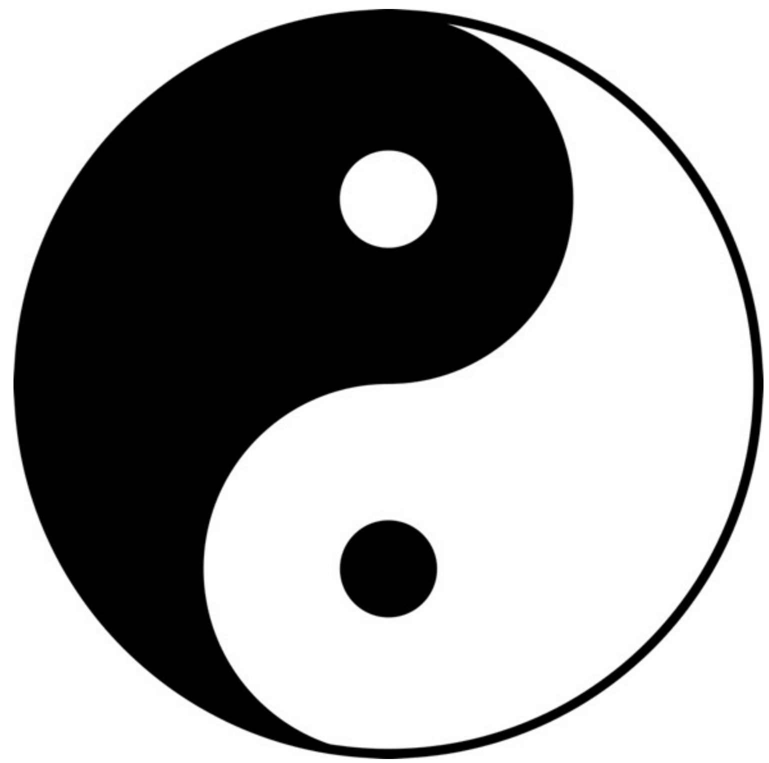
Double Field Theory



Double Field Theory

Hull & Zwiebach

- From sector of String Field Theory. Features some stringy physics, including T-duality, in simpler setting
- Strings see a doubled space-time
- Necessary consequence of string theory
- Needed for non-geometric backgrounds
- What is geometry and physics of doubled space?

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p, w)$
- Fourier transform to doubled space: $\psi(x, \tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Subsector? e.g. $g_{ij}(x, \tilde{x}),\ b_{ij}(x, \tilde{x}),\ \phi(x, \tilde{x})$
- T-duality is a manifest symmetry

Double Field Theory

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical*
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Recover Siegel's duality covariant formulation of (super)gravity

Extra Dimensions

- Torus compactified theory has charges arising in SUSY algebra, carried by BPS states
- P_M: Momentum in extra dimensions
- Z_A: wrapped brane & wound string charges
- But P_M ,Z_A related by dualities. Can Z_A be thought of as momenta for extra dimensions?
- Space with coordinates X^M, Y^A ?

Extended Spacetime

- Supergravity can be rewritten in extended space with coordinates X^M, Y^A. Duality symmetry manifest.
- But fields depend only on X^M (or coords related to these by duality).
- Gives a geometry for non-geometry: T-folds
- Actual stringy symmetry of theory can be quite different from this sugra duality: **Background dependence?**
- In string theory, can do better... DOUBLE FIELD THEORY, fields depending on X^M, Y_M.

M-Theory

- 11-d sugra can be written in extended space.
- Extension to full M-theory?
- If M-theory were a perturbative theory of membranes, would have extended fields depending on X^M and 2-brane coordinates Y_{MN}
- But it doesn't seem to be such a theory
- Don't have e.g. formulation as infinite no. of fields. Only implicit construction as a limit.
- Extended field theory gives a duality-symmetric reformulation of supergravity

Double Field Theory

- String field theory gives complete formulation of perturbative closed string in CFT background — Zwiebach
- Iterative construction of infinite number of interactions
- Non polynomial. Homotopy Lie algebra (violation of Jacobi's associativity etc)
- String field theory for torus gives infinite set of fields depending on doubled coordinates

