Signatures of extra dimensions in gravitational waves

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Based on arXiv:1704.07392 with G. Lucena Gómez

Workshop on Testing Fundamental Physics Principles,
27/09/2017, Corfu, Greece

Introduction

Direct detection of gravitational waves by LIGO and Virgo Scientific Collab. [arXiv:1602.03837], [arXiv:1606.04855], [arXiv:1706.01812]

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Variety of models: number, size, shape of extra dimensions...
Previous literature: typically very model dependent
⇒ here, remain as **generic** as possible.

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$\Rightarrow$ **Two effects**:

1. Breathing mode: new polarization mode in massless wave.
2. Additional (massive) waves of high frequencies.

$\hookrightarrow$ Observable in a near future?

General equations for gravitational waves

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In D dimensions

General Relativity: $S = \frac{1}{2\kappa_D} \int \mathrm{d}^D x \sqrt{|g_D|}\ (\mathcal{R}_D - 2\Lambda_D)$

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Background + fluctuation: $g_{D\,MN} = g_{MN} + h_{MN}$
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1^{st} order: $-\frac{1}{2}\, \square^{(0)}_D\ h_{MN} + \mathcal{R}^{(0)\,S}{}_{MNP}\, g^{PQ} h_{QS} + \nabla^{(0)}_{(M} \mathcal{G}_{N)} = 0$
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de Donder (Lorenz) gauge fixing: $\mathcal{G}_N = 0$

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Split into $4+N$ dimensions

Background: $\mathrm{d}s^2 = e^{2A(y)}\tilde{g}_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(y)\mathrm{d}y^m \mathrm{d}y^n$

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D-dimensional wave equation: $_{MN}$ components:

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D-dimensional de Donder gauge:

$$e^{-2A}\tilde{g}^{\pi\rho}\tilde{\nabla}_\pi h_{\rho\nu} - \frac{e^{-2A}}{2}\tilde{\nabla}_\nu\tilde{h}_4 - \frac{1}{2}\nabla_\nu h_N + \nabla^q h_{q\nu} + 2h_{p\nu}g^{pq}e^{-2A}\partial_q e^{2A} = 0$$
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Massless modes

Focus on zero-mode: $m_{\mathbf{0}} = 0$. Properties: $\omega_{\mathbf{0}}$ unique, constant.

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discrete basis, label $\mathbf{k}$: $\Delta_{\mathcal{M}}\, \omega_{\mathbf{k}} = -m_{\mathbf{k}}^2\, \omega_{\mathbf{k}}$ (e.g. T^N: $\omega_{\mathbf{k}}(y) = e^{\mathbf{i k \cdot y}}$)

Field (Kaluza–Klein mode) decomposition:
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Massless modes

Focus on zero-mode: $m_{\mathbf{0}} = 0$. Properties: $\omega_{\mathbf{0}}$ unique, constant.
Equations with 4d wave $h^{\mathbf{0}}_{\mu\nu}$:

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Both decouple from other fields/modes $\rightarrow$ analyse this system.

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$$\equiv h^\times_{ij} + h^+_{ij} + h^\bigcirc_{ij}$$

$\hookrightarrow$ G. R. $h^\times_{ij}, h^+_{ij}$ polarization modes and **breathing mode $h^\bigcirc_{ij}$** .

Massive modes

Focus on $h_{\mu\nu}^{\mathbf{k}\neq\mathbf{0}}$: equations: $\Box_4 h_{\mu\nu}^{\mathbf{k}} - m_{\mathbf{k}}^2\, h_{\mu\nu}^{\mathbf{k}} = 0$ + gauge cond.

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All six polarization modes, only 5 independent ones.

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Two effects and observability

1. New polarization mode in massless wave: **breathing mode**.
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Each polarization mode → **specific space deformation**
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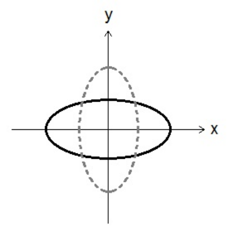

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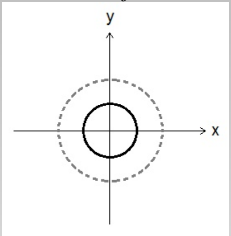

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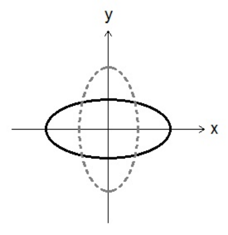

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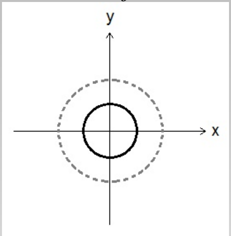

Breathing mode: need several detectors, different orientations

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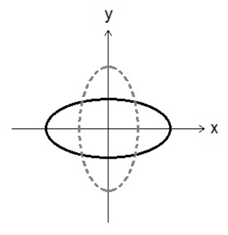

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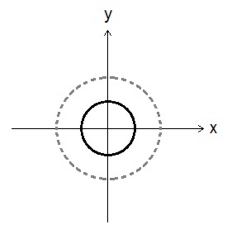

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Amplitude? Related to that $h^{\mathbf{0}}_N$... Emission?

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Table-top experiment bound: $r_N \lesssim 10^{-4}\,\mathrm{m}$ (about $10^{-3}\,\mathrm{eV}$)
$\Rightarrow \nu \sim 10^{12}\,\mathrm{Hz} \gg$ upper bound of LIGO $\sim 10^3$-$10^4\,\mathrm{Hz}$
$\hookrightarrow$ unobservable.

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All six polarization modes → various space deformations.

Angular frequency: $\omega_{\mathbf{k}}^2 = m_{\mathbf{k}}^2 + \vec{p}_{\mathbf{k}}^{\,2}$.
$\vec{p}_{\mathbf{k}}$: Minkowski spatial components, governed by 4d physics
$\Rightarrow ||\vec{p}_{\mathbf{k}}|| \sim 1/\lambda_4$
But $m_{\mathbf{k}} \sim 1/r_N$, (Kaluza–Klein) internal length r_N.

$\hookrightarrow r_N \ll \lambda_4$ so that $m_{\mathbf{k}} \gg ||\vec{p}_{\mathbf{k}}|| \quad \Rightarrow \quad$ **$\omega_{\mathbf{k}} \sim m_{\mathbf{k}}$ very high**.

Table-top experiment bound: $r_N \lesssim 10^{-4}\,\mathrm{m}$ (about $10^{-3}\,\mathrm{eV}$)
$\Rightarrow \nu \sim 10^{12}\,\mathrm{Hz} \gg$ upper bound of LIGO $\sim 10^3$-$10^4\,\mathrm{Hz}$
$\hookrightarrow$ unobservable.

Worse in future (planned) detectors.
Energy → amplitude is low...

Conclusion

Done:

- Propagation equations on most general $4 + N$-dimensional background
- On Minkowski$_4$ $\times$ compact Ricci-flat $\mathcal{M}_N$: two effects of extra dimensions on 4d gravitational waves
- The new polarization mode in the massless 4d wave, the breathing mode, could be observed with more detectors.

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- Compare to scalar-tensor models and their emission constraints
- Study emission
- Start from D-dimensional supergravity, get more involved $\mathcal{M}_N$ and a mass for h^0_N?

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Thank you for your attention!