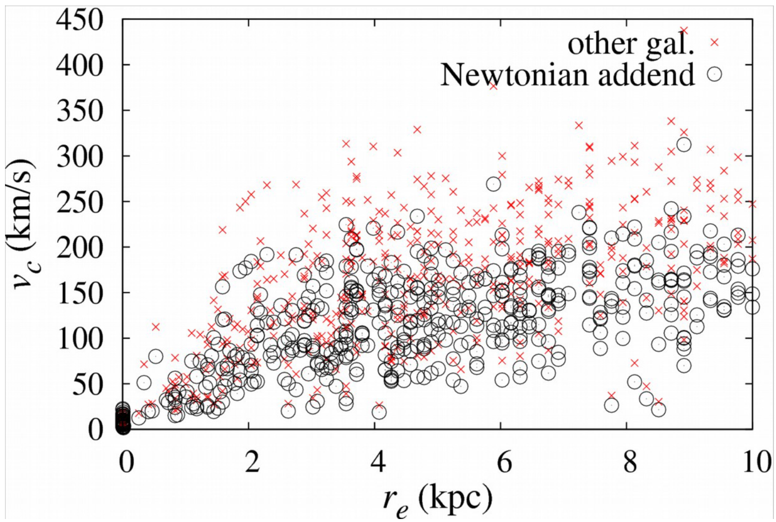


elliptical galaxies (full circles)

other galaxies (crosses)