String Field Theory on Minkowski Space

Closed SFT: Zwiebach

String field $$\Phi[X(\sigma), c(\sigma)]$$

$$X^i(\sigma) \to x^i, \text{ oscillators}$$

Expand to get infinite set of fields

$$g_{ij}(x), b_{ij}(x), \phi(x), \ldots, C_{ijk\ldots l}(x), \ldots$$

Integrating out massive fields gives field theory for

$$g_{ij}(x), b_{ij}(x), \phi(x)$$

String Field Theory on a torus

String field $\qquad \Phi[X(\sigma), c(\sigma)]$

$$X^i(\sigma) \to x^i, \tilde{x}_i, \ \text{oscillators}$$

Expand to get infinite set of double fields

$$g_{ij}(x, \tilde{x}), b_{ij}(x, \tilde{x}), \phi(x, \tilde{x}), \dots, C_{ijk\dots l}(x, \tilde{x}), \dots$$

Seek **double field theory** for

$$g_{ij}(x, \tilde{x}), b_{ij}(x, \tilde{x}), \phi(x, \tilde{x})$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

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$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

Laplacian for metric

$$ds^2 = G_{ij} dx^i dx^j + G^{ij} d\tilde{x}_i d\tilde{x}_j$$

$$L_0 - \bar{L}_0 = 0$$

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Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Laplacian for metric

$$ds^2 = dx^i d\tilde{x}_i$$

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

$$N = \bar{N} = 1$$

$$p^2 + w^2 = 0$$

$$p \cdot w = 0$$

“Double Massless”

DFT gives O(D,D) covariant formulation

O(D,D) Covariant Notation

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} \qquad \partial_M \equiv \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$$

$$\eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad M = 1, \ldots, 2D$$

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} = \frac{1}{2}\partial^M \partial_M$$

Constraint $\partial^M \partial_M A = 0$

on all fields and parameters

Weak Constraint or weak section condition

Arises from string theory constraint

$$(L_0 - \bar{L}_0)\Psi = 0$$

- Weakly constrained DFT non-local. Constructed to cubic order Hull & Zwiebach

- ALL doubled geometry dynamical, evolution in all doubled dimensions

- Restrict to simpler theory: STRONG CONSTRAINT

- Fields then depend on only half the doubled coordinates

- Locally, just conventional SUGRA written in duality symmetric form

Strong Constraint for DFT

Hohm, H &Z

$$\partial^M \partial_M (AB) = 0 \qquad (\partial^M A)\,(\partial_M B) = 0$$

on all fields and parameters

If impose this, then it implies weak form, but product of constrained fields satisfies constraint.

This gives **Restricted DFT**, a subtheory of DFT

Locally, it implies fields only depend on at most half of the coordinates, fields are restricted to null subspace N.
Looks like conventional field theory on subspace N

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- Siegel's duality covariant form of (super)gravity

- In string theory, T-duality acts on torus or fibres of torus fibration, relates local modes and winding

- Winding modes: doubling of torus or fibres

- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality. No doubling?

- DFT 'background independent' HHZ. Can write on doubling of any space. What is double if not derived from string theory?

Generalised T-duality transformations: HHZ

$$X'^M \equiv \begin{pmatrix} \tilde{x}'_i \\ x'^i \end{pmatrix} = hX^M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

h in O(d,d;Z) acts on toroidal coordinates only

$$\mathcal{E}_{ij} = g_{ij} + b_{ij}$$

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}$$

$$d'(X') = d(X)$$

Buscher if fields independent of toroidal coordinates
Generalisation to case without isometries

$$X^M = \begin{pmatrix} \tilde{x}_m \\ x^m \end{pmatrix} \qquad \xi^M = \begin{pmatrix} \tilde{\epsilon}_m \\ \epsilon^m \end{pmatrix}$$

Linearised Gauge Transformations

$$\delta h_{ij} = \quad \partial_i \epsilon_j + \partial_j \epsilon_i \; + \; \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \,,$$

$$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) - (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) \,,$$

$$\delta d \; = - \, \partial \cdot \epsilon + \, \tilde{\partial} \cdot \tilde{\epsilon} \,.$$

Invariance needs constraint

Diffeos and B-field transformations mixed.

If fields indep of $\tilde{x}_m$, conventional theory $g_{ij}(x), b_{ij}(x), d(x)$

ϵ^m parameter for diffeomorphisms

$\tilde{\epsilon}_m$ parameter for B-field gauge transformations

Generalised Metric Formulation

Hohm, H &Z

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix}.$$

2 Metrics on double space $\mathcal{H}_{MN}, \ \eta_{MN}$

$$\mathcal{H}^{MN} \equiv \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}$$

Constrained metric $\mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M{}_N$

Generalised Metric Formulation

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Constrained metric $\quad \mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^{M}{}_{N}$

Covariant O(D,D) Transformation

$$h^{P}{}_{M}h^{Q}{}_{N}\mathcal{H}'_{PQ}(X') = \mathcal{H}_{MN}(X)$$

$$X' = hX \qquad h \in O(D,D)$$

O(D,D) covariant action

$$S \;=\; \int dx d\tilde{x}\, e^{-2d}\;\, L$$

$$L \;=\; \frac{1}{8}\,\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK}$$
$$-\, 2\,\partial_M d\,\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$

- Lagrangian L CUBIC in fields!
- Indices raised and lowered with η_{MN}
- O(D,D) covariant (in $\mathbb{R}^{2D}$)

2-derivative action

$$S \;=\; S^{(0)}(\partial,\partial) + S^{(1)}(\partial,\tilde{\partial}) + S^{(2)}(\tilde{\partial},\tilde{\partial})$$

Write $S^{(0)}$ in terms of usual fields

Gives usual action (+ surface term)

$$\int dx\sqrt{-g}e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{12}H^2\right]$$

$$S^{(0)} = S(\mathcal{E}, d, \partial)$$

2-derivative action

$$S \;=\; S^{(0)}(\partial,\partial)+S^{(1)}(\partial,\tilde\partial)+S^{(2)}(\tilde\partial,\tilde\partial)$$

Write $S^{(0)}$ in terms of usual fields

Gives usual action (+ surface term)

$$\int dx\sqrt{-g}e^{-2\phi}\left[R+4(\partial\phi)^2-\frac{1}{12}H^2\right]$$

$$S^{(0)}=S(\mathcal{E},d,\partial)$$

$$S^{(2)}=S(\mathcal{E}^{-1},d,\tilde\partial)$$ T-dual!

$S^{(1)}$ strange mixed terms

O(D,D) covariant action

$$S = \int dx d\tilde{x}\, e^{-2d}\ L$$

$$L = \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK}$$
$$- 2\,\partial_M d\,\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$

Gauge Transformation

$$\delta_\xi\mathcal{H}^{MN} = \xi^P\partial_P\mathcal{H}^{MN}$$
$$+ (\partial^M\xi_P - \partial_P\xi^M)\,\mathcal{H}^{PN} + (\partial^N\xi_P - \partial_P\xi^N)\,\mathcal{H}^{MP}$$

Write as "Generalised Lie Derivative"

$$\delta_\xi\mathcal{H}^{MN} = \widehat{\mathcal{L}}_\xi\mathcal{H}^{MN}$$

Generalised Lie Derivative

$$A^{M_1\ldots}{}_{N_1\ldots}$$

$$\widehat{\mathcal{L}}_\xi A_M{}^N \equiv \xi^P \partial_P A_M{}^N$$
$$+(\partial_M \xi^P - \partial^P \xi_M) A_P{}^N + (\partial^N \xi_P - \partial_P \xi^N) A_M{}^P$$

Usual Lie derivative, plus terms involving η_{MN}

$$\widehat{\mathcal{L}}_\xi A_M{}^N = \mathcal{L}_\xi A_M{}^N$$
$$- \eta^{PQ} \eta_{MR}\, \partial_Q \xi^R\, A_P{}^N$$
$$+ \eta_{PQ} \eta^{NR}\, \partial_R \xi^Q\, A_M{}^P$$

Strong Constraint: Gauge symm ~ diffeos and b-field trans

O(D,D) $X' = hX$

Symmetry for flat doubled space $M = \mathbb{R}^{2D}$

B-shifts and $GL(D, \mathbb{R})$ arise from local symmetries.