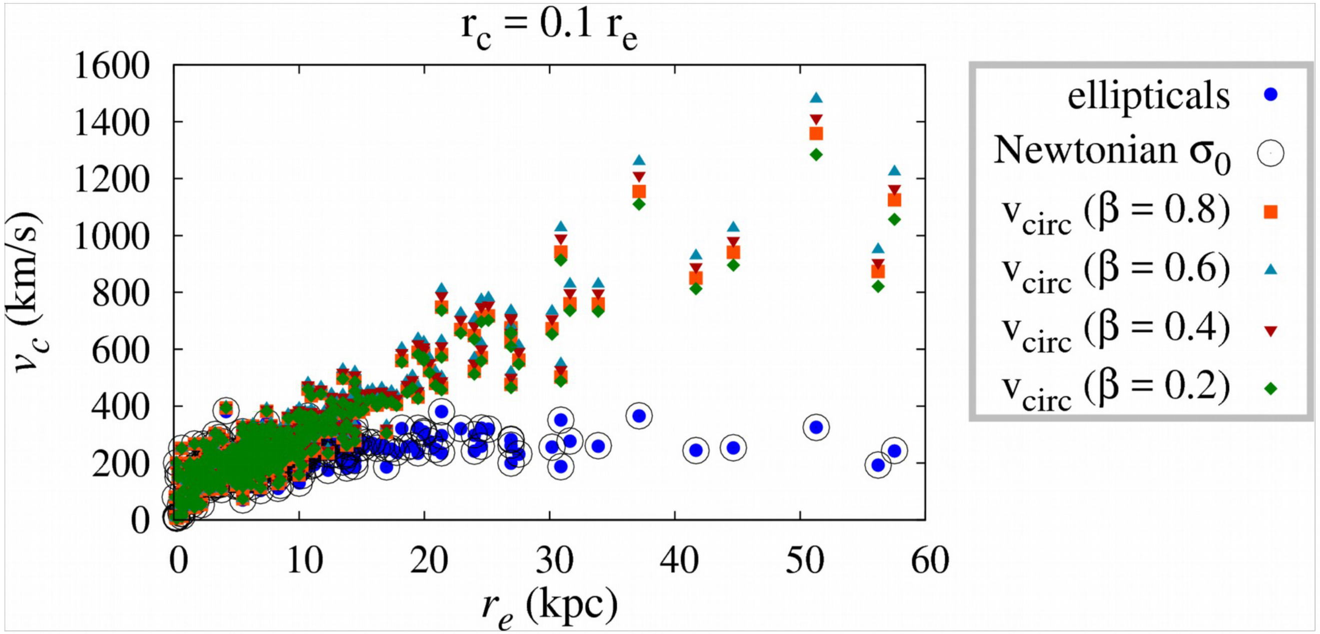
zoomed part of the figure



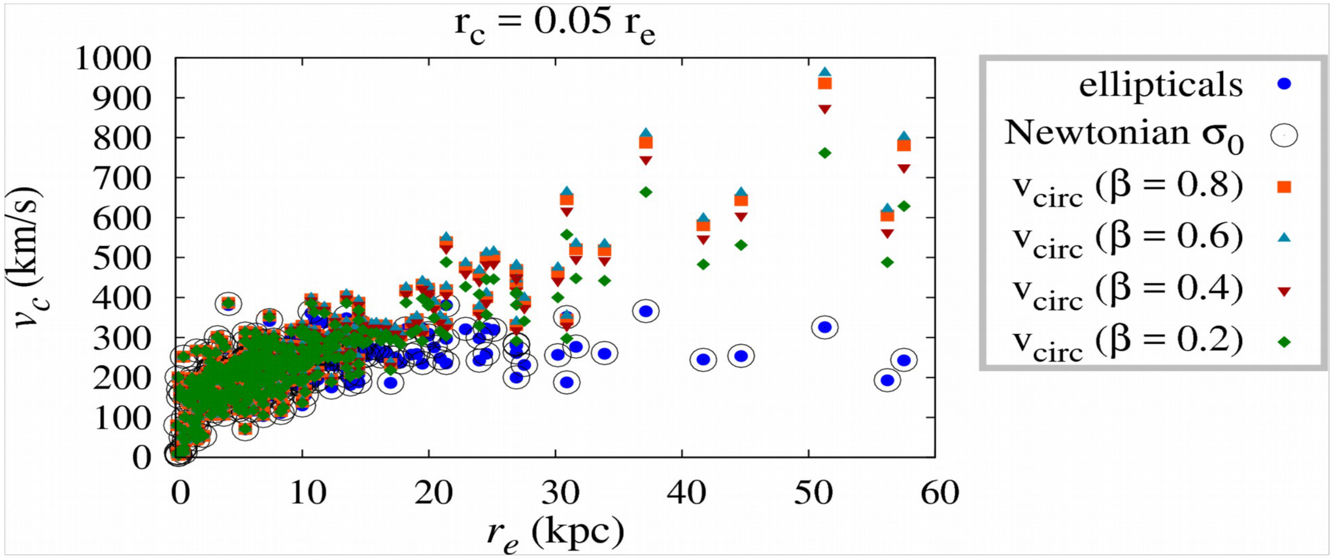
Results

- relation between $\boldsymbol{r_c}$ and $\boldsymbol{r_e}$ and for some values of parameter β

($\beta = 0$ Newtonian case)

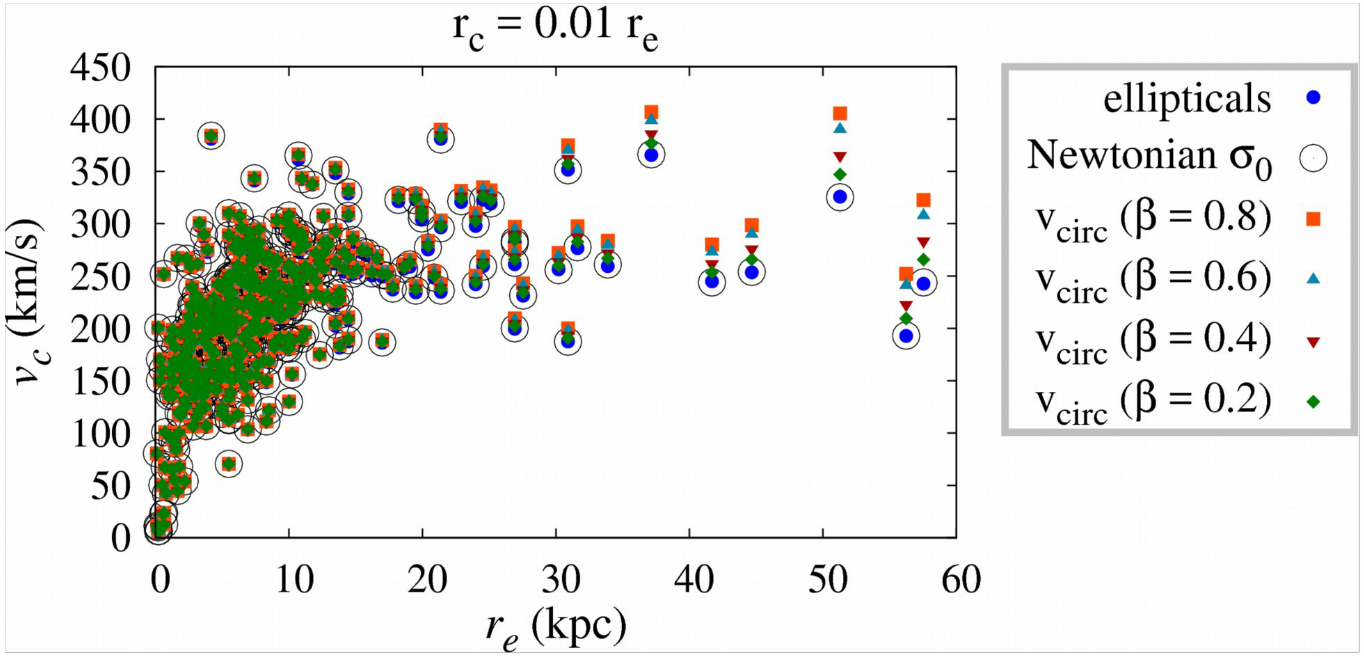


$r_c / r_e = 0.1$

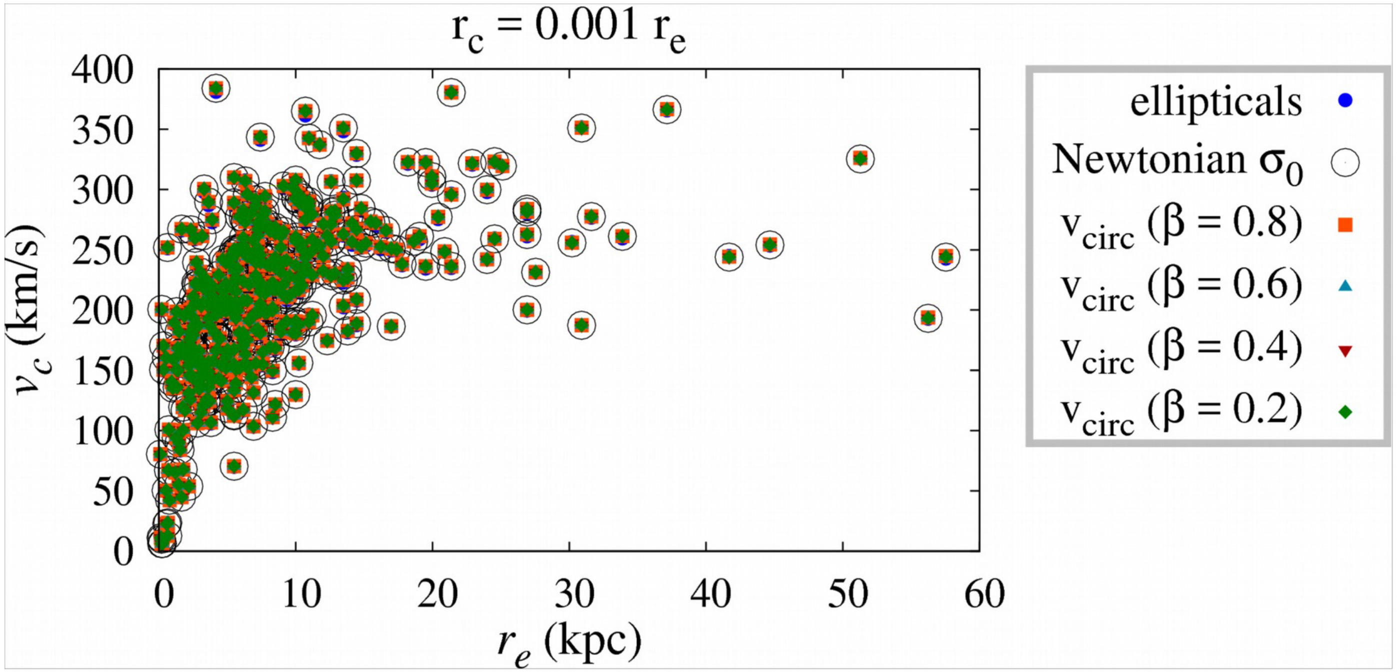


$r_c / r_e = 0.05$

Results



***r_c* / *r_e* = 0.01**



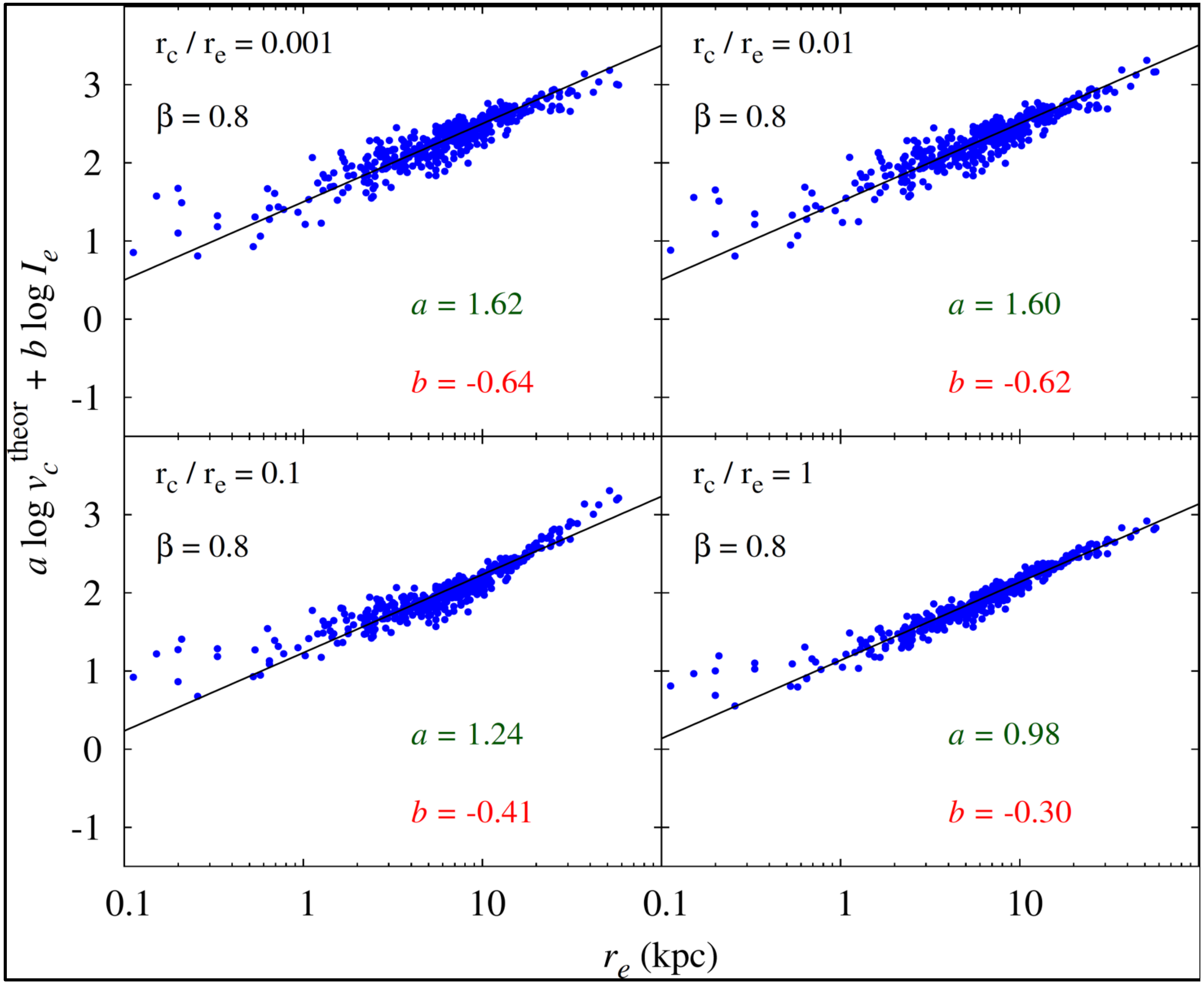
***r_c* / *r_e* = 0.001**

Results

The empirical FP relation $\log r_e = a \log \sigma_0 + b \log I_e + c$ from *f(R)*

r_e - effective (half-light) radius
σ_0 - central velocity dispersion
I_e - mean surface brightness within r_e

FP of elliptical galaxies with calculated circular velocity: dependence of FP parameters (a,b) on parameters of *f(R)* gravity.



Discussion and Conclusions

- We used power-law *f(R)* gravity to demonstrate the existence of a new fundamental gravitational radius.

- This radius plays an analog role, in the case of weak gravitational field at galactic scales (IR scales) as the Schwarzschild radius in the case of strong gravitational field in the vicinity of compact massive objects (UV scales).

- The radius emerges as a conserved quantity from Noether's symmetries that exist for any power-law *f(R)* function.

- Using this new gravitational radius, *f(R)* gravity is able to explain the baryonic Tully-Fisher relation of gas-rich galaxies without DM hypothesis.

- MOND is a particular case of *f(R)* gravity in the weak field limit.

Discussion and Conclusions

- The same radius is useful to address the FP of elliptical galaxies.

- The range $0.5 \leq \beta \leq 0.8$ (corresponding to $1.5 \leq n \leq 3.5$) is in a good agreement with observations. These values agree with observational constraints on β obtained by fitting FP and MOND. We do not need DM to explain baryonic Tully-Fisher relation, and even more, ΛCDM is not in satisfactory agreement with observations.
- For elliptical galaxies $\mathbf{r_c}$ is proportional to $\mathbf{r_e}$ that is $\mathbf{r_c \sim 0.01\, r_e}$.

- Considering the definition of $\mathbf{r_e}$, we can say that the effective radius (defined photometrically as the radius containing half of the luminosity of a galaxy) is led by gravity.
- In perspective, the whole galactic dynamics can be addressed by Extended Gravity.
- Work in progress for Faber-Jackson relation, galactic potentials, Boltzmann-Vlasov relation, and Virial Theorem.

References

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[4] S. Capozziello, P. Jovanović, V. Borka Jovanović, D. Borka, *JCAP* **1706,** 044 (2017) .