Isometries: if fields indep of some coords, more of O(D,D) can arise from local symmetries HHZ

Torus spacetime $N = \mathbb{R}^{n-1,1} \times T^d$ $M = \mathbb{R}^{2n-2,2} \times T^{2d}$

O(D,D) broken to subgroup containing B-shifts and

$$O(n, n) \times O(d, d; \mathbb{Z})$$

General Spacetime: No natural action of O(D,D)

Strong Constraint: Gauge symm ~ diffeos and b-field trans

<u>O(D,D)</u> $X' = hX$

Symmetry for doubled space $M = \mathbb{R}^{2D}$

<u>Torus spacetime</u> $N = \mathbb{R}^{n-1,1} \times T^d$ $M = \mathbb{R}^{2n-2,2} \times T^{2d}$

O(D,D) broken to subgroup containing B-shifts and

$$O(n,n) \times O(d,d;\mathbb{Z})$$

B-shifts and $GL(n,\mathbb{R}) \times GL(d,\mathbb{Z})$
arise from <u>local</u> symmetries.

General Spacetime: No natural action of O(D,D)

Generalized scalar curvature

$$\mathcal{R} \equiv \; 4\,\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\,\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\,\partial_N d + \frac{1}{8}\,\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL}$$

$$S = \int dx\, d\tilde{x}\, e^{-2d}\,\mathcal{R}$$

Gauge Symmetry

$$\delta_\xi\mathcal{R} \; = \widehat{\mathcal{L}}_\xi\mathcal{R} = \; \xi^M\partial_M\mathcal{R}$$

$$\delta_\xi\, e^{-2d} = \partial_M(\xi^M e^{-2d})$$

Generalized scalar curvature

$$\mathcal{R} \equiv \; 4\,\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN}$$
$$-\,4\,\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\,\partial_N d$$
$$+\,\frac{1}{8}\,\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL}$$

$$S = \int dx\, d\tilde{x}\, e^{-2d}\,\mathcal{R}$$

Gauge Symmetry

$$\delta_\xi \mathcal{R} \;= \widehat{\mathcal{L}}_\xi \mathcal{R} = \; \xi^M \partial_M \mathcal{R}$$
$$\delta_\xi\, e^{-2d} = \partial_M(\xi^M e^{-2d})$$

Field equations give gen. Ricci tensor

Gauge Algebra

Parameters Σ^M

Gauge Algebra $$[\delta_{\Sigma_1}, \delta_{\Sigma_2}] = \delta_{[\Sigma_1, \Sigma_2]_C}$$

$$\Big[\, \widehat{\mathcal{L}}_{\xi_1}\, , \widehat{\mathcal{L}}_{\xi_2}\, \Big] = -\widehat{\mathcal{L}}_{[\xi_1, \xi_2]_{\rm C}}$$

C-Bracket:

$$[\Sigma_1, \Sigma_2]_C \equiv [\Sigma_1, \Sigma_2] - \frac{1}{2}\eta^{MN}\eta_{PQ}\,\Sigma^P_{[1}\,\partial_N\,\Sigma^Q_{2]}$$

Lie bracket + metric term

Parameters $\Sigma^M(X)$ restricted to N
Decompose into vector + 1-form on N
C-bracket reduces to Courant bracket on N

Same covariant form of gauge algebra found in similar context by Siegel

Jacobi Identities not satisfied!

$$J(\Sigma_1, \Sigma_2, \Sigma_3) \equiv [\,[\Sigma_1, \Sigma_2]\,, \Sigma_3\,] + \text{cyclic} \neq 0$$

for both C-bracket and Courant-bracket

How can bracket be realised as a symmetry algebra?

$$[\,[\delta_{\Sigma_1}, \delta_{\Sigma_2}]\,, \delta_{\Sigma_3}\,] + \text{cyclic} = \delta_{J(\Sigma_1, \Sigma_2, \Sigma_3)}$$

Symmetry is Reducible

Parameters of the form $\Sigma^M = \eta^{MN} \partial_N \chi$
do not act

Gauge algebra determined up to such transformations

cf 2-form gauge field $\delta B = d\alpha$
Parameters of the form $\alpha = d\beta$
do not act

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Parameters of the form $\alpha = d\beta$
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Resolution:

$$J(\Sigma_1, \Sigma_2, \Sigma_3)^M = \eta^{MN}\partial_N \chi$$

$\delta_{J(\Sigma_1,\Sigma_2,\Sigma_3)}$ does not act on fields

D-Bracket

$$[\,A,B\,]_{\mathrm{D}} \equiv \widehat{\mathcal{L}}_A B$$

$$[\,A,B\,]_{\mathrm{D}}^{M} = [\,A,B\,]_{\mathrm{C}}^{M} + \frac{1}{2}\partial^M (B^N A_N)$$

Not skew, but satisfies Jacobi-like identity

$$[A,[B,C]_{\mathrm{D}}]_{\mathrm{D}} = [[A,B]_{\mathrm{D}}],C]_{\mathrm{D}} + [B,[A,C]_{\mathrm{D}}]_{\mathrm{D}}$$

On restricting to null subspace N

C-bracket → Courant bracket

D-bracket → Dorfman bracket

Gen Lie Derivative → GLD of Grana, Minasian, Petrini and Waldram

DFT geometry

arXiv:1406.7794

- Simple explicit form of finite gauge transformations. Associative and commutative.
- Doubled space is a manifold, not flat, despite constant ‘metric’ η in DFT.
- Gives geometric understanding of ‘generalised tensors’ & relation to generalised geometry
- Transition functions give global picture
- T-folds: non-geometric backgrounds included

Further Developments

- Action for F-theory Berman, Blair, Malek, Rudolph
  Theory in 12 dimensions with SL(2) symmetry; section condition gives 12-D theory

- Superstring Hohm & Zwiebach,…

- DFT (super)geometry, curved metric η Cederwall

- Generalised paralellalizability, susy flux backgrounds, exceptional CY,… Waldram et al

- Susy AdS, exceptional Sasaki-Einstein,…
  Ashmore, Petrini, Waldram

- Extension to WZW models Blumenhagen, Hassler & Lust
  Scherk-Schwarz,... Grana et al

Conclusions

- Duality symmetries lead to extension of geometry to allow "non-geometric" solutions

- String theory on torus: T-duality symmetry. Winding modes: doubled geometry, infinite number of doubled fields

- DFT: with strong constraint, get conventional sugra in duality symmetric formulation

- More generally, this applies locally in patches. Use DFT gauge and O(D,D) symmetries in transition functions. Get T-folds etc.

- Full theory with weak constraint: non-local, stringy
- How much of this is special to tori?
- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality? No doubling?
- Duality symmetry gives deep insight into stringy geometry. But seems to be very different on different backgrounds, e.g. T^4, K3
- Much remains to be understood about string/ M theory

Type IIA Supergravity Compactified on T^4

Duality symmetry SO(5,5)
BPS charges in 16-dim rep

Type IIA Supergravity on $M_4 x M_6$
Can be written as "Extended Field Theory"
in space with 6 +16 coordinates
with SO(5,5) Symmetry

Berman, Godzagar, Perry;
Hohm, Sambtleben

Is SO(5,5) a "real" symmetry for generic solutions $M_4 x M_6$?

Similar story for $M_d x M_{10-d}$; $M_d x M_{11-d}$

Extension to String Theory?

Type IIA Superstring Compactified on T^4

U-Duality symmetry SO(5,5;Z) 6+16 dims?

But for other backgrounds, symmetry different

Type IIA Superstring Compactified on K3

U-Duality symmetry SO(4,20;Z) 6+24 dims?

Duality symmetry seems to be background dependent; makes background independent formulation problematic